Journal of Basic Physical Research ISSN: 2141-8403 Prints, 2141-8411 Online
Vol.12, No.1 pp. 1- 12 March., 2024. Available online at www.jbasicphyres-unizik.org

COUETTE FLOW IN THE PRESENCE OF VISCOUS DISSIPATIVE FLUID ALONG AN UPSTANDING CHANNEL AFFECTED BY NEWTONIAN HEATING: HOMOTOPY PERTURBATION APPROACH

Emmanuel Omokhuale[1*] and Godwin Ojemeri[2]
[1]*Department of Mathematics, Faculty of Sciences, Federal University Gusau, Zamfara State.*
[2]*Department of Mathematics, College of Sciences, Federal University of Agriculture, Zuru, Kebbi State.*
**Corresponding Author's email: emmanuelomokhuale@gmail.com*

Abstract: *This article attempts to describe the analytical implication of viscous dissipation on an electrically-conducting and incompressible fluid in a vertical channel controlled by Newtonian heating condition. The governing equations have been derived using homotopy perturbation method. The impacts of the pertinent flow parameters on velocity and temperature were graphically displayed. The rate of heat transfers and shear stress on the heated and cold plates have also been calculated. It is noteworthy to mention that the hydrodynamic and thermodynamic distributions of the fluid increase with an increase in the viscous dissipation parameter. The graphical comparison between the work of Zulkifree et al. (2019) and the present study demonstrates an excellent agreement for limiting case. This study can find relevance in science, engineering, and industrial technologies such as cooling of electrical appliances, geothermal energy, porous solids drying, thermal insulation, gas drainage, plasma physics, gas turbines, fossil fuel combustion, food processing industries, to mention a few.*
Keywords: *Free convection, Viscous dissipative fluid, Newtonian heating, Vertical channel and Homotopy perturbation method.*

Introduction

Free convection flow in a boundary layer region is a motion that results from the interaction of gravity with density differences within a fluid. These differences occur due to temperature or concentration gradients or due to their composition. Studies pertaining free convection flows in vertical parallel plate channels for a single phase of an incompressible viscous fluids have received much attention in recent years both theoretically (exact or approximate solutions) and experimentally owing to the fact that many practical applications involve natural convection heat transfer (Al-Subaie and Chamkha, 2004, Zullkifree *et al*. 2019). The laminar flow of a viscous fluid between two parallel surfaces, one of which travels tangentially with respect to the other, is known as the Couette flow. This flow can be motivated by a pressure gradient that is present in the flow direction (Mng'ang'a, 2023). With these concerns in mind, Khan et al. (2022) analyzed the second law to determine how Newtonian heating affects the Couette flow of a viscoelastic dusty fluid and the transfer of heat in a rotating frame. Umavathi *et al.* (2010) studied the Poiseuille–Couette flow with heat transfer in the inclined channel. They concluded that increases in the Grash of number, angle of inclination, and height ratio increase velocity, while increases in the Hartmann number, viscosity, and conductivity ratios decrease the velocity profiles. Beg *et al.* (2010) analyzed the oblique magnetic field in a rotating highly permeable medium. Barikbin *et al.* (2014) analyzed non-Newtonian fluid for MHD Couette flow using the Ritz–Galerkin method. Sreekala and Reddy (2014) studied the effect of an inclined magnetic field with the steady MHD Couette flow. Joseph *et al.* (2014) investigated heat transfer in inclined magnetic field with unsteady MHD Couette flow between two in6nitely parallel porous plates. The temperature-dependent transient MHD Couette flow and heat transfer of dusty fluid were studied by Mosayebidorcheh *et al.* (2015). Jha *et al.* (2015) investigated thermal radiation with unsteady MHD free convective Couette flow. Ngiangia and Okechukwu (2016) discussed the impact of variable electro-conductivity and radiation. They indicated that increases in electro-conductivity, Prandtl number, Reynolds number, and Grashof number result in an increase in velocity distribution, while increases in the magnetic 6eld result in a drop in velocity. Raju *et al.* (2016) discussed the influence of diffusion thermo and thermal di1usion on a natural convection Couette flow using the 6nite element method.

Ali *et al.* (2016) investigated convective cooling of the nanofluids in a rotating system. Job and Gunakala (2016) studied the thermal radiation for vertical permeable plates using Galerkin's finite element method. Also, Ali *et al.* (2017) analyzed the Couette flow of a maxwell fluid for three dimensional with periodic injection/suction. Hussain *et al.* (2018) analyzed instability of the MHD Couette flow of an electrically conducting fluid. Anyanwu *et al.* (2020) discussed the in5uence of radioactive and a constant pressure gradient on an unsteady MHD Couette flow. In the work of Ajibade *et al.* (2021), the effect of viscous dissipation on steady natural convection Couette flow of a heat-generating fluid in a vertical channel was investigated. Recently, Unsteady Convective Couette flow with heat sink, radiation, heat source, magnetic field and thermal property effects was research on in the works of Omokhuale and Jabaka (2022a), (2022b) and Omokhuale and Jabaka (2023).

