

Transient Control of a Non-Linear Turbo-Generator using Multiple Shooting, Differential Dynamic Programming and Quasi-Linearization Technique

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Abstract

The main objective of this paper is primarily on practical solution to transient control problem of a non-linear turbo-generator model through the application of classical optimal control principle. The well-known two point boundary value problem that usually results from such optimal control problem formulation is iteratively solved using the approach of multiple shooting algorithm. In order to focus on the advantages of proposed technique, the same two point boundary value problem has been consistently solved using the existing differential dynamic programming and quasi-linearization technique with comparison offered from the stand points of execution time and core memory requirements. The derived open-loop optimal control laws for fast valving and forced excitation resulted in significant transient performance improvement of the nonlinear turbo-generator model feeding an infinite bus system after being subjected to a three phase short circuit fault at its terminals. In addition, a suboptimal feedback control simulations yielded somewhat less satisfactory results but found to be easily implementable from practical view point. Thus a microcomputer based suboptimal feedback control concept is evolved; and specific hardware configuration for such a controller is proposed.

Keywords: Turbo-generator, transient control, multiple shooting, differential dynamic programming, quasi-linearization

1.0 Introduction

The key objective of any power system is to continuously strive to attain cheap, reliable and uninterrupted power supply to various categories of consumers. Consequently, in addition to ensuring prescribed voltage levels and frequency within specified limits at steady state, the power engineer is equally concerned with the problem of transient stability inherent in present day power systems. Broadly speaking transient stability relates to the system's operational capability to remain secured when subjected to severe transient disturbance. There are several transient stabilizing means whose possible electrical augmentation effects on power systems under stability crisis are fairly understood by power engineers. The transient stabilizing means of prime interest for power system transient stability augmentation include dynamic braking resistors, fast valving, switched series capacitors, independent pole tripping breaker applications, to mention a few.

However, this fast acting and multivariable system usually connected to the power system, is often characterized by some levels of disturbances resulting from three phase short circuit and other severe disturbances which has the consequence of loss of synchronism unless the fault is quickly cleared. When major power system disturbances occur, protection and control actions are required to stop power system instability and restore the system to a normal state by minimizing the impact of disturbance. Therefore efficient and intelligent techniques are required. This situation has in recent being controlled using fast operating circuit breakers operational in two cycles which after clearing the fault, the post fault transient oscillation remains, which need to be stamped out as soon as possible to avoid building up of large forces that may jeopardize the integrity of the entire system. Sequel to this negative development in power system, the concept of investigation of the transient performance of a

nonlinear turbo-generator was born, and the causes of these disturbances was also investigated and the solutions to them were provided in this work through the use of computer based program known as Multiple Shooting Technique which is believed to be computationally efficient in tackling the problem. Other techniques used in this work to achieve the same objective includes; Differential Dynamic Programming and Quasi-linearization Technique.

2.0 System Model Development

A Single-Machine Infinite Bus (SMIB) power system is shown in fig. 1. This consists of a

turbo-generator connected to an infinite bus through a short transmission line. The turbo-generator is equipped with the traditional Automatic Voltage Regulator (AVR) and turbine governor, whose mathematical modeling is presented bellow for the purpose of transient stability augmentation studies to be undertaken in this research work. The synchronous machine model is based on familiar Park's equations, cast for its electrical representation comprising of a circuit in each of the rotor axis i.e. 'd' and 'q' to account for transient saliency and effects of rotor induced eddy currents.

