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Variance Ratio Tests of Random Walk Hypothesis of Stock Return in Nigeria: Further Evidence

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Abstract

This study investigated whether stock returns in Nigeria equity market follows a random walk as stated by the efficient market hypothesis. Therefore, this study examined the weak-form of market efficiency in Nigeria stock market by testing the random walk hypothesis through various approaches, specifically unit root and variance ratio tests on the monthly price index of the Nigeria Stock Exchange over the period from January, 2013 until December, 2022. The empirical results rejected the random walk hypothesis at the weak-form level, indicating that the stock prices do not fully reflect all historical information. This has important implications for the fortune of equity investors: Increase market activities through reduction in transaction cost and increase in membership of the NSE, and minimize institutional restrictions on trading of securities in the bourse. This made all other markets to flow as a deregulated market. It is therefore, recommended that there is need for policy makers to enlighten potential investors of the opportunities that are available in the stock market. Such enlightenment should seek to stimulate their interest in capital market activities and thus increase the breadth and depth of the capital market.

Keywords: Nigeria Stock Exchange, Weak-Form, Efficient Market Hypothesis, Unit Root Tests, Variance Ratio Test.

INTRODUCTION

The stock market is the collection of exchanges and other venues where the buying, selling, and issuance of shares of publicly held companies take place (Afolabi, 1998). The shares, also known as equities, are fractional ownership in a company (Olowe, 1996; Levinson, 2006; Alfred, 2007; Bhalla, 2012). Academic research suggest that share prices follow a random walk. That is, successive price changes are independent of each other. The search for an explanation of this apparent randomness led to the formation of Efficient Market Hypothesis (EMH) (Adams et al. 2003). A Stock market is said to be efficient if security (share) prices at any time "fully reflect" all available, relevant information (Fama, 1970 1991); reflect information to the point where the marginal benefits would not exceed marginal costs (Fama, 1991, Jensen, 1978); and it will be impossible for an investor to beat the market (Fama, 1970, 1991). A precondition for this strong version is that the benefits of acting on the information equals the cost of collecting it, always 0 (Grossman and Stiglitz, 1980; Adams et al 2003).

Eugene Fama developed the idea of efficient market hypothesis as an academic concept of study; and classified it into three versions according to the levels of information, namely: weak-form (How well do past returns predict future returns?); semi-strong form (How quickly do share prices reflect public information announcements?); and strong-form efficiencies (Do any investors have private information that is not fully reflected in the market prices?) (Fama, 1970; 1991; Roberts, 1959). As we move from weak-form to strong-form we are referring to progressively more information. Efficient market is a market which "adjust rapidly to new information" (Fama *et al* 1969). In an efficient capital market, there should not exist a significant correlation between the share prices over time (Brealey, 1969). Fama (1970) used the term "random walk" as a synonym for weak-form efficiency.

Random walk properties of stock prices have long been a prominent topic in the study of stock returns (see Summers. 1996, Fama and French, 1988, Lo and macKinlay, 1988; Liu and He, 1991; Malkiel, 2005). Several studies attempt to address the RWH and market efficiency in emerging markets, with mixed results

Although there are an abundance of empirical studies concerning testing the Random Walk Hypothesis (RWH) (Liu and He (1991), Huang (1995), Poshakwale (1996), Islam and Khaled (2005), etc.), the interest in the market efficiency still remains in academicians and practitioners. Academicians would like to better know the return patterns of financial assets. Practitioners attempt to identify the market inefficiency to develop global trading strategies. Today, the availability of new market data, the longer study period, and more methodologies satisfy academicians' and practitioners' interest.

Among methodologies available to test RWH, variance ratio tests are considered powerful RWH test methodologies. Lo and MacKinlay (1988) initiate the conventional variance ratio test. Later, Chow and Denning (1993) modify Lo-MacKinlay's test to form a simple multiple variance ratio test and Wright (2000) proposes a non-parametric ranks and signs-based variance ratio tests to address the potential limitation of Lo-MacKinlay's conventional variance ratio test.

The random walk hypothesis is important in economics and, particularly in empirical finance and applied macro-econometrics, one is often interested in testing the absence of temporal dependence. A popular approach among practitioners is the variance-ratio analysis. This type of analysis introduced by Lo and MacKinlay (1988) and Poterba and Summers (1988) is often used to test the hypothesis that a given time series or its first difference is a collection of independent and identically distributed observations (iid.) or that it follows a martingale difference sequence (mds). It exploits the fact that aggregation of data sampled at various frequencies verifies an interesting property under the iid. Hypothesis. This study therefore sought to test the random behavior of stock returns in Nigeria stock market applying the Lo and Mackinlay variance ratio test.

Overview of Nigerian Capital Market

The Nigerian Capital Market is a channel of mobilizing long-term funds by providing mechanism for private and public savings through financial instruments (equities, debentures, bonds and stocks) with major components consisting of the Security and Exchange Commission (SEC) and the Nigerian Stock Exchange (NSE). Founded in 1960, the NSE is the second largest market in sub-Saharan Africa with fully automated exchange that provides the listing and trading services as well as electronic Clearing, Settlement and Delivery (CSD) services through Central Securities Clearing System (CSCS). The exchange keeps on evolving as a competitive market and meeting the needs of investors. It operates fair, orderly and transparent markets with over 200 listed equities and 258 listed securities, and had attracted the best of African enterprises as well as the local and global investors (NSE, 2013). The market has become an integral part of the global economy such that any shock in the market has contagious consequences. Moreover, the Nigeria's capital market has enjoyed a decade of unprecedented growth. The market capitalization increased by over 90.0% from 2003 to 2008. However, from a peak in March 2008, the market capitalization went declined spirally by about 46% in 2009 (SEC Report, 2009).

The Nigerian capital market is an integral part of the Nigerian financial system. Other sectors of the Nigerian financial system include: the money market, the insurance market and the pensions. Each of these markets has a statutory regulatory institution namely: CBN, SEC, NAICOM and PENCOM for the money, capital, insurance and pension markets respectively. These regulatory institutions are empowered by statutes (laws) to supervise the various markets and facilitate the exchange of funds between the surplus and deficit units.

The convergence of global economy makes all countries and all markets sensible to the happenings in other countries (the contagious effect). The 2008 global financial meltdown originated from the United States of America (USA) had varying degree of impacts on different capital markets in various countries. This situation is compounded with the continuous volatility in the global oil price which in theory adversely and significantly affecting capital markets (Njiforti, 2015; Asaolu and Ilo, 2016). Nigeria recently experienced economic recession as a consequence of the 2014-2016 global oil price downturn. In view of these, the various SEC reports came with several recommendations to reposition the Market as a world class institution. The main recommendations are; the development of an enforcement framework to prevent market manipulation, and the establishment of principles for risk management for capital market operators.