Generally, the problems of free convection flows in parallel plate are usually modeled under the assumptions of constant surface temperature, ramped wall temperature, or constant surface heat flux (Rajput 2011a, 2011b, Narahari 2012, Singh and Sarehu 2015). However, in many practical situations where the heat transfer from the surface is taken to be proportional to the local surface temperature, the above assumptions fail to work. Such types of flows are termed as conjugate convective flows, and the proportionally condition of the heat transfer to the local surface temperature is termed as Newtonian heating (Lesnic *et al.* 2004). Natural convection involving Newtonian heating or cooling through various geometrical shapes and channels have reported by several scholars. The natural convection boundary layer flow over a vertical surface with Newtonian heating was first studied by Merkin (1994). In view of this, Shehzad *et al.* (2014) investigate three-dimensional flow of Jeffrey fluid with Newtonian heating. Akbar *et al.* (2013) were discussing effect of Newtonian heating for mixed convective magneto-hydrodynamics peristaltic flow of Jeffrey fluid while Hamza (2016) investigate the influence of Navier Slip and Newtonian heating in transient flow of an exothermic fluid in vertical channel. Lesnic *et al.* (1999, 2000) studied free convection boundary layer flows along vertical and horizontal surfaces in a porous medium generated by Newtonian heating. Subsequently, a free convection boundary layer flow above a nearly horizontal surface in a porous medium with Newtonian heating was reported by Lesnic *et al.* (2004). Abid *et al.* (2013) studied heat and mass transfer past an oscillating vertical plate with Newtonian heating where the governing equations of the problem were solved by the Laplace transform technique. Subsequently, Abid *et al.* (2014) studied an unsteady boundary layer MHD free convection flow in a porous medium with constant mass diffusion and Newtonian heating analytically by the Laplace transform technique. An unsteady hydro-magnetic natural convection flow past an impulsively moving vertical plate with Newtonian heating in a rotating system was studied by Seth *et al.* (2015). Das *et al*. (2015) performed a numerical analysis on unsteady heat and mass transfer of hydro-magnetic Casson fluid across a vertical plate in the involvement of chemical reaction and thermal radiation. Qayyum *et al.* (2017) described the heat and mass transfer of water-B nanofluid across a stretching sheet in the coexistence of chemical reaction and thermal radiation. Hayat *et al.* (2018) discussed the impact of MHD flow of Powell-Eyring fluid by a stretching cylinder with thermal radiation through an inclined magnetic field under the Newtonian heating effect. Zin *et al.* (2018) presented an exact and numerical solutions of unsteady MHD heat and mass transfer flow of Jeffrey fluid flowing through an oscillatory vertical plate provoked by thermal radiation and Newtonian heating factors.

The influence of viscous dissipation along upward looking channel using homotopy perturbation method have not been examined in a single work in any of the above-mentioned literature, which prompted the interest for this research. Thus, motivated by the above knowledge gap, the focus of this work is to expand the work carried out by Zulkiflee *et al.*

(2019) by investigating the impact of viscous dissipation on Couette flow inside two upward parallel plates in the involvement of Newtonian heating using homotopy perturbation technique. Various illustrative graphs have been sketched to demonstrate the flow pattern of the pertinent parameters embedded in the flow regime. This research can find relevance in engineering and industrial technologies namely cooling of atomic reactors, geothermal supplies, porous solids drying, thermal insulation, gas drainage, plasma physics, gas turbines, fossil fuel combustion, and food processing industries and so forth.

Mathematical Formulation of the Problem

We consider a fully developed Laminar MHD flow of an incompressible viscous fluid passing through two vertical parallel plates with Newtonian heating instigated by viscous dissipation effect. As shown in Figures 1, the wall at $y_0 = 0$, is instigated by Navier slip effect while the no-slip surface is kept at $y_0 = H$. The flow is subjected to uniform transverse magnetic field in the presence of thermal buoyancy effects. All the fluid properties are assumed to be constants. The flow variables are functions of space y only. Following Zullkifree *et al.* (2019), the leading equations for the current problems in dimensional, employing the Boussinesq buoyancy approximation with boundary conditions and assuming that the fluid is affected by viscous dissipation effect, can be modeled as follows:

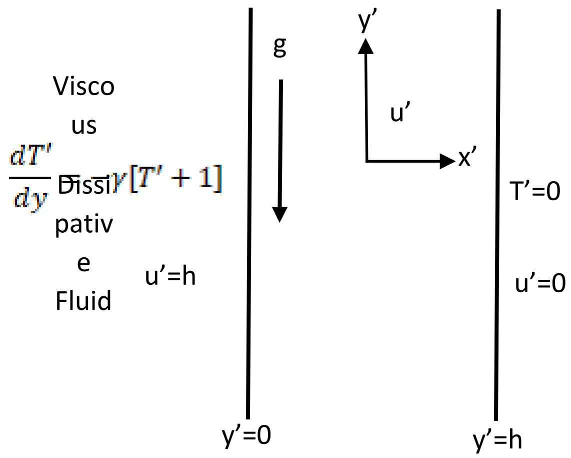

Figure 1: Physical Configuration of the Flow System

$$V\frac{d^2u'}{dy'^2} + g\beta(T' - T_O) = 0$$
(.1)

$$\frac{k}{\rho c_\rho}\frac{d^2T'}{dy'^2} + \frac{v}{c_\rho}\left(\frac{du'}{dy'}\right)^2 = 0 \qquad (.2)$$

$$\left.\begin{array}{l} \left.\begin{array}{l} u' = U_0 \\ \frac{dT'}{dy'} = -\frac{h_s}{k}T' \end{array}\right\} at\ y' = 0 \\ \left.\begin{array}{l} u' = 0 \\ T' = T_h^0 \end{array}\right\} \qquad at\ y' = h \end{array}\right\} \qquad (3)$$

To solve eqns (1) to (3), we employ the dimensionless quantities and parameters:

$$u = \frac{u'}{U}, y = \frac{y'}{h}, \theta = \frac{T' - T_0}{T_w - T_0}, x = \frac{x'v}{Uh^2}, B_r = \frac{hH}{k}\ , Gr = \frac{g\beta(T_w - T_0)h^3}{v^2} \qquad (4)$$

Using the dimensionless quantities in eqn (4), the basic eqns (1) to (3) becomes:

$$\frac{d^2U}{dy^2} + Gr\theta = 0 \tag{5}$$

$$\frac{d^2\theta}{dy^2} + Br\left(\frac{du}{dy}\right)^2 = 0 \tag{6}$$

The initial and boundary conditions in dimensionless forms are:

$$\left.\begin{array}{l} \left.\begin{array}{r} \frac{d\theta}{dY} = -\gamma(\theta+1) \\ U = 1 \end{array}\right\} at\ Y = 0 \\ \\ \left.\begin{array}{r} \theta = 0 \\ U = 0 \end{array}\right\} at\ Y = 1 \end{array}\right\} \tag{7}$$

Where Gr is thermal Grash of number, Br = EcPr is the Brinkman number.