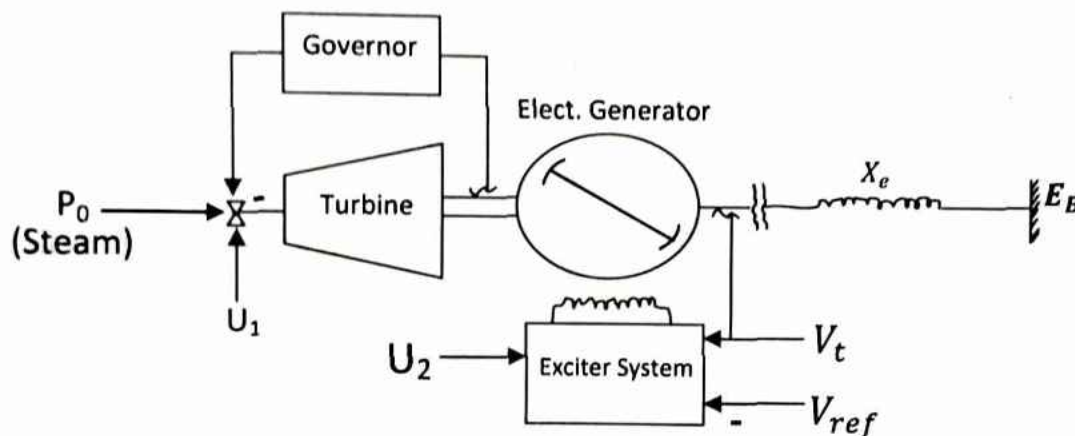


Figure 1: Governing system with infinite bus

where X_E is the transmission line reactance, E_B is the infinite bus voltage, P_e is reference of Power in p.u., P_c is active power of the generator, V_{ref} is the reference Voltage, U_1 is Generator Valve

Steam is produced in the boiler by unleashing the heat of the combustion system, this is then expanded into the prime mover otherwise known as steam turbine and is then condensed into the condenser to be fed into the boiler again [1]. The steam turbine drives the alternator which converts the mechanical energy of the turbine into electrical energy. The greater the temperature differences the greater the heat energy is being converted into mechanical energy. This is necessary to keep the steam in the condenser at the lowest temperature but it is obvious that the greater the temperature difference, the greater the amount of heat loss. Considering to dynamic operations of a turbo-generator system in thermal power generating

stations, the stability of the output power should be the ultimate focus of a power system Engineer.

2.1 System steady state operation

When a synchronous machine feeding a network is operating at steady state, its performance could be analyzed by means of phasor diagram. The steady state analysis is usually started with a load flow calculation to determine the voltage at key nodes. Fig. 2 is the phasor diagram for the generator steady state operation and the infinite bus voltage is the reference vector. The component currents I_d and I_q provides component voltage drops jI_dX_d for a lagging power factor [2].

The generator terminal voltage is given by equation 1b in terms of its d - and q - components.

$$|V_t| = \sqrt{V_q^2 + V_d^2} \quad (1)$$

Expression of V_q and V_d in terms of the infinite bus and the state variables is deferred to appendix 1, $V_{ref} = |V_t|$ at the steady state.

The initial steady state vector defined in equation 1 can be calculated by solving equation (2).

$$X_0 = [X_{01}, X_{02}, X_{03}, X_{04}, X_{05}, X_{06}, X_{07}]^T \quad (2)$$

where X_{01} to X_{07} are the steady state vectors.

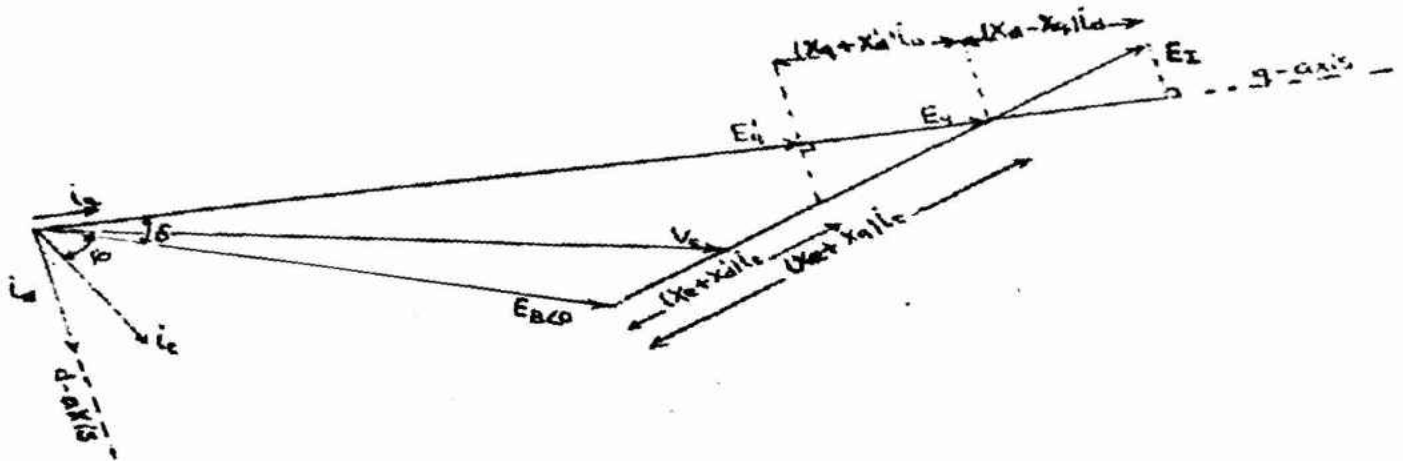


Fig 2. Phasor Diagram of Generator (salient pole)