The Stock Exchange is a market for the buying and selling of stocks, shares and securities. It is essentially a secondary market in that only existing securities as opposed to new issues, could be traded on (Afolabi, 1998). It is a secondary market in the sense that the shares are already in existence, so that trade takes place between investors and need not directly involve the corporations themselves (Bailey, 2005). Stock exchanges provide a more organized way to trade shares (Levinson, 2006).

The Nigerian Stock Exchange ("The Exchange" or "NSE") operates fully electronic marketplaces for Equities, Bonds, Exchange Traded Products, with plans to include Derivatives trading shortly. The NSE operates an Automated Trading System (ATS) platform with a central order book which allows Dealing Members to participate on equal terms, competing on the hierarchical basis of Price, Cross and Time priority. The Exchange runs a hybrid market, allowing Dealing Members to submit orders and Market Makers to submit two-sided quotes into the order book (NSE, 2019).

HISTORY OF RANDOM WALK AND EFFICIENT MARKET HYPOTHESIS

The idea of random walk was introduced in sixteenth century by Italian Mathematician, Girolamo Cardano in his book "The book of chance" in which he mentioned that equal condition is the fundamental principle of all gambles. If inequality exists in favor of you, you are unjust and if it is in favor of your opponent, you are fool. Many other scientists, especially

mathematicians have contributed to this concept in later years. For the first application in stock
markets, in 1863, a French stockbroker, Jules Regnault claimed that there is a direct relationship between the price deviation and the square root of time. Later, in 1889, Gibson introduced the concept of efficient market in his book "The Stock Markets of London, Paris and New York" (Sewell, 2008). Mathematical finance emerged with Bacheliier in 1900. In his doctoral thesis, he mentioned:"
The influences that determine fluctuations on the exchange are innumerable; past, present, and even discounted future events are reflected in market prices, but often show no apparent relation to price changes…. The determination of these fluctuations depends on infinite numbers of factors; therefore, it is impossible to aspire to mathematical prediction of prices" (Bachelier L., 1900). In all, the main message of his work was that the expected profit for the speculator is zero.
Karl Pearson introduced random walk concept in 1905. In the year 1905, Albert Einstein unaware of Bachelier's result, extended the equations for Brownian motion. Some years later, Keynes in 1923 mentioned that investors are rewarded based on their risk baring and not for knowing the future better and he concluded that this is a consequence of EMH. In 1925, Frederick McCauley found a similarity between the fluctuation of stock market and throwing a dice and Cowles, in 1933, after analyzing the performance of forecasters, pointed out that prices could not be forecast. Working, in 1934, found the same result and assert that the behavior of stock returns look like numbers from lottery.
In his book. The general theory of employment, interest and money‟, John Maynard Keynes claimed that investors make decision in stock market based on „animal spirit‟. Once again, in 1944, Cowles came to conclusion that forecasters did not beat the stock market and Working, in 1958 showed that forecasters could not predict price changes in an ideal future market. Working on 22 time series, Kendall (1953) has found that stock prices at weekly intervals were random. In 1962, Paul H. Cootner, perhaps for the first time, found

that stock market prices did not follow random walk and Arnold B. Moore found a slight positive serial correlation for the index. Later, Granger and Morgenstern claimed that market prices followed the simple random walk in the short lag but did not obey the simple random walk in a long range (Sewell, 2008). These researches followed by Steiger (1954) paper in which he claimed that stock prices did not follow a random walk. Before 1965, many empirical works validated the random walk (Walter, 2003). Later, interplays between academics and practitioners started around the predictability versus random walk and this clash is still not completely reduced. For example, Williams in his guidebook „the theory of investment value‟ mentioned that for individual was possible to outperform when she had the superior information.

In 1960‟s, the first midterm solution brought by some studies. For example, Fama claimed "now in fact, we can probably never hope to find a time series that is characterized by perfect independence. Thus, strictly speaking, the random walk theory cannot be a completely accurate description of reality. For practical purposes, however, we may be willing to accept the independence assumption of the model as long as the dependence in the series of successive price changes is not above some "minimum acceptable" level. The independence assumption is an adequate description of reality as long as the actual degree of dependence in the series of price changes is not sufficient to allow the past history of the series to be used to predict the future in a way which makes expected profits greater than they would be under a naïve buy-and-hold model. The issue of predictability seemed closed, leaving behind two more or less opposing and irreconcilable concepts" (Fama, 1965).

Besides, firstly applying the random walk hypothesis, Samuelson (1965) provided economic argument for efficient market. Efficient market framework was built with underlying probabilistic assumptions. With these assumptions efficient marker hypothesis lost its general nature. For example, if we assume short period of compensation, efficiency will be rejected. Besides, the efficiency is limited by the specific restraining characteristics of probability laws (Walter, 2003). Later, Harry Roberts (1967) divided the EMH‟s tests to weak and strong form tests. Fama and his group continued doing research with event study and came to the conclusion that the stock market was efficient (Fama et al., 1969).

In 1970, Fama (1970) defined the efficient market as a market in which available information is fully reflected in prices. Random walk testing continued with Kemp and Reid (1971) paper in which they claimed that stock prices were conspicuously nonrandom. Besides, in EMH tests, Beja (1977) found that real market was impossibly efficient; Grossman and Stiglitz (1980) showed that perfect informationally efficient market was impossible; LeRoy and Porter (1981) rejected market efficiency; De Bondt and Thaler (1986) , in the first behavioral finance paper, found that stock prices overreact and that market is not efficient in a weak form.

In an outstanding article in random walk, Lo and MacKinlay (1988), using variance-ratio test for a weekly data, strongly rejected the random walk hypothesis. In an international context, Eun and Shim (1989) found that stock markets were not informationally efficient. Later, Fama concludes that, “market efficiency survives the challenge from the literature on long-term return anomalies” (Fama, 1998) but later Shleifer in 2000, challenged the assumption of investor rationality and perfect arbitrage in EMH (Sewell, 2008).

Finally, Pesaran claims that “it is often argued that if stock markets are efficient then it should not be possible to predict stock returns, namely that none of the variables in the stock market regression (1) should be statistically significant. Some writers have even gone so far as to equate stock market efficiency with the non-predictability property. But this line of argument is not satisfactory and does not help in furthering our understanding of how markets operate. In fact, it is easily seen that stock market returns will be non-predictable only if market efficiency is combined with risk neutrality (Pesaran, 2003)”.