Method of Solution

Assuming that the solutions of U and θ are:

$$U = u_0 + pu_1 + p^2u_2 + p^3u_3 \ldots$$
$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_3 \ldots \tag{8}$$

Constructing a convex homotopy on eqns (5-7) and substituting eqn (8) into eqns (5-7), we have:

$$H(1-p)\frac{d^2(u_0+pu_1+p^2u_2+p^3u_3)}{dy^2} = p[-Gr(\theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_3)] \tag{9}$$

$$H(1-p)\frac{d^2(\theta_0+p\theta_1+p^2\theta_2+p^3\theta_3)}{dy^2} + p\left[Br\left(\frac{d(u_0+pu_1+p^2u_2+p^3u_3)}{dy}\right)^2\right] = 0 \tag{10}$$

The boundary conditions in dimensionless forms now becomes:

$$\left.\begin{array}{r} \frac{d(\theta_0+p\theta_1+p^2\theta_2+p^3\theta_3)}{dY} = -\gamma[(\theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_3) + 1] \\ (u_0 + pu_1 + p^2u_2 + p^3u_3)(Y) = 1 \end{array}\right\} at\ Y = 0 \tag{11}$$

and

$$\left.\begin{array}{r} (\theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_3)(Y) = 0 \\ (u_0 + pu_1 + p^2u_2 + p^3u_3)(Y) = 0 \end{array}\right\} at\ Y = 1 \tag{12}$$

Comparing the coefficients of the power of p, we have

$$p^0: \frac{d^2u_0}{dy^2} = 0 \tag{13}$$

$$p^1: \frac{d^2u_1}{dy^2} = -Gr\theta_0 \tag{14}$$

$$p^2: \frac{d^2u_2}{dy^2} = -Gr\theta_1 \tag{15}$$

$$p^3: \frac{d^2u_3}{dy^2} = -Gr\theta_2 \tag{16}$$

$$p^0: \frac{d^2\theta_0}{dy^2} = 0 \tag{17}$$

$$p^1: \frac{d^2\theta_1}{dy^2} = -Br\left[\frac{du_0}{dy}\right]^2 \tag{18}$$

$$p^2: \frac{d^2\theta_2}{dy^2} = -2Br\left[\frac{du_0}{dy}.\frac{du_1}{dy}\right] \tag{19}$$

$$p^3: \frac{d^2\theta_3}{dy^2} = -2Br\left[\frac{du_0}{dy}.\frac{du_2}{dy}\right] \tag{20}$$

So that the transformed boundary condition now becomes

$$\left.\begin{aligned} \frac{d\theta_0}{dy} &= -\gamma(\theta_0+1) \\ \frac{d\theta_1}{dy} &= -\gamma(\theta_1) \\ \frac{d\theta_2}{dy} &= -\gamma(\theta_2) \\ \frac{d\theta_3}{dy} &= -\gamma(\theta_3) \end{aligned}\right\} at\ y = 0 \tag{21}$$

And

$$\theta_0 = \theta_1 = \theta_2 = \theta_3 = 0\ at\ y = 1 \tag{22}$$

The set of analytical solutions for velocity and temperature have been derived as follows:

$$u_0 = A_1 y + A_2 \tag{23}$$

$$\theta_0 = B_1 y + B_2 \tag{24}$$

$$u_1 = -Grb_1\left(\frac{y^3}{6} - \frac{y^2}{2}\right) + A_3 y + A_4 \tag{25}$$

$$\theta_1 = \frac{-Bry^2}{2} + B_3 y + B_4 \tag{26}$$

$$u_2 = -Gr\left(-\frac{Bry^4}{24} + \frac{B_3 y^3}{6} + B_4\frac{y^2}{2}\right) + A_5 y + A_6 \tag{27}$$

$$\theta_2 = -2Br\left[\left[Grb_1\left(\frac{y^4}{24} - \frac{y^3}{6}\right) - A_3\frac{y^2}{2}\right]\right] + B_5 y + B_6 \tag{28}$$

$$u_3 = -Gr\left[b_2\left(\frac{y^6}{720} - \frac{y^5}{120}\right) + b_3\frac{y^4}{24} + B_5\frac{y^3}{6} + B_6\frac{y^2}{2}\right] + A_7 y + A_8 \tag{29}$$

$$\theta_3 = -2Br\left[Gr\left(\frac{-Bry^5}{120} + \frac{B_3 y^4}{24} + B_4\frac{y^3}{6}\right) - A_5\frac{y^2}{2}\right] + B_7 y + B_8 \tag{30}$$

Setting p=1 from eqn (8), the velocity and temperature solutions becomes

$$U = u_0 + u_1 + u_2 + u_3 \ldots \tag{31}$$

$$\theta = \theta_0 + \theta_1 + \theta_2 + \theta_3 \ldots \tag{32}$$

The rate of heat transfer at both plates otherwise known as Nusselt number is obtained as follows:

$$\left.\frac{d\theta}{dy}\right|_{y=0} = b_1 + B_3 + B_5 + B_7 \tag{33}$$

$$\left.\frac{d\theta}{dy}\right|_{y=1} = b_1 - Br + B_3 - \frac{b_2}{3} + b_3 + B_5 - 2Br\left[-\frac{GrBr}{24} + \frac{GrB_3}{6} + \frac{GrB_4}{2} - A_5\right] + B_7 \tag{34}$$