The resulting generator mathematical model bears similarity with model II as described by [3]. Equations 3 and 4 which describe the variations of the armature flux linkages in d and q axis respectively are derived from park's equations.

$$\frac{\partial E_{q'}}{\partial t} = \frac{1}{T_{d0}} \left[E_{td} - \frac{X_e + X_d E_{q'}}{X_e + X_{d'}} + \frac{X_d - X_{d'} E_B \cos \delta}{X_e - X_{d'}} \right] \quad (3)$$

$$\frac{\partial E_{d'}}{\partial t} = \frac{1}{T_{q0}} \left[\frac{(X_e + X_q) E_{d'}}{X_e + X_q} - \frac{X_e - X_q \sin \delta}{X_e + X_q} \right] \quad (4)$$

Equation of motion for the rotating solid rotor mass is expressed thus:

$$\frac{2\delta}{t^2} = \frac{1}{M} (P_m - P_e - D_w) \quad (5)$$

The use of per unit power (P_e, P_m) instead of the torque (T_e, T_m) is informed of the fact that the normalized speed deviation is negligible in the normal region of transient stability; hence, $T_e = T_m$ and $P_m = T_m$.

2.2 Governing System and Turbine Performance Equations

The governing system and turbine is usually simpler in that only gains and time constants are used to represent the time response of the various elements in the governor sub-system [4]. The two first order differential equations (6) and (7) respectively govern the time response of the steam and mechanical power.

$$\frac{\partial P_m}{\partial t} = \frac{1}{T_s} (P_v - P_m) \quad (6)$$

$$\text{where, } P_{min} \leq P_m \leq P_{max}$$

$$\frac{\partial P_v}{\partial t} = \frac{1}{T_g} \left(P_0 U_1 \frac{w}{R} \right) \quad (7)$$

where $U_{1min} \leq U_{1max}$, i.e., the input U_1 is bounded.

2.3 Excitation System Model

The generator excitation system which incorporates a voltage regulator and supplementary excitation control input, U_2 , the limits imposed on the exciter output which account for saturation effects in the generator. Equation 8 for the automatic voltage regulator (AVR) is given by:

exciter unit derived from its block diagram is expressed as follows:-

$$\frac{\partial E_{fd}}{\partial t} = \frac{CE}{T} - (V_{ref} - |V_t| + U_1) - \frac{x_7}{T} \quad (8)$$

where $E_{fd \min} \leq E_{fd} \leq E_{fd \max}$

Table 1 shows the parameters of the subsystems of the turbo-generator infinite bus system. The system parameters used are typical of any modern steam power station such as Egbin Power Station, one of the thermal power stations feeding the Nigerian National Grid. Table 2 is the initial steady state loading condition obtained using the method outlined above.

Table 1: System Parameters

S/N	Subsystem Generator	Parameter	Per-unit (P.U) Value*
1		X_d	1.98
		X_d^l	0.262
		X_q	1.87
		X_q^l	0.45
		T_{do}	9.0
		T_{qo}	1.0
2	Turbine and Governing system	T_g	0.3
		T_s	0.5
		D	0.005
		R	20.0
		M	0.018
3	Exciter	C_e	3.0
		T_e	0.2sec.
4	Transmission line and Infinite bus	X_E	0.4

Table 2: The Base Case Load-flow solution for the study system

S/N	System parameters	Initial values
1	E_B	1.0
2	V_t	1.08
3	δ_o	61.8°
4	P	0.80
5	Q	0.0
6	P_m	0.80

The computer Program for determination of critical time was written and executed based on the system initial steady state summarized in Table 2. The critical clearing time under the above condition with exciter and governing system action was computed.