The Random Walk Hypothesis and Market Efficiency

The concept of market efficiency is often linked to random walk hypothesis, a very strong statistical hypothesis that might be applied to any variable for which observations over time are possible. In spite of the odd implications of the name, the random walk hypothesis does not maintain that security prices are random in the sense of being uncaused. Instead, the random walk hypothesis consists of two hypotheses:

i. It asserts that successive returns are independent, which implies that the correlation between one period's returns and the next is zero; and

ii. It asserts that the distribution of returns in all periods is identical. This implies, for example, that the chances of 50 percent loss are the same in every period.

Random Walk Theory

A random walk is defined by the fact that successive price changes (one-period returns) are independent of each other; and identically distributed (Fama, 1970,1991; Brealey et al, 2005). The random walk hypothesis states that consecutive price changes are independent and

identically distributed (hereafter iid). The random walk theory suggests that share price movements are independent of each other and that today's share price cannot be used to predict tomorrow's share price. Therefore, the movement of a share price follows no predictable pattern, but moves in a random fashion with no discernable trend (Davies et al 2008). The RWH is a form of the martingale hypothesis but stricter, because the martingale hypothesis states that the expected price of tomorrow is the price of today while the RWH demands that the distribution is identical for all lags. A random walk with drift is defined as:

$$p_t = \mu + p_{t-1} + \varepsilon_t \qquad (2.1)$$

where p is the natural logarithm of the price, the subscript t denotes time, μ is a drift parameter and ε_t is the error term which is iid with a mean of 0 and a variance of σ^2.

Equation (2.1) can be rewritten as a first difference to illustrate the concept of a random walk:

$$\Delta p_t = \mu + \varepsilon_t. \qquad (2.2)$$

As Equation (2.2) shows, the only thing that changes the price is the drift parameter (μ) and random shocks (ε_t). The series will therefore wander randomly and unpredictably.

Because the consensus is that financial data rarely has an identical distribution over time, the RWH in its original form is too strict (Campbell et al., 1997). However, this does not make the concept of random walks as tests for market efficiency useless. It is possible to formulate a random walk model with less restrictive assumptions. Campbell et al. (1997) divide the random walk model into three levels: RW1, RW2 and RW3. RW1 is the pure random walk as described above, following an iid. RW2 is independent but not identically distributed, which allows the error terms to be heteroscedastic, a weaker condition than the RW1. RW3 is the weakest form of random walk, allowing dependencies in the error term, except for linear dependence. For example, $Cov\ (\varepsilon_t, \varepsilon_{t-q}) \neq 0$ is not allowed but $Cov\ (\varepsilon_t^2, \varepsilon_{t-q}^2) \neq 0$ is. Hence, the RW3 is neither strictly independent nor identically distributed but it is linearly independent. The RW3 is the form of the RWH mostly tested in the literature, and the one most test in this study assume.

The Random Walk Models

The random walk model as presented in the seminal work of Lo *et al.* (see Alvarez-Ramirez, et al. 2012) organize various versions of the random walk by considering various kinds of dependence that can exist between an asset's prices $p_t = InP_t$ and $p_{t+k} = InP_{t+k}$ at two dates t and $t + k$. The random walk model is the basic model of stock prices based on the assumption of market efficiency. The basic idea is that returns can be represented as unforecastable fluctuations around some mean return. This assumption applies that the distribution of the returns at time t is independent from, or at least uncorrelated with, the distribution of return in previous moment. Campbell, Lo and Mackinlay (1997) summarize three versions of RWH based on the characteristics of increments.

Within the random walk hypothesis, three successively more restrictive sub-hypotheses with sequentially stronger tests for random walks exist (Campbell et al. 1997). That is, Campbell et al. (1997) divide the random walk model into three levels: RW1, RW2 and RW3. RW1 is the pure random walk, following an iid. RW2 is independent but not identically distributed, which allows the error terms to be heteroscedastic, a weaker condition than the RW1. RW3 is the weakest form of random walk, allowing dependencies in the error term, except for linear dependence. The least restrictive of these is that in a market that complies with a random walk it is not possible to use information on past prices to predict future prices. That is, returns in a market conforming to this standard of random walk are serially uncorrelated, corresponding to a random walk hypothesis with dependent but uncorrelated increments. However, it may still be possible for information on the variance of past prices to predict the future volatility of the market. A market that conforms to these conditions implies that returns are serially uncorrelated, corresponding with a random walk hypothesis with increments that are independent but not identically distributed. Finally, if it is not possible to predict either future price movements or volatility on the basis of information from past prices then such a market complies with the most restrictive notion of a random walk. In this market, returns are serially uncorrelated and conform to a random walk hypothesis with independent and identically distributed increments.

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Independent and identically distributed increments (IID increment)

The first and the most simplistic version of the random walk model is that the disturbance term is independently drawn from the same distribution with the same mean and variance. Normal and IID disturbance terms in asset prices can be modeled by building the relationship between the natural logarithm of prices follows a random walk with normally distributed increments:

$$P_t = \mu + p_{t-1} + \epsilon_t$$

$$p_t - p_{t-1} = \mu + \epsilon_t \tag{2.3}$$

$$r_t = \mu + \epsilon_t, \qquad \epsilon_t \sim IID\ (0,\ \sigma^2)$$

Random walk I (hereafter RW1) implies that price increments are independent and identically distributed. This implies that increments are uncorrelated and any nonlinear functions of the increments are also uncorrelated. The assumption of identically distributed increments, however, is not plausible for financial assets prices over long periods of time spans because of the changes in probability distributions of financial assets returns resulting from changes in the economic, technological, institutional and regulatory environment surrounding the asset prices (Emenike, 2017).

Independent increments

(Independent but not Identically Distributed (INID) increments)

The second version of the random walk hypothesis relaxes the assumption of identically distributed increment allowing heteroskedastic disturbance terms. It still assumes that all increments are independent but they are not drawn from the same distribution, it accounts for each disturbance term being drawn from different distributions. Modifying equation (2.9) gives:

$$r_t = \mu + \epsilon_t, \qquad \epsilon_t \sim INID\ (0, \sigma^2)$$

(2.4)

which is a broader definition that includes (2.8). What is important however is that moving from the first model to the second allows for unconditional heteroskedasticity in the disturbance terms – it permits time varying fluctuation if and only if the disturbance terms are independent and not correlated or satisfy the conditional probability density function:

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$$pdf(\epsilon_{t+k}|\epsilon_t) = pdf(\epsilon_{t+k}) \tag{2.5}$$

As a result of implausibility of identically distributed increments, Random walk II (RW2) assumes independent but not identically distributed (INID) increments and thus allows for heteroscedasticity in increments. The RW2 therefore allows for unconditional heteroscedasticity, which is a particularly useful feature of time variation in volatility of many financial assets. Relaxing the identical distribution assumption in RW2 does not change the main economic property of increments, that is, prediction of future price increments cannot be estimated using past price increments (Campbell, Lo & Mackinlay, 1997).

