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MAGNETOHYDRODYNAMIC (MHD) CONVECTIVE FLOW IN A POROUS MEDIUM WITH SUCTION/ INJECTION AND DUFOUR EFFECTS

Salisu Saleh[1], Sheriff Abdulazeez[1] and Omokhuale Emmanuel[2]
[1]*Department of Mathematics, Faculty of Science, Zamfara State University Talata Mafara, Nigeria*
[2]*Department of Mathematics, Faculty of Science, Federal University Gusau, Zamfara State, Nigeria*
***Corresponding Author's Email**: salehsalisuagp@gmail.com*

Abstract
Here MHD convective flow with Dufour and suction/injection effects through a porous medium is explored. The equations governing the flow are: continuity, momentum, energy and mass equations that are modeled using partial differential equations (PDEs) in non-dimensionless form. The PDEs explains the unsteady flow of an incompressible electrically conducting fluid. Furthermore, the PDEs were transformed to dimensionless form employing suitable variables. Finite Difference Method (FDM) was used to derived approximations for the velocity, temperature and concentration. Numerical computations were performed to investigate and discuss the influence of physical parameters embedded in the fluid flow. It was noticed that the concentration, temperature and velocity of the fluid rises with increase in injection parameter and an opposite trend was found when suction parameter became significant. It is also seen that the momentum and thermal boundary layers becomes higher as the Dufour number increases.
***Keywords:** Suction/injection, porous medium, Dufour number, MHD, Finite Difference Method.*

Introduction

Suction/injection method was first introduced as one of the means for preventing or delaying boundary layer separation. Suction/injection is one of the methods of boundary layer control, which have the aim of reducing losses of energy in channels. Injection is defined as the administering a fluid in to a system as in the case of blood transfusion while in the case of suction is defined as the exclusion of fluid from a system. If the two runs at the same time, then the opposite sides of the plates are porous which allow the fluid to move in and out. Suction/injection of fluid channels has gained a special concerned due to its paramount applications of different field such as Science, engineering, petroleum drilling industries and food processing industries etc. (Usman *et al.,* 2023).

Magnetohydrodynamics is the phenomenon of fluid flow under the influence of a magnetic field. The magnetic field generally acts on fluids (plasma, blood, mercury, liquid metals, electrolytes) by the Lorentz force. This force increases the temperature and the concentration but decreases the speed of the flow. Magnetohydrodynamics finds its application in biomedicine, MHD generators, MHD pumps; nuclear reactors, electronic devices etc. However, it should be noted that convective mixed flow in presence of a magnetic field is a fundamental problem in several applications involving heat transfer by convection (Ketchate *et al.*, 2024). Adnan *et al.* (2024) considered heat and mass transfer analysis of magnetohydrodynamic (MHD) Carreau fluid flow along a stretching sheet in a permeable medium with impacts of melting condition, heat generation, double diffusion, and variable thermal conductivity. Rath & Nayak (2024) explored the transient magnetohydrodynamics gravity-driven flow of a viscous, incompressible, electrically conducting fluid past a permeable exponentially accelerated vertical plate in the presence of thermal radiation and suction. Due to several applications in different areas, the influence of Hall current, Dufour number, heat source parameter, and chemically reactive species diffusion are vital and are incorporated in the study, which is the novelty of their work. Their model equations were derived and resolved by utilizing an analytical technique called the Laplace transform procedure. Further studies on MHD can be found in (Omokhuale and Jabaka, 2020a; Omokhuale and Jabaka, 2020b).

Porous media has garnered significant attention in heat transfer, and it removes heat from reactors, heat exchangers, solar energy, etc. Over the past two decades, numerous researchers have dedicated their efforts to studying heat transfer phenomena specifically within porous media. A porous medium is a substance that contains pores, or spaces between solid materials through which liquid or gas can pass. Omowaye *et al.* (2015) presented an analytical method of solution to steady two-dimensional hydromagnetic flow of a viscous incompressible, electrically conducting fluid past a semi-infinite moving permeable plate embedded in a porous medium. It is assumed that the fluid properties are constant except for the fluid viscosity which vary as an inverse linear function of temperature. The boundary layer equations were transformed in to a coupled ordinary differential equation with the help of similarity transformations. They solved the resulting coupled ordinary differential equations were solved using the Homotopy Analysis Method (HAM). The influence of Joule heating and induced magnetic field on magnetohydrodynamics generalised Couette flow of Jeffrey fluid in an inclined channel through a porous medium with Soret and Dufour effects has been investigated. The mathematical model in Cartesian coordinate system takes into account the effect of viscous dissipation. The nonlinear partial differential equations governing the flow are approximated numerically using the finite difference method. The profiles of the flow variables such as velocity, induced magnetic field, thermal field and concentration field are studied and the results are presented through graphs (Mng'ang'a and Richard Onyango, 2024).