The skin friction at both plates is derived as follows:

$$\left.\frac{dU}{dy}\right|_{y=0} = -1 + A_3 + A_5 + A_7 \tag{35}$$

$$\left.\frac{dU}{dy}\right|_{y=1} = -1 + \frac{Grb_1}{2} + A_3 - Gr\left(-\frac{Br}{6} + \frac{B_3}{2} + B_4\right) + A_5 - Gr\left[-\frac{b_2}{30} + \frac{b_3}{6} + \frac{B_5}{2} + B_6\right] + A_7 \tag{36}$$

Results and Discussion

The steady state analysis of free convection Couette flow and viscous dissipation of an incompressible electrically-conducting fluid traveling vertically within an isothermally heated parallel plate micro-channel is performed, with one surface having super-hydrophobic slip and temperature jump and the other having no slip. The homotopy perturbation method is used to evaluate the steady state governing equations. Several graphs were drawn to demonstrate the effects of various settings on the flow configuration and energy profile. The default values chosen for this research are (γ = 0.01, Br=0.001 and Gr=50), except otherwise stated, as it relates to real life situation. The graph showcasing the comparison between the work of Zulkifree *et al.* (2019) and the present study is demonstrated in Figure 2 for limiting case. It is interesting to report the comparison reveals an excellent agreement as clearly seen from the figure.

Figures 3 and 4 showcase the impact of the Brinkman number (Br) on dimensionless thermal and momentum distributions. It is evident from these figures that the temperature and velocity of the fluid increase drastically as the local Brinkman number (Br) increases. Higher Brickman values indicate stronger convective heating at the lower channel surface, resulting in a better temperature and velocity at the lower plate. Makinde and Aziz (2011) infer that uplifting the local Brinkman numbers makes the convective heating at the lower channel wall stronger. This leads to higher surface temperatures and lets the thermal effect go deeper into the still fluid.

Figure 5 and 6 display the function of the Newtonian heating parameter γ on the temperature velocity profiles. As expected, an increase in γ leads to a reduction in the fluid density as a result of amplification of the momentum boundary layer thickness. Consequently, an increase in the Newtonian heating parameter tends to increase the fluid velocity.

Figure 7 demonstrates the effect of the thermal Grashof number *Gr* on the velocity profiles. Physically, *Gr* represents the effect of free convection current i.e., $Gr = 0$ indicates the absence of free convection currents. It is seen that an increase in *Gr* tends to enhance the fluid velocity in the boundary layer.

Figure 8 shows the impact of the Newtonian heating parameter γ against Brinkman number (Br) on the rate of heat transfer coefficient. Heat transfer rate upsurges for growing values of Brinkman number in both walls at the lower region of the vertical channel whereas it diminishes at the upper vicinity of the flow channel. However, there exist a point of intersection at 0.7 at both walls

Figure 9 illustrates the action of thermal Grashof number versus Br on the velocity gradient. The effect of large Gr is seen to strengthen the frictional force (at y = 0) as shown in Fig. 8a, whereas a reverse trend is obtained (at y = 1) as depicted in Figure 8b.

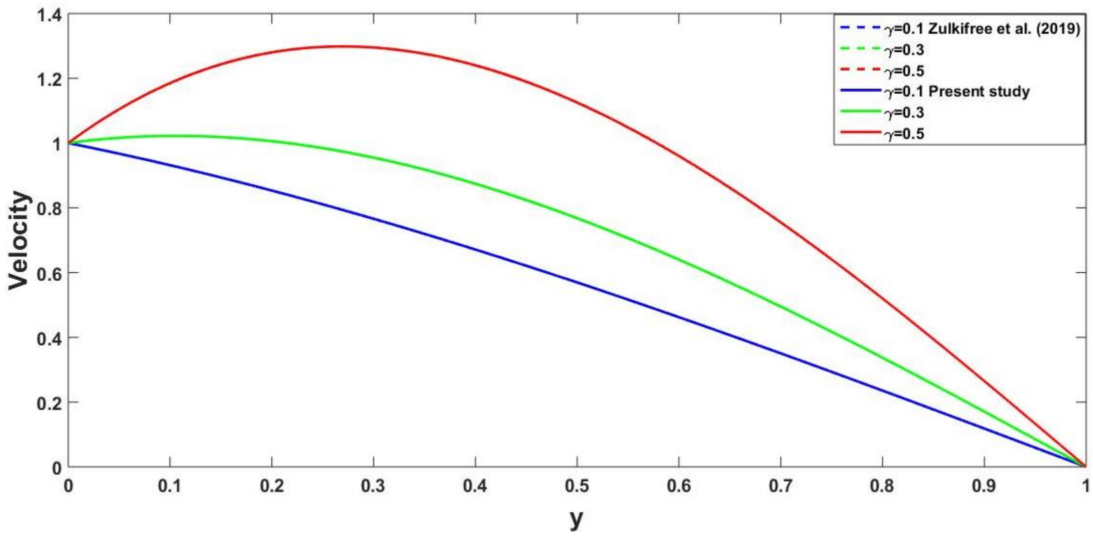

Figure 2: Graphical comparison between the work of Zulkifree *et al.* (2019) and the present study