Uncorrelated increments

In this third and final form of the random walk hypothesis considered, the assumption of uncorrelated increments still holds but that increments need not be independent and permit processes with dependent but uncorrelated increments. Campbell, Lo, et al. (1997) propose that all possible versions of the random walk model can be captured by the orthogonality condition with two arbitrary functions $f(.)$ and $g(.)$ (and for disjoint t, k):

$$cov[f(\epsilon_t), g(\epsilon_{t+k})] = 0 \tag{2.6}$$

but if functions $f(.)$ and $g(.)$ were restricted to linear functions, then (2.6) linearly implies that they are *not* serially correlated but they are transformed by a linear function making them dependent.

Random walk III (RW3) is obtained by relaxing the independence assumption of RW2 to include processes with dependent but uncorrelated increments. It only imposes a lack of correlation between subsequent increments.

Variance Ratios Test

The variance ratio test has been suggested in the finance literature as a test of the random walk

model (RWM) where ε_t's is assumed to be identically and independently distributed with mean

0 and variance σ^2.

$$y_t = \mu + y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim IID, (0, \sigma^2), \; t = 1, 2, \; . \; . \; ., \tag{2.7}$$

This test is developed by Cochrane, (1988) and Lo and MacKinlay (1988, 1989) for testing the randomness of stock prices.

The variance ratio test of Lo and MacKinlay (1988) is based on the property that the variance of increments of a random walk X_t is linear in its data interval. That means, the variance of $(X_t - X_{t-q})$ is q times the variance of $(X_t - X_{t-l})$. Therefore, the RWH can be checked by comparing $1/q$ times the variance of $(X_t - X_{t-q})$ to the variance of $(X_t - X_{t-l})$.

In the study of the RWH in emerging markets, VR tests have been by far the most widely used econometric tools since the pioneering work of Lo and MacKinlay (1988). The gist of VR tests is that if a stock's return is purely random, the variance of k-period return is k times the variance of one-period return (Al-Khazali, Ding and Pyuu, 2007). Hence, the VR, defined as the ratio of $1/k$ times the variance of the k-period return to the variance of the one-period return, should be equal to one for all values of k.

The VR approach has gained popularity and has become the standard tool in random-walk testing. We can apply the test to both, the stock price index and to the individual stocks (Urrutia, 1995). Lo and MacKinlay (1988) show that the variance ratio test is more powerful than the unit root tests. Also, Ayadi and Pyun (1994) argue that the variance ratio has more appealing features than other procedures.

The test is demonstrated to be more reliable and as powerful as or more powerful than the unit root test (Lo and MacKinlay, 1988). The test is based on the assumption that the variance of increments in the random walk series is linear in the sample interval. Specifically, if a series follows a random walk process, the variance of its q-differences would be q times the variance of its first differences.

METHODOLOGY

Population of the Study

As at May 31, 2018, The Nigerian Stock Exchange has more than 200 listed companies with a total market capitalization of over ₦13-trillion (NSE, 2019). All listings are included in The Nigerian Stock Exchange All- Shares index. In terms of market capitalization, The Nigerian Stock Exchange is the largest stock exchange in Africa (NSE, 2019).

Sample Size and Sampling Procedure

The sample size is ten (10) stocks. The study employs monthly raw stock prices of ten (10) companies, continuously traded in the Nigerian Stock Exchange (NSE) over the period January 2013 to December 2022. The choice of the period is motivated by the fact that this timeline (2013) captures the year the NSE all-share index rose astronomically by 42.8 percent, thereby enforcing attraction of investments to further deepen the market.

The companies were randomly selected based on their ability to trade frequently on the market and absorb the shocks of tin trading with irregular hiking. They include: Dangote cement, Dangote sugar, Guinness, J. Berger, Neimeth, Okomu Oil, PZ, UPDC, Vita form, and Zenith Bank. Consequently, only those stocks that were listed before 1st January 2003 were considered.

Sources of Data Collection

Data was collected from 10 stocks listed on the Nigerian Stock Exchange. Monthly stock price data were obtained from the exchange database over the ten (10) years trading period. The start date is 2nd January, 2013 to 31 December, 2022.

The All-share Index is used to measure the stock market in terms of the magnitude and direction

of general price movement. It also indicates the total market index, which reflects the behavior of stock in the market. (Udom and Richard, 2019). The All-Share Index (ASI), also serves as useful indicator of stock market efficiency. Secondary data were collected on the Nigerian Stock Exchange All-share historical weekly returns, various issues of the statistical Bulletin and Annual Reports of the Central Bank of Nigeria (CBN), text books, Journals and unpublished materials. Data on stock prices for the ten (10) sampled securities as well as the All-share Index of the Nigerian Stock Exchange were also obtained from the internet (using the website of cash-craft asset management). Owing to the absence of consistent real series, nominal values of the variables under consideration were used. The main data for stock price changes for all the securities traded on the floor of the Nigerian stock Exchange is the All-share Index of the Nigerian Stock Exchange. The choice of the All-share index is informed by the fact that it is the aggregation of all price gains and losses for a given period. To this end, the All-share index for the period January, 2013-December, 2022 is used to compute market statistics.

Week-end closing prices were deliberately chosen. The essence was to ensure uniformity in time lag as well as avoid contamination of results, which may occur as a result of thin market structure (Heman, 2001). There is an underlying assumption of the Efficient Market Hypothesis that deals are continuously consummated in securities. When this assumption is withdrawn, a longer differencing interval would need to be applied to adjust for lags in trading time.

Sampling Design

In view of the problem definition, the variables surveyed are the stock prices of 10 selected securities for the period January 2013 – December 2022. From a population of over two hundred (200) quoted companies, a sample of ten (10) securities were randomly selected based on their ability to trade frequently on the market and absolve the shocks of tin trading with irregular hiking. The age of the securities on the floor of the Nigerian Stock Exchange was also considered. Consequently, only those securities that were listed before 1st January 2003 were considered.

Background of the Study Area

The weak form of EMH states that Securities prices are essentially random and there is no chance of speculation in the stock market based on the assumption that successive price changes are independent of each other and follow a random walk. In other words, it means no individual can make abnormal profit from trading in securities. The mathematical/statistical approaches followed in this study include: (a) Test for normality of distribution of prices of selected stocks and (b) the variance ratio test examines the random walk hypothesis.