2.4 Multiple Shooting Technique

One major problem in the use of the initial value method for solving two point boundary value is in the accumulation of error when the system differential equation is numerically integrated in the interval t_0, t_f especially if the interval is relatively long. The state and co-state variables may in this case grow so large as to prevent convergence of the algorithm or even cause an over- flow of a computer's arithmetic facilities. To circumvent this problem, Keller [5] introduced a variant of the initial method which he christened 'multiple shooting'. In the multiple shooting techniques, the transient oscillation period is divided into a number of subintervals and the unknown variables are estimated at the boundary and all the interior points in preparation for integration of system equations as an initial value problem. The initial value problem at each subinterval is integrated and the boundary iterates are then simultaneously adjusted in order to satisfy the boundary as well as continuity conditions. To illustrate the general case of multiple shooting, the two point boundary problem is written in equation 9 with a generalized boundary condition;

$$\bar{y} = f(y)Ay(t_0) + BY(t_f) = \alpha \quad (9)$$

where $y = (x, \lambda)^T$

$$\alpha = [X_0, 0]^T \quad (10)$$

'Y' is the dependent vector and h is the independent variable, time in this case, and it is bounded in (t_0, t_f) . As a result of the subdivision the variable h_i in each subinterval is defined as

$$t_0 \equiv h_0 < h_1 < h_2 < \dots < h_{j-1} < h_j = t_f \quad (11)$$

The interval $h_{i-1} < h < h_i$ is referred to as the i th subinterval and its length is $h_i - h_{i-1}$. Suppose on each subinterval a new independent variable t , and a new dependent variable $y_i(t)$ is introduced as follows

$$t = \frac{h - h_{i-1}}{\Delta}, y(h) = y(h_{i-1} + t_i)\Delta_i \quad (12)$$

Thus on each sub-interval i th new independent variable can only vary between 0 and 1 in the

case of forward integration and between 0 and -1 in the case of backward integration. With the above variable definition and interval subdivision, equation (9) becomes

$$\frac{dy_i}{dt} = f_i(t, y_i(t)) \quad 0 < t < 1 \quad (13)$$

and $i = 1, 2, \dots, J$

and the general boundary condition of equation (9) becomes

$$AY_1(0) + BY_J(0) = \alpha \quad (14)$$

Continuity of the solution of equation at each interior point, h_i is expressed by the conditions in equation (8).