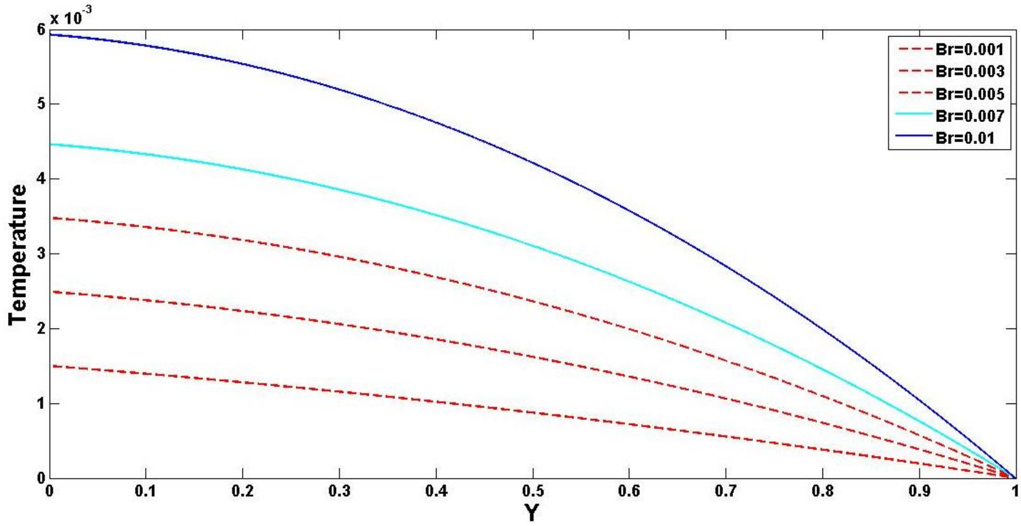

Figure 3. Effect of Br on temperature

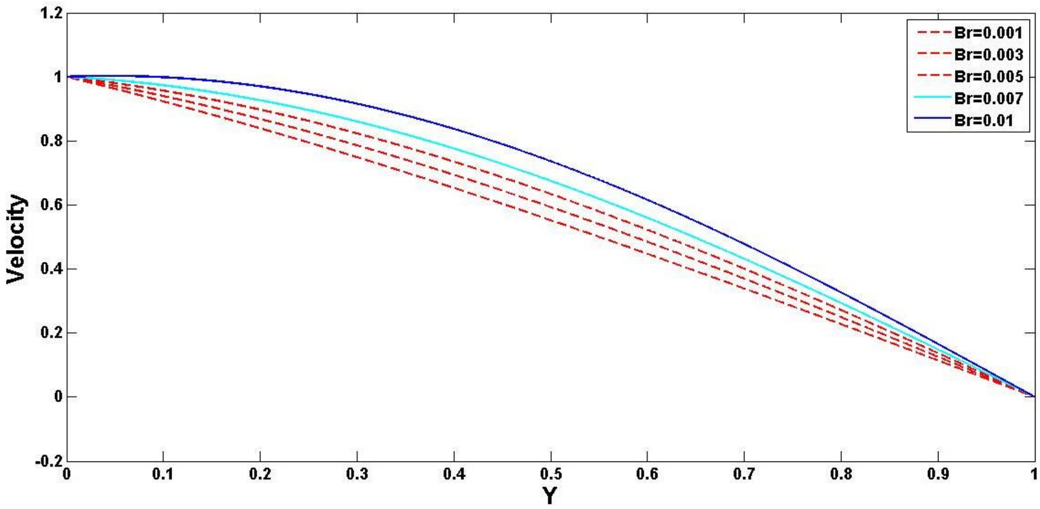

Figure 4. Effect of Br on velocity

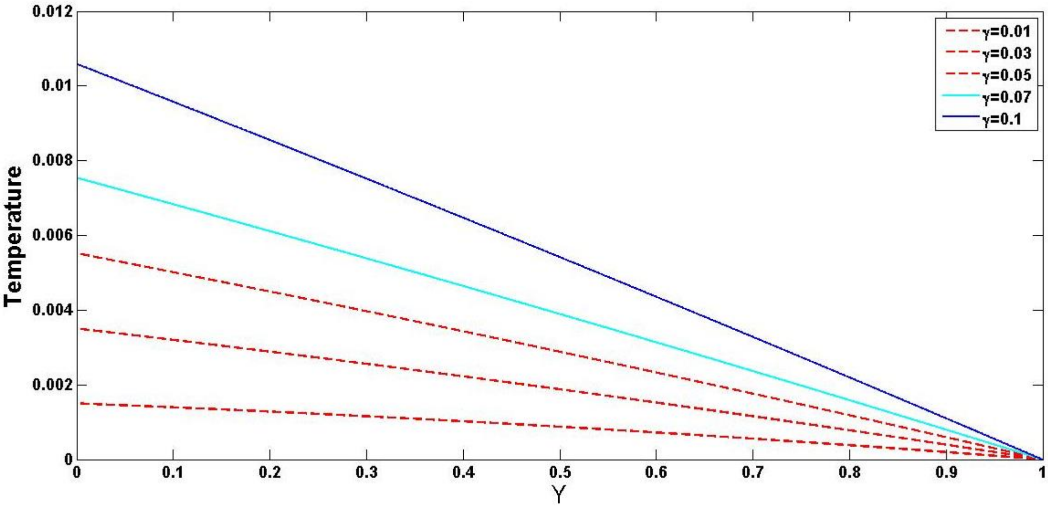

Figure 5. Effect of γ on temperature

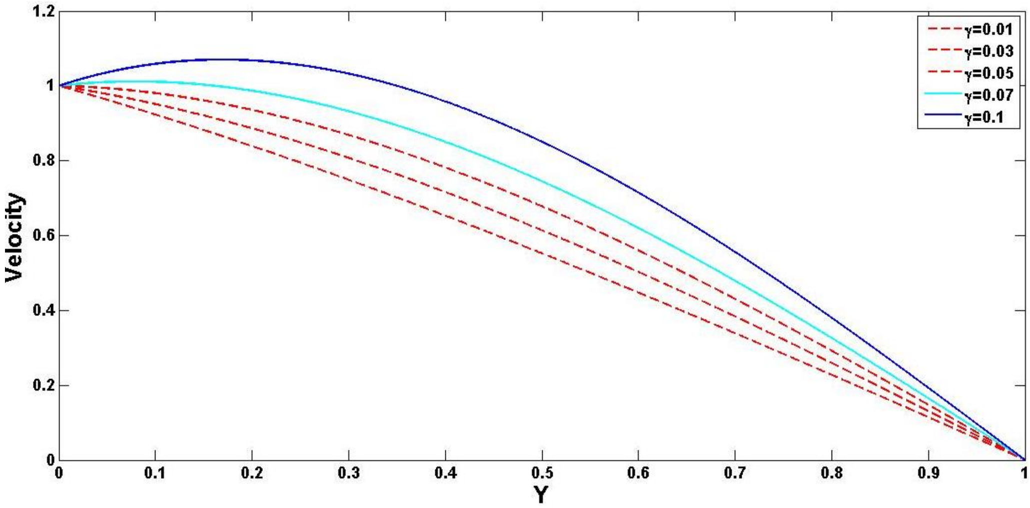

Figure 6. Effect of γ on velocity

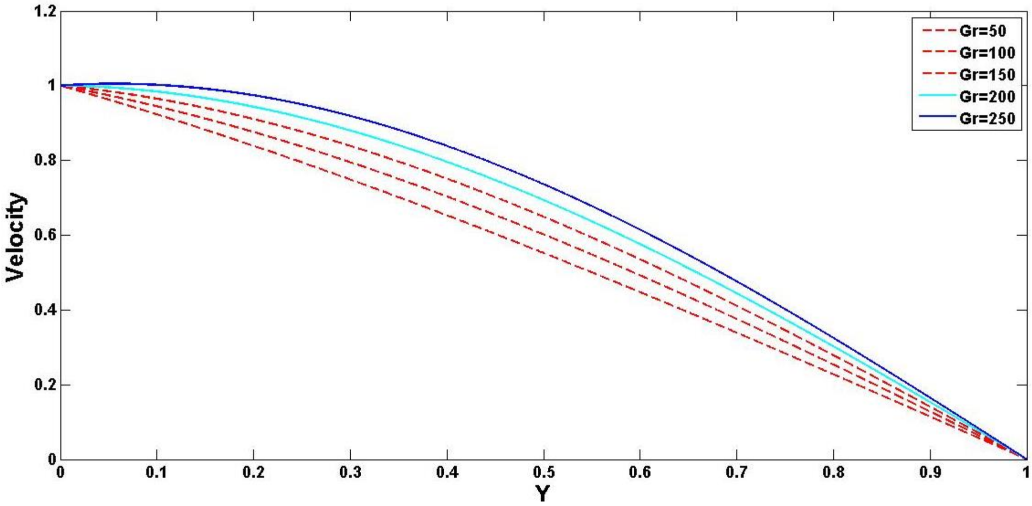

Figure 7. Effect of Gr on velocity

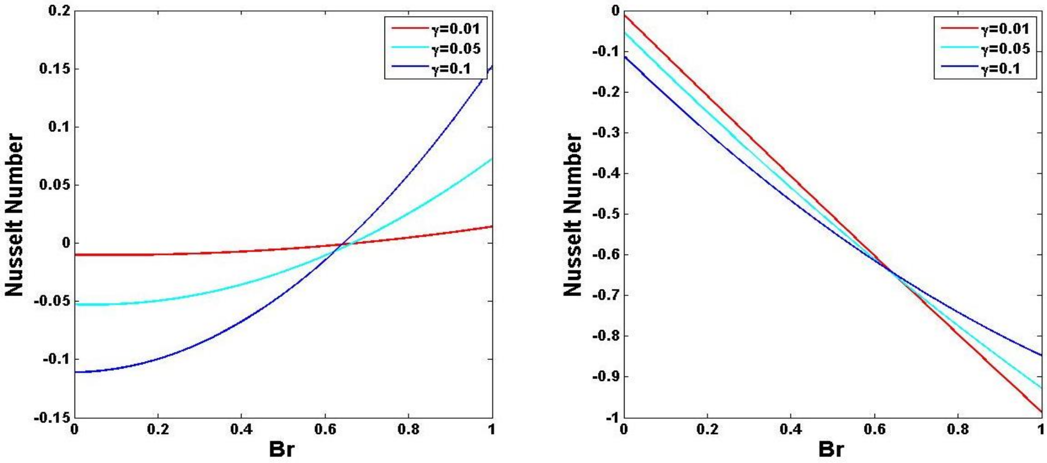

Figure 8. Effect of γ versus Br on Nusselt number

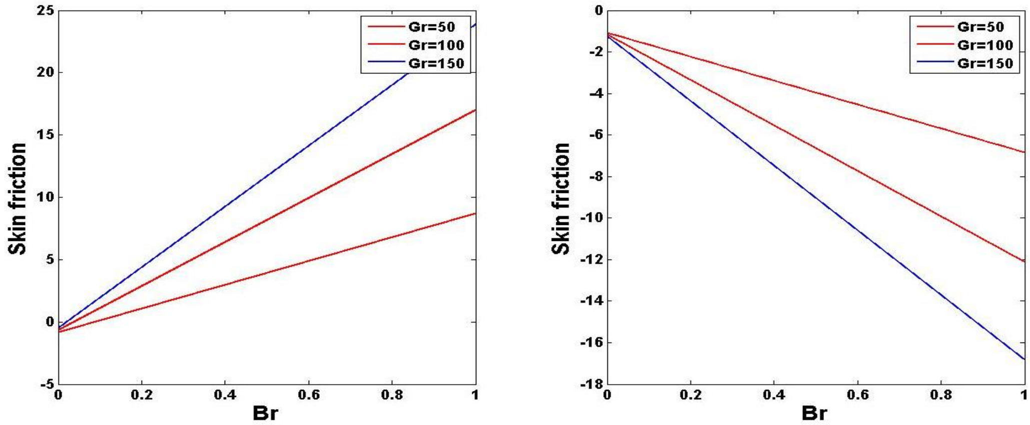

Figure 9. Effect of Gr versus Br on skin friction

Conclusion

The current study looked at the consequences of viscous dissipation on steady state Couette flow and Newtonian heating on an incompressible electrically-conducting fluid, moving vertically through two heated parallel vertical plates. Homotopy perturbation method was employed to generate the steady state solutions for temperature, velocity, rate of heat transfers and sheer stress. The influence of pertinent parameter dictating the flow configuration is discussed in detail using various plots. It is concluded from this study that:

(i) Raising the Newtonian heating parameter leads to a substantial growth in the temperature profile and the fluid acceleration respectively.

(ii) As the Brinkman number parameter Br, goes up, the temperature and the velocity of the fluid is significantly improved.

(iii) By uplifting the values of Br, the heat transfer rate is encouraged at the no-slip plate, whereas at the super-hydrophobic surface, the trend is the opposite.

(iv) The employed method, homotopy perturbation, demonstrates an excellent potential in respect to accuracy and convergence for simulating fluid flow

(v) The function of increasing the Brinkman number is to speed up the skin friction at the plate y = 1 while the reverse phenomenon happens at y = 0.