We reinforce this evidence of randomness by conducting variance ratio test. The three assumptions of randomness are Independent and Identical Distributed (IID) Increment, Independent Increment and Uncorrelated Increment.

The IID Increments-This assumes that all increments or price changes/returns are independently drawn from the same distribution with unconditional mean and variance. By expression this can be defined as.

$$S_t = S_{t-1} + \varepsilon_t \tag{3.1}$$

$$S_t - S_{t-1} = \varepsilon_t \tag{3.2}$$

$$S_t - S_{t-1} = r_t = \varepsilon_t : \varepsilon_t \sim IID(0, \delta^2) \tag{3.3}$$

Where: S_t price at time t, S_{t-1} price at time t-1, $S_t - S_{t-1}$ price changes/increments, r_t return at time t and ε_t randomness assumed to be IID with zero mean and constant variance.

The Independent Increments (independent but not identically distributed *(INID)* increments)- This assumes that not only the disjointed increments are uncorrelated, but any of the non-linear functions of the increments are uncorrelated too. That is.

$$\text{cov}(f(r_h), g(r_s)) = 0 \tag{3.4}$$

The Uncorrelated Increments- This is more general than the Independent Increments, and obviously the weakest form of the random walk hypothesis. It assumes the disjointed increment are uncorrelated $\text{cov}(r_h, r_s) = 0$, but the non-linear functions are correlated $\text{cov}(r_h^2, r_s^2) \neq 0$. Despite the differences in their definitions, the three components of the random walk still share some features such as conditional mean and variance.

$$E(S_t / S_0) = S_0 + ut \qquad (3.5)$$

$$Var(S_t / S_0) = \delta_t^2 \qquad (3.6)$$

As stated in equation 14, it is now assumed that the variances of the increments dependent or conditioned on the length of time. The variance ratio test makes use of this feature by comparing the variances of increments of the different time intervals/lengths to the test the null of RWH against the alternate hypothesis of stationary. The test can be conducted by constructing an estimator for k-period variance ratio [VR(k)] statistic.

$$VR(k) = \frac{\delta^2(k)}{\delta^2(1)} \qquad (3.7)$$

Where $\delta^2(1)$ is one-period return variance that is estimated using the one-period return $S_t - S_{t-1}$ defines.

$$\delta^2(1) = \frac{1}{T-1}\sum_{t=1}^{T-1}(r_t - \bar{r})^2 \qquad (3.8)$$

Or

$$\delta^2(1) = \frac{1}{T-1}\sum_{t=1}^{T-1}(S_t - S_{t-1} - \bar{r})^2 \qquad (3.9)$$

With $\bar{r} = \frac{1}{T-1}\sum_{t=1}^{T-1} r_t$ as the estimated average of the one-period return.

According to Lo and MacKinlay (1988) and Campbell et al. (1997) $\delta^2(k)$ can be defined as.

$$\delta^2(k) = \frac{1}{m}\sum_{t=k}^{T}(S_t - S_{t-k} - \bar{r}k)^2 ;\ m = k(T-k+1)(1-kT^{-1}) \qquad (3.10)$$

To reject the alternate hypothesis that return follows a stationary process, $\delta^2(k) \approx \delta^2(1)$.

Hypothesis and p-value

A hypothesis is a statement temporarily accepted as true in the light of what is, at the time, known about a phenomenon, and it is employed as a basis for action in the search of new truth. It is a shrew and intelligent guess, a supposition, inference, hunch, provisional statement or tentative generalization as to the existence of some facts, condition or relationship relative to some phenomenon which serves to explain already known facts in a given area of research and guide the research for new truth on the basis of empirical evidence.

More formally, as the parameter space is partitioned into two disjoint sets H_0 and H_1, H_0 is the null hypothesis while H_1 the alternative hypothesis. Hypothesis testing is like a legal trial. We assume someone is innocent unless the evidence strongly suggests that he is guilty. Similarly, we retain H_0 unless there is strong evidence to reject H_0.

Reporting "reject H_0" or "retain H_0" is not very informative. Instead, we could ask, for every α, whether the test rejects at that level. Generally, if the test rejects at level α it will also reject at level $\alpha' > \alpha$. Hence, there is a smallest α at which the test rejects and we call this number the p- value.

Informally, the p-value is a measure of the evidence against H_0: the smaller the p-value, the stronger the evidence against H_0. The p-value is the smallest α at which we do reject H_0. If the evidence against H_0 is strong, the p-value will be small. According to DeFusco et al (2001), the p-value is the smallest level of significance at which the null hypothesis can be rejected. Typically, researchers use the following evidence scale:

Table 3.1 p-value evidence scale

p-value	evidence
$< .01$	very strong evidence against H_0
.01 - .05	strong evidence against H_0
.05 - .10	weak evidence against H_0
$>$ .1	little or no evidence against H_0

It should be noted that a large p-value is not strong evidence in favor of H_0. That is, the p-value is the smallest level at which we can reject H_0. A large p-value can occur for two reasons: (1) H_0 is true or (2) H_0 is false but the test has low power. The p-value is not the probability that the null hypothesis is true.

EMPIRICAL RESULTS

Data Presentation, Model Specifications and Findings

The study employs monthly raw stock prices of ten (10) companies, continuously traded in the Nigerian Stock Exchange (NSE) over the period January 2013 to December 2022. Monthly time plots of the data obtained is provided in Figure 4.1 which allows for a visual interpretation of the changes in price experienced by the Stock Exchange.

The companies were randomly selected based on their ability to trade frequently on the market and absolve the shocks of tin trading with irregular hiking (and they include: Dangote cement, Dangote sugar, Guinness, J. Berger, Neimeth, Okomu Oil, PZ, UPDC, Vita form, and Zenith Bank). Thin (or infrequent) trading occurs when stocks do not trade at every consecutive interval, and thin or infrequent trading of stocks can produce statistical biases in the time series of stock prices. The bias of thinly traded shares is caused by prices that are recorded at the end of one period, but can represent the outcome of a transaction in an earlier period, inducing serial correlation. [Several studies investigate the consequences of thin trading (e.g., Lo and MacKinlay, 1989; Stoll and Whaley, 1990; Miller, Muthuswamy, and Whaley, 1994)]

We were prompted to begin the investigation of the behavior of the selected stocks in the year 2013. This timeline captures the year the NSE all-share index rose astronomically by 42.8 percent, thereby enforcing attraction of investments to further deepen the market. A sight view of the trajectories of the selected company prices and returns are shown in the figures below.