$$Y_i(1) - y_{i+1}^2(1) = 0 \quad (15)$$

$i = 1, 2, \dots, J - 1$

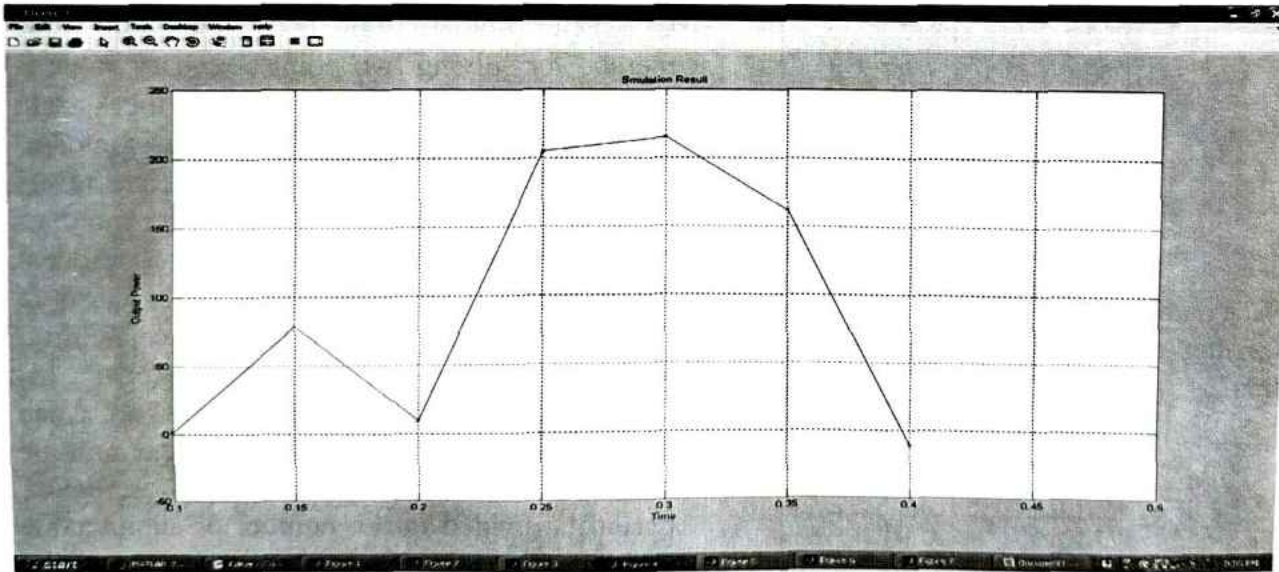


Figure 3: Multiple Shooting Optimal control

Figure 3 shows the result of the Multiple Shooting algorithm. When the program was executed on a microcomputer it converged at fourteenth iteration, at a time of 0.38 second. The result shows the ability of the shooting technique to react positively to a disturbance at such time.

2.5 Differential Dynamic Programming

Equation (16) is the general expression for the cost function, V , of a dynamic system in transition from t_0 to t_f

$$V(X_0, t_0) = \int_{t_0}^{t_f} L(x, u, t) dt \quad (16)$$

L is the quadratic performance index defined above. The optimal cost function $V^0(x, t_f)$ for

the system in equation 17 obeys Bellman partial differential equation as given below

$$-\frac{\partial V^0}{\partial t} = (x, I) U_{\min} [1, (x, u, I) 1(x, t); f(x, u, I)] \partial t \quad (17)$$

Analytical solution of equation (17) is very difficult and the numerical solution requires an impractically high amount of storage. The use of differential dynamic programming to solve (18) involves the forward integration of (18) and backward integration of (19) to (21)

$$\dot{X} = f(x, u, t) \quad X(t_0) = X_0 \quad (18)$$

$$\dot{\alpha} = H - H(\dot{X}, \dot{U}, V_x; t) \quad (19)$$

$$\tilde{V}_x = H_x + B^T H_u \quad (20)$$

$$\tilde{V}_{xx} = H_{xx} + H_{uu}^{-1} f_u^T V_{xx} \quad (21)$$

[6], define the variables in the above equations as follows: $H(x, u, V_x, t) = L(x, u; t) + V_x^T f(x, u, t)$. Much like the Hamiltonian shown above, V_x and V_{xx} are the first and second terms for expansion of the cost function v about some nominal x , the stat trajectory

$$\beta^T = -H_u(H_{ux} + f_u^T V_{xx}) \quad (22)$$

' β ', represents the predicted improvement in the cost at each iteration.