References

Ajibade A. O., Umar A. M. and Kabir T. M. (2021). An analytical study on effects of viscous dissipation and suction/injection on a steady mhd natural convection couette flow of heat generating/absorbing fluid, *Advances in Mechanical Engineering*, 13(5), 1–12

Ali, A. O., Makinde, O. D. and Nkansah-Gyekye, Y. (2016). Numerical study of unsteady MHD Couette flow and heat transfer of nanofluids in a rotating system with convective cooling, *International Journal of Numerical Methods for Heat and Fluid Flow*, 26, 12-20

Ali, Y. Rana, M. and Shoaib, M. (2017). Magneto-hydrodynamic three-dimensional Couette flow of a Maxwell fluid with periodic injection/suction, *Mathematical Problems in Engineering*, vol. 2017, Article ID 1859693, 19 pages.

Al-Subaie, M. A. and Chamkha, A. J. (2004). Transient natural convection flow of a particulate suspension through a vertical channel, *Heat and Mass Transfer,* 40(9): 707–713.

Anyanwu, E. O. Olayiwola, R. O. Shehu, M. D. and Lawal, A. (2020). Radiative e1ects on unsteady MHD Couette flow through a parallel plate with constant pressure gradient, *Asian Research Journal of Mathematics*, 1–19.

Barikbin, Z. Ellahi, R. and Abbasbandy, S. (2014). The Ritz-Galerkin method for MHD Couette flow of non-Newtonian fluid, https://www.sid.ir/en/journal/ViewPaper.aspx?id= 389767.

Beg, O. A. Ghosh, S. K. and Narahari, M. (2010). Mathematical modeling of oscillatory MHD Couette 5ow in a rotating highly permeable medium permeated by an oblique magnetic field, *Chemical Engineering Communications*, 198(2), 235–25.

Das, M., Mahato, R. and Nandkeolyar, R. (2015). Newtonian heating eff ect on unsteady hydromagneticcasson fluid flow past a flat plate with heat and mass transfer, *Alexandria Engineering Journal*, 54(4), 871–879.