The relationship between fluxes and driving materials grows more complicated while heat and mass transfer become obvious simultaneously via MHD Casson fluid. The diffusion-thermo (Dufour) effect emerges from the concentration gradient and produces an energy flux in addition to temperature gradients. Equivalently, the thermal-diffusion (Soret) effect expedites mass flux in response to temperature gradients. Although, the Soret-Dufour effects are typically neglected in many heat and mass transfer examinations owing to their smaller magnitude related to Fourier and Fick’s laws. However, Soret-Dufour effects can play a decisive role in the fields such as petrology, chemical engineering, geosciences, geothermal energy, and nuclear actor research. For instance, the Soret effect is practiced in isotope severance between the mixtures with gases of different molecular weights (Reddy, 2024). Ahmad *et al.* (2024) studied Non-similar solutions for radiative bioconvective flow with Soret and Dufour impacts. Ahmed & Gohain (2024) researched on Heat and Mass Transfer Analysis of a Three-Dimensional MHD Convective Flow with Sinusoidal Suction in the Presence of Radiation Absorption and Diffusion Thermo Effect. They highlighted that both radiation absorption and the Dufour effect contribute to the enhancement of shear stress and heat transfer rate at the plate. Furthermore, it was established that increasing the Reynolds number decelerates the momentum transfer rate while enhancing both the heat and mass transfer rates.

The finite difference method (FDM) stands as one of the most commonly employed numerical techniques, particularly in the context of fluid flow problems. FDM replaces derivatives in the governing equations with discrete difference approximations, utilizing the values of the solution at discrete mesh points within the study domain, a process known as discretization. This discretized representation is systematically utilized to formulate algebraic systems of equations, relying on unknown values at mesh points. However, a notable limitation of this approach arises when dealing with complex domains, as it may lead to inaccuracies, although such challenges can be mitigated through the utilization of coordinate transformation techniques (Khan *et al.*, 2024). Some researches using FDM can be seen in (Omokhuale and Dange, 2023a; Omokhuale and Dange, 2023).

The aim of the present analysis is to study Unsteady MHD convective flow through porous medium with effects of Dufour and suction/injection. Equations of momentum, energy and diffusion, which govern the fluid flow, heat and mass transfer are solved by using FDM. The effects of various physical parameters on fluid velocity, temperature, concentration are derived, discussed and shown through graphs.

Mathematical Formulation

We considered an unsteady flow of an incompressible, viscous, electrically conducting and heat-absorbing fluid through a semi-infinite vertical plate. A uniform transverse magnetic field of magnitude B_0 is applied in the presence of temperature and concentration buoyancy effects in the direction of the $y^* -$ axis. The transversely applied magnetic field and magnetic Reynolds number is assumed to be very small so that the induced magnetic field is negligible. We assumed that there is no applied voltage which implies the absence of an electric field. The physical variables are independent of the x^* $-$axis. We further assume that there exist a homogeneous first order chemical reaction between the diffusing species and the fluid. The presence of constant injection/suction is considered.

The governing equations under the usual Boussinesq's and boundary layer approximations are:

Continuity equation:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + g\beta(T^* - T^*_\infty) + g\beta^*(C^* - C^*_\infty) + \upsilon\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\upsilon}{K^*}u^* - \frac{\sigma {B_0}^2}{\rho}u^* \tag{2}$$

Energy equation:

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{k_T}{\rho Cp}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{Q_0}{\rho Cp}(T^* - T^*_\infty) + \frac{D_m k_T}{C_s C_p}\frac{\partial^2 C^*}{\partial y^*} \tag{3}$$

Mass Equation:

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D_m\frac{\partial^2 C^*}{\partial y^{*2}} - K^*_c(C^* - C^*_\infty) \tag{4}$$

The initial and boundary conditions associated with the flow are given as:

$$u^* = 0, T^* = T^*_w, C^* = C^*_w \text{ at } y^* = 0 \tag{5}$$

$$u^* \to 0, T^* \to T^*_\infty, C^* \to C^*_\infty \text{ as } y^* \to \infty \tag{6}$$

where x, y are the distances along and perpendicular to the plate, respectively, u, v are the components of velocities along the perpendicular plates, respectively, ν is the kinematic viscosity, g is the acceleration due to gravity, ρ is density, β is the coefficient of volume expansion with temperature, β^* is the volumetric coefficient of expansion with concentration, T^*, T^*_w, T^*_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, C^*, C^*_w, C^*_∞ are the corresponding concentrations, K^* is the Darcy permeability, σ is electric conductivity, C_p is the specific heat at constant pressure, k_T is the thermal diffusion ratio, c_s is the concentration susceptibility, $Q_0(T^* - T^*_\infty)$ is assumed to be the amount of heat generated or absorbed per unit volume, Q_0 is a constant which may take either positive or negative values, D_m is the coefficient of mass diffusivity.

where $v^* = \pm v_0$ from equation (1) and $-\frac{1}{\rho}\frac{\partial p}{\partial x} = \lambda$.