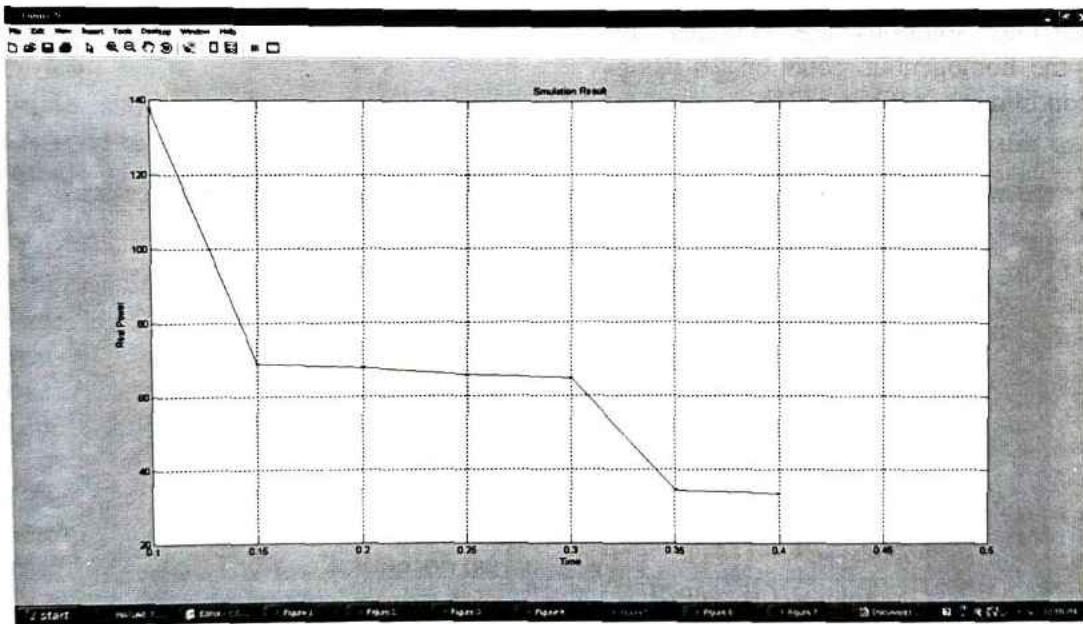


Figure 4: Differential Dynamic Programming (DDP) Optimal control

Figure 4 above shows the result of Differential Dynamic Programming (DDP) Technique. When the algorithm was executed on a Microcomputer, the result shows that the program converged at eight iteration time of 0.4 seconds. This shows that differential dynamic programming (DDP) has the ability to react to any disturbance within 0.4 second.

2.6 Quasi-linearization Technique

In the quasi-linearization Technique the non-linear ordinary differential equations of the two point boundary problem are solved as a set of linear equations which are easier to handle numerically. Linearization of the equations is

achieved by a two term Newton-Raphson expansion of the nonlinear functions about the nominal trajectory [7]. Let (22), (23) and (24) be the nonlinear equations and the two point boundary conditions respectively.

$$\frac{dx}{dt} = f(x, t) \quad (22)$$

$$X_i(o) = x_{oj} \quad j = 1, 2, \dots, m \quad (23)$$

$$X_j(tf) = X_{fj} \quad j = m + 1, m + 2, \dots, 2m \quad (24)$$

Equation 25 is the linearized version of (22)

$$\frac{dx_{n+1}}{dt} = F(X_{nt}) + J(X_n)(X_{n+1} - X_n) \quad (25)$$

The 'n' denotes the n^{th} iteration which results in the $n+1$ th trajectory. $J(x_n)$ is the jacobian matrix defined below

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_{2m}} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_{2m}} \end{bmatrix} \quad (26)$$

$$X_j^p(0) = 0; \quad j = 1, 2, \dots, m \quad (27)$$

$$X_j^p(0) = X_0; \quad j = m + 1, \dots, 2m \quad (28)$$

The homogenous solutions are obtained by integrating the homogenous equation 28 using the initial conditions of (30) and (31)

$$\frac{\partial x_{n+1}}{\partial t} - j(X_n)X_{n+1} = 0 \quad (29)$$

$$X_{ij}^h(0) = 1 \quad j \neq i \quad (30)$$

$$X_{ij}^h(0) = 0 \quad j = i \quad (31)$$

The complete solution is formed by superposing the particular solution on the homogenous solution using (32)

$$X_{n+1} = X^{p_{n+1}} + \sum_{j=1}^m c_j X^{h_{n+1}} \quad (32)$$

The 'm' is integration constants used in (32), C_j , are calculated from the algebraic equation (33)

$$X_i = x_{ip}(t_f) + \sum_{j=1}^M c_j X^{hi} J(t_f) I = m + 1 \dots 2m \quad (33)$$