Hamza M. M. (2016). Free convection slip flow of an exothermic fluid in a convectively heated vertical channel, *Ain Shams Engineering journal*, http://dx.doi.org/ 10.1016/j.asej. 2016.08.011

Hayat, T., Hussain, Z., Farooq, M. and Alsaedi, A. (2018). Magnetohydrodynamic flow of powelleyring fluid by a stretching cylinder with Newtonian heating, *Thermal Science*, 22(1B), 371–382.

Hussain, Z. Hussain, S. Kong, T. and Liu, Z. (2018). Instability of MHD Couette flow of an electrically conducting fluid, *AIPAdvances*, 8(10), Article ID 105209.

Jha, B. K. Isah, B. and Uwanta, I. (2015). Unsteady MHD free convective Couette 5ow between vertical porous plates with thermal radiation, *Journal of King Saud University Science*, 27(4), 338–348.

Job V. M. and Gunakala, S. R. (2016). Unsteady MHD free convection Couette flow between two vertical permeable plates in the presence of thermal radiation using galerkin’s finite element method, *International Journal of Mechanical Engineering*, 2, 99–110.

Joseph, K. Daniel, S. and Joseph, G. (2014). Unsteady MHD Couette flow between two in6nite parallel porous plates in an inclined magnetic 6eld with heat transfer, *International Journal of Mathematics and Statistics Invention*, 2, 103–110.

Jumanne Mng'ang'a (2023). Effects of Chemical Reaction and Joule Heating on MHD Generalized Couette Flow between Two Parallel Vertical Porous Plates with Induced Magnetic Field and Newtonian heating/cooling, *International Journal of Mathematics and Mathematical Sciences*, Volume 2023, Article ID 9134811, 14 pageshttps://doi.org/10.1155/2023/9134811

Khan D., Kumam, P. A., Rahman *et al.* (2022). The outcome of Newtonian heating on Couette flow of viscoelastic dusty 5uid along with the heat transfer in a rotating frame: second law analysis, *Heliyon*, 8(9), Article ID e10538.