$v_0 > 0$ signifies suction and $v_0 < 0$ explains injection.

In order to derive dimensionless form of the governing equations and the boundary condition. The following dimensionless quantities in equation (7) are introduced into equations (1) to (4).

$$y = \frac{U_0 y^*}{\upsilon}, u = \frac{u^*}{U_0}, t = \frac{t^* U_0^2}{\upsilon}, \eta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \xi = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*} \quad (7)$$

Therefore, equations (1) to (4) become:

$$\frac{\partial u}{\partial t} \pm \phi \frac{\partial u}{\partial y} = \lambda + G_r \eta + G_m \xi + \frac{\partial^2 u}{\partial y^2} - \frac{1}{D_a} u - Mu \quad (8)$$

$$\frac{\partial \eta}{\partial t} \pm \phi \frac{\partial \eta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \eta}{\partial y^2} - \gamma \eta + Du \frac{\partial^2 \xi}{\partial y^2} \quad (9)$$

$$\frac{\partial \xi}{\partial t} \pm \phi \frac{\partial \xi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \xi}{\partial y^2} - \delta \xi \quad (10)$$

The corresponding boundary conditions (5) and (6) in dimensionless form are:

$$u = 0, \eta = 1, \xi = 1 \; at \; y = 0 \quad (11)$$

$$u = 0, \eta = 0, \xi = 0 \; as \; y \to \infty \quad (12)$$

Here G_r is the Grashof number for heat transfer, G_m is the Grashof number for mass transfer, γ is the heat absorption, λ is the pressure term, D_a is the Darcy permeability parameter, M is the Magnetic field parameter, Pr is the Prandtl number, Sc is the Schmidt number, δ is the chemical reaction parameter, ϕ is suction or injection parameter, Du is Dufour number, u, η, ξ, t are the dimensionless velocity, temperature, concentration and time, respectively.

where $\delta = \frac{K_c^* \upsilon}{U_0^2}, \phi = \frac{v_0}{U_0}, Pr = \frac{\mu C_p}{k_T}, \gamma = \frac{Q_0 v}{\rho C_p U_0^2}, Du = \frac{D_m k_T C_w^* - C_\infty^*}{C_s C_p v (T_w^* - T_\infty^*)}, Gr = \frac{g\beta v (T_w^* - T_\infty^*)}{U_0^3}$,
$Gm = \frac{g\beta v (C_w^* - C_\infty^*)}{U_0^3}, D_a = \frac{K^* U_0^2}{v^2}, M = \frac{\sigma B_0{}^2 v}{\rho U_0^3}$

Numerical Solution

The unsteady, nonlinear coupled equations (8) to (10) with conditions (11) and (12) are solved by an explicit finite difference scheme. Consider a rectangular region with y varying from 0 to $y_{max} = 6$, where y_{max} corresponds to $y = \infty$ at which lies well outside the boundary layers. The region to be studied in (y, t) space is covered by a rectilinear grid with sides parallel to axes with Δy and Δt, the grid spacing in y and t directions, respectively. The grid points (y, t) are given by

$(i\Delta y, k\Delta t)$. The finite difference equations corresponding to equations (8) to (10) are as follows:

equations (8) to (10) are non-linear and coupled PDEs. In order to obtain solutions for the velocity, temperature and concentration of the fluid, we assume solution of the form:

$$\frac{u_i^{k+1} - u_i^k}{\Delta t} - \phi \frac{u_i - u_{i-1}}{\Delta y} = \lambda + G_r \eta_i + G_m \xi_i + \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta y^2} - \frac{1}{Da} u_i - M u_i \quad (13)$$

$$\frac{\eta_i^{k+1} - \eta_i^k}{\Delta t} - \phi \frac{\eta_i - \eta_{i-1}}{\Delta y} = \frac{1}{Pr} \frac{\eta_{i+1} - 2\eta_i + \eta_{i-1}}{\Delta y^2} - \gamma \eta_i + Du \frac{\xi_{i+1} - 2\xi_i + \xi_{i-1}}{\Delta y^2} \quad (14)$$

$$\frac{\xi_i^{k+1} - \xi_i^k}{\Delta t} - \phi \frac{\xi_i - \xi_{i-1}}{\Delta y} = \frac{1}{Sc} \frac{\xi_{i+1} - 2\xi_i + \xi_{i-1}}{\Delta y^2} - \delta \xi_1 \quad (15)$$

The corresponding initial and boundary conditions (11) and (12) become:

$$u_i = 0, \eta_i = 1, \; \xi_i = 1 \; at \; y = 0 \quad (16)$$

$$u_i = 0, \eta_i = 0, \xi_i = 0 \; as \; y \to \infty \quad (17)$$

The coefficient appearing in difference equations are treated as constants. The finite-difference equations at every internal nodal point on a particular n-level constitute a tri-diagonal system of equations. These equations are solved by using the Thomas Algorithm. Computations are carried out until the steady-state solution is reached when the absolute difference between the values of velocity, temperature and concentration at three consecutive time steps are less than 10^{-8} at all grid points.