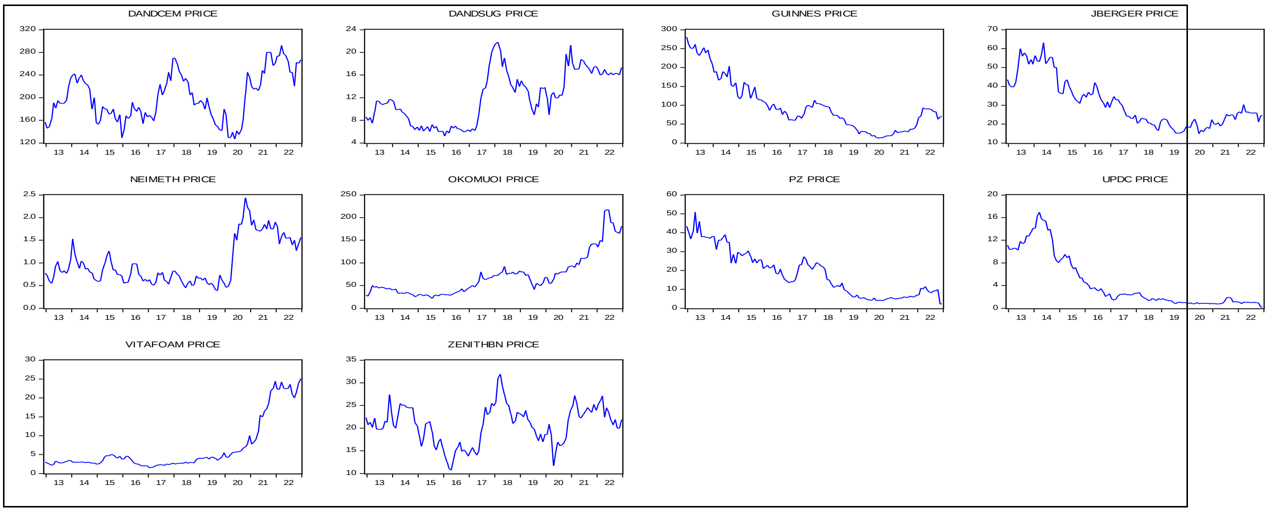

Figure 4.1-Prices of Selected Stocks over the Period Jan 2013 to Dec 2022

The trajectories of prices for four of the companies (Guinness, J-Berger, UPDC and PZ) started declining since 2013. The prices of other companies showed fluctuations, indicating gain or loss over the sample period.

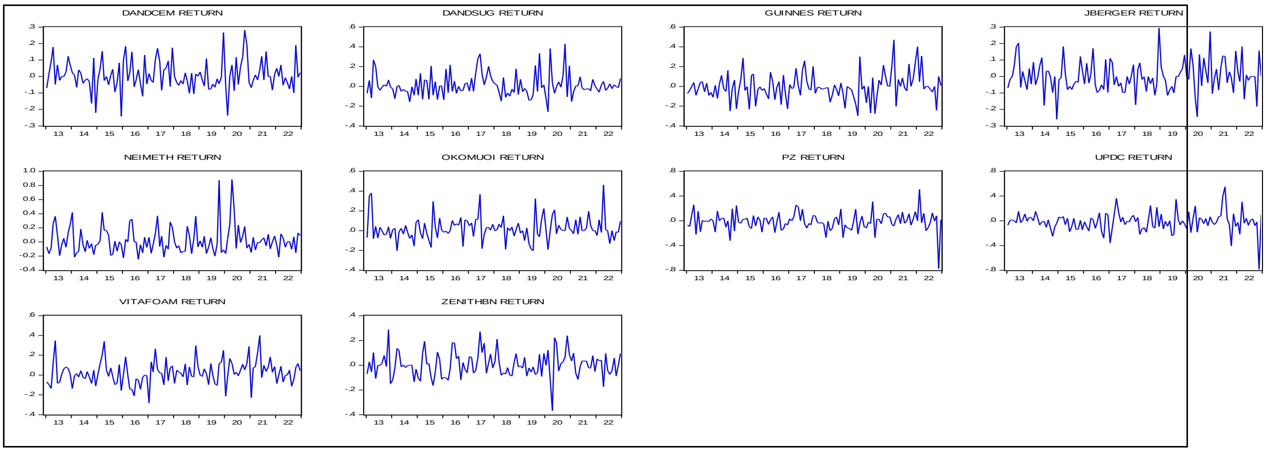

Figure 4.2-Increments/Returns of Selected Stocks over the Period Jan 2013 to Dec 2022

Normality Test

It is usually assumed that the populations from where the samples are collected are normally distributed. A graphical test for normality was conducted using Q-Q plot (by comparing a time series; see figure 4.3). The data points rest on the transfer lines for each case, implying that the stock returns of the companies follow normal distribution process, despite that they appear noisy in figure 4.2.

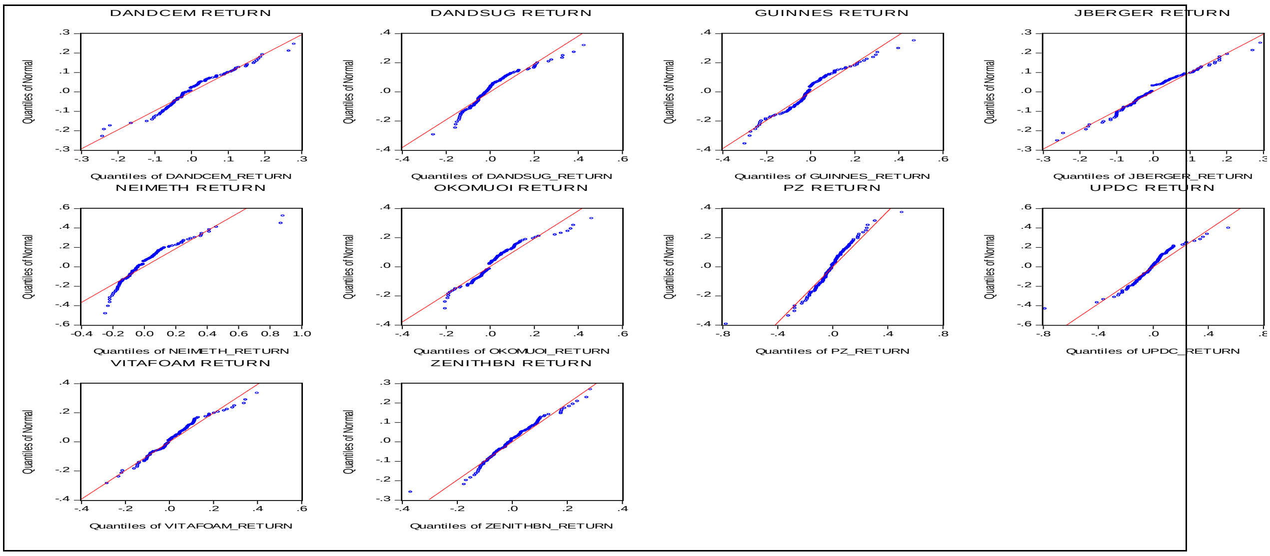

Figure 4.3-Q-Q Plots of Selected Stocks over the Period Jan 2013 to Dec 2022

Stationarity

Typically share prices have a unit root, that is, the mean and volatility is not independent through time but this can be accounted for by using the definition of returns as the logarithmic difference of lagged raw prices as defined in conceptual review. The price process is a difference-stationary process, that is taking the difference of the lagged logarithmic raw price data will convert the non-stationary process that has a unit root to a stationary process which does not have a unit root.