Lesnic, D., Ingham, D., Pop, I. and Storr, C. (2004). Free convection boundary-layer flow above a nearly horizontal surface in a porous medium with Newtonian heating, *Heat and Mass Transfer*, 40(9), 665–672.

Merkin J.H. (1994). Natural convection boundary layer flow on a vertical surface with Newtonian heating, *International Journal for Heat and Fluid Flow*, 15, 392-398.

Mosayebidorcheh, S. Mosayebidorcheh, T. Hatami, M. Ganji, D. D.and Mirmohammadsadeghi, S. (2015). Investigation of transient MHD Couette 5ow and heat transfer of dusty fluid with temperature-dependent properties, *Journal of Applied Fluid Mechanics*, 8(4), 921–929.

Narahari, M. (2012). Transient free convection flow between long vertical parallel plates with ramped wall temperature at one boundary in the presence of thermal radiation and constant mass diff usion. *Meccanica*, 47(8), 1961–1976.

Ngiangia A. T. and Okechukwu, A. (2016). Influence of variable electroconductivity and radiation on MHD Couette flow, *World Scientific News*, 47, 241–253.

Omokhuale, E. and Jabaka, M. L. (2022). Unsteady Convective Couette flow with heat sink and radiation effects. *FUDMA Journal of Sciences,* 6(1), 266 – 277. Doi:10.33003/fjs-2022-0601-873

Omokhuale, E. and Jabaka, M. L. (2022). Unsteady convective Couette flow with heat source. *International Journal of Science for Global Sustainability,* 8(1), 25 – 37. Doi/10.57233/ijsgs.v8i1.317.

Omokhuale E. and Dange M. S. (2023). Natural convection Couette flow in the presence of magnetic field and thermal property. *International Journal of Science for Global Sustainability,* 9(2), 10 – 20. Doi.org/10.57233/ijsgs.v9i2.453.

Qayyum, S., Hayat, T., Shehzad, S. A. and Alsaedi, A. (2017). Eff ect of a chemical reaction on magnetohydrodynamic (mhd) stagnation point flow of walters-b nanofluid with Newtonian heat and mass conditions, *Nuclear Engineering and Technology,* 49(8), 1636–1644.

Rajput, U. and Sahu, P. (2011). Combined eff ects of chemical reactions and heat generation/absorption on unsteady transient free convection mhd flow between two long vertical parallel plates through a porous medium with constant temperature and mass diff usion, *Elixir Appl. Math.* 39, 4855–4859.

Rajput, U. and Sahu, P. (2011). Transient free convection mhd flow between two long vertical parallel plates with constant temperature and variable mass diff usion, *Journal of Math. Analysis,* 34(5), 1665–1671.

Raju, R. S., Jithender Reddy, G. Rao, J. A. and Rashidi, M. M. (2016): thermal diffusion and diffusion thermo effects on an unsteady heat and mass transfer magneto-hydrodynamic natural convection Couette flow using FEM, *Journal of computational Design and Engineering*, 3(4), 349–362.

Shehzad, S., Hayat, T., Alhuthali, M. and Asghar, S. (2014). Mhd three-dimensional flow of Jeff rey fluid with Newtonian heating, *Journal of Central South University*, 21(4), 1428–1433.

Singh, A. and Sarveshanand M. (2015). Magnetohydrodynamic free convection between vertical parallel porous plates in the presence of induced magnetic field, *SpringerPlus,* 4(1), 333.

Sreekala L. and Reddy, E. K. (2014). Steady MHD Couette 5ow of an incompressible viscous fluid through a porous medium between two infinite parallel plates under the effect of inclined magnetic field, *International Journal of Engineering Science,* 3, 18–37.

Umavathi, J. C. Liu, I. C. and Kumar, J. P. (2010). Magneto-hydrodynamic Poiseuille-Couette flow and heat transfer in an inclined channel, *Journal of Mechanics*, 26(4), 525–532.

Zin, N. A. M., Khan, I. and Shafie, S. (2018). Exact and numerical solutions for unsteady heat and mass transfer problem of Jeff rey fluid with mhd and Newtonian heating eff ects, *Neural Computing and Applications,* 30(11), 3491–3507.

Zulkiflee F., Mohamad A. Q., Shafie S. and Khan A. (2019). Unsteady Free Convection Flow between Two Vertical Parallel Plates with Newtonian Heating, *Matematika MJIAM,* 35(2), 117–127.

Nomenclature

B_0 = Constant magnetic flux density [$kg/s^2.m^2$]
g=Gravitational acceleration [m/s^2]
h= Width of the channel [m]
C_pC_v=Specific heats at constant pressure and constant volume [$Jkg^{-1}K^{-1}$]
γ = Navier slip parameter
Br = Viscous Dissipation parameter
Nu=Dimensionless heat transfer rate
T=Dimensionless temperature of the fluid [K]
T_0=Reference temperature [K]
u=Dimensionless velocity of the fluid [ms^{-1}]
y= Dimensionless distance between plates
U_0=Reference velocity [ms^{-1}]

Greek letters

β =Thermal expansion coefficient [K^{-1}]
$\beta_t\beta_v$ =Dimensionless variables
μ =Variable fluid viscosity [$kgm^{-1}s^{-1}$]
k=Thermal conductivity [$m.kg/s^3.K$]
α =Thermal diffusivity [m^2s^{-1}]
γ_s = Ratios of specific heats (C_pC_v)
σ =Electrical conductivity of the fluid [s^3m^2/kg]
ρ = Density of the fluid [kgm^{-3}]
v =Fluid kinematic viscosity [m^2s^{-1}]