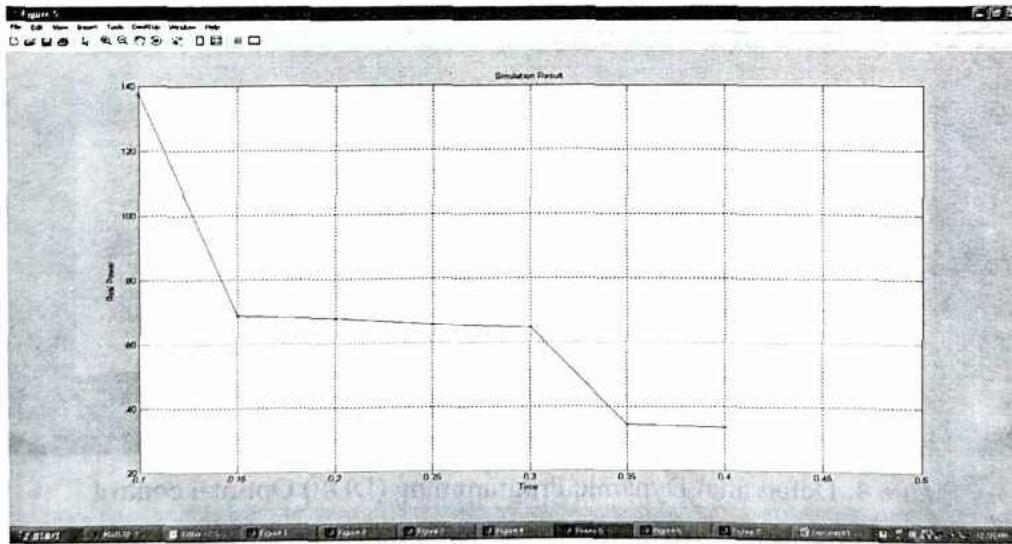


Figure 5: Quasi-linearization Optimal control

Figure 5 is the result of quasi-linearization technique. When the algorithm was executed on a microcomputer, the result shows that the program converged at seventh iteration with the execution time of 0.4 second. The result shows that the program or technique has the ability to clear a disturbance at the time of 0.4 second.

3.0 Result and discussion

From the results of the three programs, it is observed that the Differential Dynamic

Programming (DDP) and Quasi-linearization technique has shorter convergence but slightly longer execution time in comparison with the Multiple Shooting Technique. Similarly, though the seventh order differential dynamic programming (DDP) converged in eighth iteration against multiple shooting's fourteen, the former took a 0.4 second execution time against the latter's 0.38 second. Therefore, Multiple Shooting technique can be confidently said to be the best optimal transient control technique.

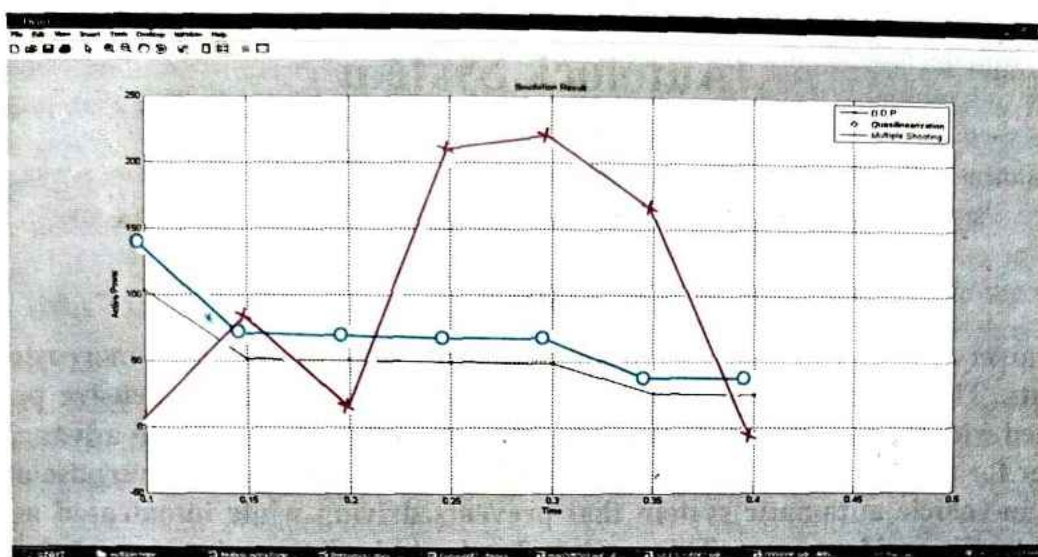


Figure 6: Simulation results of Multiple Shooting Technique, Differential Dynamic Programming and Quasi-linearization Technique.

4.0 Conclusion

The multiple shooting algorithm has been successfully used to solve a two point boundary problem involving a nonlinear turbo-generator model whose transient oscillations are controlled via the fast valving and excitation inputs. Again, the application of the optimal controls on the machine model improved its transient performance over the uncontrolled transient oscillations. The results of the variation studies on the multiple shooting program revealed that: Even for short circuit faults of critical time duration, the turbo-generator may not need optimal control strategy if the generator pre-fault load is small, <0.5 p.u. The application of the multiple shooting, differential dynamic programming and quasi-linearization algorithms showed that the multiple shooting algorithm is the best from the stand points of execution time and computer memory as it demanded the least memory requirement and the shortest execution time.

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