To investigate the random walk hypothesis (RWH), unit root test of stationarity (absence of randomness) is very necessary. Unit root test is a statistical test for investigating stationarity in time series. In this study Augmented Dickey-Fuller (ADF) unit root test, Phillips-Perron (P-P) unit root test and The Kwiatkowski, Phillips, Schmidt and Shin Test (KPSS) has been employed for this purpose.

The null hypothesis of ADF and Phillips and Perron (1988) tests is the presence of a single unit root in the autoregressive representation of a process. An alternative approach would be to take the 'stationarity' as the null hypothesis Such a test is developed by Kwiatkowski, Phillips, Schmidt, and Shin (1992, KPSS).

ADF, PP, and PKSS tests were carried on the log of the index of the Nigeria stock exchange market. The tests were performed in levels, and first differences, and trend. The results of ADF and PP tests are reported in Tables 4.1 and 2, while the results of KPSS are reported in table 4.3.

The ADF test results are shown in table 4.1 where raw price data and returns are subject to the augmented Dickey-Fuller test. Table 4.1 shows that the null hypothesis of the presence of a unit root is rejected for all data sets consisting of raw prices.

Table 4.1-Unit Root Tests - ADF-Test Result

	ADF-Test	
Company	Stat	CV @ 5%
DANGCEM	-11.22	-2.89
DANGSUG	-6.27	-2.89
GUINNES	-10.70	-2.89
JBERGER	-11.53	-2.89
NEIMETH	-9.02	-2.89
OKOMUOI	-11.17	-2.89
PZ	-12.89	-2.89
UPDC	-10.51	-2.89
VITAFOAM	-9.84	-2.89
ZENITHBN	-9.70	-2.89

ADF Unit Root Test results on the Logarithm of the Nigeria Exchange Index at first difference.

The random walk hypothesis (RWH) is supported if the stock market returns series contain unit root and are non-stationary.

The null hypothesis is that δ = 0; that is, there is a unit root, the time series is nonstationary. The alternative hypothesis is that δ is less than zero. Failing to reject H_0 implies that we do not reject that the time series has the properties of a random walk.

As mentioned earlier, it is assumed that the return series are stationary, but it has been tested by a Dickey-Fuller test. In the table above, the Dickey-Fuller test is displayed. From this, we can see that the null hypothesis is strongly rejected for all indices in the period under study periods. The ADF statistics is (in absolute terms) larger than the associated 5% critical value for each of the company returns. This confirms the rejection of the null hypothesis that return has a unit root. This test confirms the assumption regarding stationarity in time series and thereby also that the returns have the characteristics of possible random walk.

The random walk hypothesis (RWH) is supported if the stock market returns series contain unit root and are non-stationary. Since the unit root null hypothesis is not accepted by the ADF unit root test across the study periods, it means that the stock market return series of the NSE are not covariance-stationary and hence weak-form efficient and investors cannot predict the stock market returns.

Table 4.2-Unit Root Tests - PP-Test Result

	PP-Test	
Company	Stat	CV @ 5%
DANGCEM	-11.23	-2.89
DANGSUG	-11.53	-2.89
GUINNES	-10.89	-2.89
JBERGER	-11.96	-2.89
NEIMETH	-8.88	-2.89
OKOMUOI	-11.17	-2.89
PZ	-12.79	-2.89

UPDC	-10.50	-2.89
VITAFOAM	-9.81	-2.89
ZENITHBN	-9.70	-2.89

PP Unit Root Test results on the Logarithm of the Nigeria Exchange Index at first difference.

The PP- Test statistics is in absolute terms larger than the associated 5% critical value for each of the company returns. This confirms the rejection of the null hypothesis that return has a unit root.

Table 4.3-Unit Root Tests - KPSS-Test Result

	KPSS-Test	
Company	Stat	CV @ 5%
DANGCEM	0.06	0.46
DANGSUG	0.08	0.46
GUINNES	0.26	0.46
JBERGER	0.16	0.46
NEIMETH	0.08	0.46
OKOMUOI	0.08	0.46
PZ	0.06	0.46
UPDC	0.06	0.46
VITAFOAM	0.63	3.12
ZENITHBN	0.06	0.46

KPSS Unit Root Test results on the Logarithm of the Nigeria Exchange Index at first difference. The KPSS statistic shows lower values than the 5% critical values.

The ADF and PP statistics are in absolute terms larger than the associated 5% critical value for each of the company returns. This confirms the rejection of the null hypothesis that return has a unit root.

Large value of KPSS leads to rejection of the stationarity null hypothesis, since that means the series deviate from its mean. In the (KPSS) unit root test hypotheses are stated as follows:

H_0: there is not any unit root in the series (stationary). H_1: there is a unit root in the series. The KPSS statistic shows lower values than the 5% critical values. Thus, there is a strong evidence that the underlying series is stationary at levels or prices of the selected companies are I(1) variables.

Table 4.4 ADF, PP and KPS Tests Results

Company	ADF-Test		PP-Test		KPSS-Test	
	Stat	CV @ 5%	Stat	CV @ 5%	Stat	CV @ 5%
DANGCEM	-11.22	-2.89	-11.23	-2.89	0.06	0.46
DANGSUG	-6.27	-2.89	-11.53	-2.89	0.08	0.46
GUINNES	-10.70	-2.89	-10.89	-2.89	0.26	0.46
JBERGER	-11.53	-2.89	-11.96	-2.89	0.16	0.46
NEIMETH	-9.02	-2.89	-8.88	-2.89	0.08	0.46
OKOMUOI	-11.17	-2.89	-11.17	-2.89	0.08	0.46
PZ	-12.89	-2.89	-12.79	-2.89	0.06	0.46
UPDC	-10.51	-2.89	-10.50	-2.89	0.06	0.46
VITAFOAM	-9.84	-2.89	-9.81	-2.89	0.63	3.12
ZENITHBN	-9.70	-2.89	-9.70	-2.89	0.06	0.46

The results of ADF, PP, as well as that of KPSS provide evidence that the Nigeria index are nonstationary at level, and stationary for the first and second differences. Therefore, the results are consistent with the random walk hypothesis.

Variance Ratio Test Results

In this section, the random walk hypothesis is examined by applying the variance-ratio test. We propose the test for this study, and the results are reported in table 4.1.

Table 4.5-Variance Ratio Test Results

Company	Period	Var. Ratio	Std. Error	z-Statistic	Probability
DANDCEM	2	0.517708	0.126269	-3.819568	0.0001
DANDSUG	2	0.372639	0.139784	-4.488082	0.0000
GUINNES	2	0.484274	0.114762	-4.493873	0.0000
JBERGER	2	0.490001	0.113685	-4.486083	0.0000
NEIMETH	2	0.666126	0.153766	-2.171307	0.0299
OKOMUOI	2	0.530744	0.120418	-3.896901	0.0001
PZ	2	0.325200	0.156205	-4.319956	0.0000
UPDC	2	0.493389	0.145475	-3.482448	0.0005
VITAFOAM	2	0.578492	0.113801	-3.703908	0.0002
ZENITHBN	2	0.644981	0.119694	-2.966051	0.0030

Note that our test is based on 2 periods, though similar results are obtained in longer periods.

The p-value measures the evidence against H_0 (the null hypothesis). The p-value is the smallest α at which we do reject H_0. the smaller the p-value, the stronger the evidence against H_0. If the evidence against H_0 is strong, the p-value will be small.

Here, randomness suggests that price movement on the floor of the Nigerian Stock Exchange does not tend to follow any particular pattern. Table 4.5 explores the result of variance ration test of the index returns of Nigeria stock exchange does not follow the random walk. The variance ratio statistic for each of the company is associated with probability value of 0%. This suggests the rejection of the null hypothesis of sustainable random process. Therefore, our test evidence shows that the returns of these companies are not random; rather they are stationary and predictable.

The VR tests assumption are strongly rejected with marginal p-values of 0.0000. This means that the NSE stock returns do not follow a random walk. Based on the results of variance ratio tests, it is concluded that the Nigerian daily stock return series do not move in a random fashion and hence the null hypothesis of a random walk is rejected, thus the Nigerian Stock Market is a weak-form inefficiency market.

5. SUMMARY AND CONCLUSION

This study examined the random walk behavior and efficiency of the Nigeria stock market using unit root tests and variance ratio test. The empirical results reject the random walk hypothesis at the weak-form level, implying that stock prices do not fully reflect all historical information. The results of this paper conform to the results of most previous findings of Ekechi (2002), Inegbedion (2009). Emenike (2008, 2017); Gimba (2012); Afego (2012); Goudarzi (2013) and Ogbulu (2016), and Ogbonna and Ejem (2020).

This has important implications for the fortune of equity investors:

To increase market activities through reduction in transaction cost and increase in membership of the NSE; and minimize institutional restrictions on trading of securities in the bourse. This will make all other markets to flow as a deregulated market. It is therefore, recommended that there is need for policy makers to enlighten potential investors of the opportunities that are available in the stock market. Such enlightenment should seek to stimulate their interest in capital market activities and thus increase the breadth and depth of the capital market.

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Appendix

Null Hypothesis: DANDCEM_RETURN is a martingale
Date: 01/01/24 Time: 07:52
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.517708	0.126269	-3.819568	0.0001

Test Details (Mean = 0.000776242065926)

Period	Variance	Var. Ratio	Obs.
1	0.01678	--	119
2	0.00869	0.51771	118

Null Hypothesis: DANDSUG_RETURN is a martingale
Date: 01/01/24 Time: 07:56
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.372639	0.139784	-4.488082	0.0000

Test Details (Mean = 0.00126692711425)

Period	Variance	Var. Ratio	Obs.
1	0.02883	--	119
2	0.01074	0.37264	118

Null Hypothesis: GUINNES_RETURN is a martingale
Date: 01/01/24 Time: 07:58
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.484274	0.114762	-4.493873	0.0000

Test Details (Mean = 0.000671163461861)

Period	Variance	Var. Ratio	Obs.
1	0.03581	--	119
2	0.01734	0.48427	118

Null Hypothesis: JBERGER_RETURN is a martingale
Date: 01/01/24 Time: 08:00
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.490001	0.113685	-4.486083	0.0000

Test Details (Mean = 0.000586281024038)

Period	Variance	Var. Ratio	Obs.
1	0.01933	--	119
2	0.00947	0.49000	118

Null Hypothesis: NEIMETH_RETURN is a martingale
Date: 01/01/24 Time: 08:02
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.666126	0.153766	-2.171307	0.0299

Test Details (Mean = 0.00135022296445)

Period	Variance	Var. Ratio	Obs.
1	0.05994	--	119
2	0.03993	0.66613	118

Null Hypothesis: OKOMUOI_RETURN is a martingale
Date: 01/01/24 Time: 08:04
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.530744	0.120418	-3.896901	0.0001

Test Details (Mean = 0.00140624537341)

Period	Variance	Var. Ratio	Obs.
1	0.02837	--	119
2	0.01505	0.53074	118

Null Hypothesis: PZ_RETURN is a martingale
Date: 01/01/24 Time: 08:06
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.325200	0.156205	-4.319956	0.0000

Test Details (Mean = 0.000735013083233)

Period	Variance	Var. Ratio	Obs.
1	0.04986	--	119
2	0.01621	0.32520	118

Null Hypothesis: UPDC_RETURN is a martingale
Date: 01/01/24 Time: 08:09
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.493389	0.145475	-3.482448	0.0005

Test Details (Mean = 0.00142661715849)

Period	Variance	Var. Ratio	Obs.
1	0.04839	--	119
2	0.02388	0.49339	118

Null Hypothesis: VITAFOAM_RETURN is a martingale
Date: 01/01/24 Time: 08:11
Sample: 2012M12 2022M12

Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.578492	0.113801	-3.703908	0.0002

Test Details (Mean = 0.000971435085662)

Period	Variance	Var. Ratio	Obs.
1	0.02508	--	119
2	0.01451	0.57849	118

Null Hypothesis: ZENITHBN_RETURN is a martingale
Date: 01/01/24 Time: 08:12
Sample: 2012M12 2022M12
Included observations: 119 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.644981	0.119694	-2.966051	0.0030

Test Details (Mean = 0.00138460035177)

Period	Variance	Var. Ratio	Obs.
1	0.01790	--	119
2	0.01154	0.64498	118