Discussion of Results

The effects of various flow parameters associated with the flow and transport properties are examined by assigning some specific values. The results are demonstrated from Figures 4.1 to 4.12.

Figures 4.1 and 4.2 show the effects of thermal and mass Grashof numbers on the velocity profiles. It is seen that there is rise in the velocity because of the enhancement of thermal buoyancy force. Also, the peak value of the velocity is higher near the porous plate and decays smoothly for free steam velocity. It is further observed that the fluid velocity increases and the peak value is more distinctive due to significance in species buoyancy force. Thus, the velocity distribution attains a distinctive maximum value in the vicinity of the plate and then reduces properly to approach free stream value. The effect of Dufour number on the velocity profiles is as illustrated in Figure 4.3. It is observed that the velocity of the fluid rises with increasing Dufour numbers.

It is clear from Figures 4.4 and 4.5 that higher values of injection and suction parameters cause rise in the fluid velocity and a reverse trend is seen as suction becomes significant, respectively. Therefore, it indicates that there is an enhancement in the fluid velocity as injection progresses. This is because suction/injection parameter controls the fluid flow.

Figure 4.6 depicts the effect of injection on the temperature profiles. It is shown that the temperature becomes higher as the values of injection parameter increases. An opposite is observed as presented in Figure 4.7 where increase in suction reduces the momentum boundary layer. In case of suction the fluid at ambient conditions is brought closer to the surface and reduces the thermal boundary layer thickness. The same principle used but in opposite direction in case of injection. Figure 4.8 displays the effect of Dufour on the energy boundary layer. It is observed that the temperature of the fluid increases as Dufour number is higher. An increment in the Dufour number indicates a comprehensive rise in concentration gradient over temperature gradient. Hence, increasing concentration gradient upsurges the temperature field. Effect of γ on the temperature profiles is shown in Figure 4.9. It is found that the temperature reduces as γ is increased. Figure 4.10 shows the effect of chemical reaction on the concentration of the fluid. It is clear that the concentration becomes lower as chemical reaction is increased. The logic is that as chemical reaction increases, the quantity of solute molecules undergoing them to grows, this causes concentration to drop, which in turn reduces the like hood of destructive chemical reaction.

The effect of both suction and injection parameters on the concentration of the fluid are illustrated in Figures 4.11 and 4.12 respectively. It is noticed that the mass boundary layer lower as suction is significant and the reverse is seen as injection increases

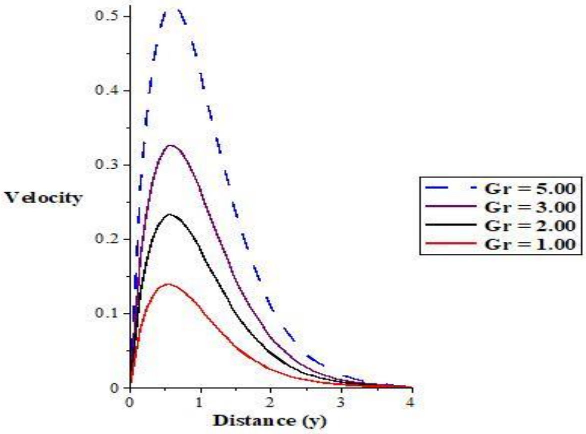

Figure 4.1: Effect of Gr on the velocity profiles

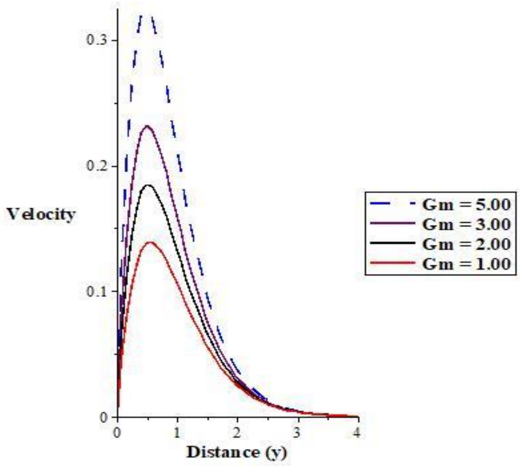

Figure 4. 2: Effect of Gm on the velocity profiles

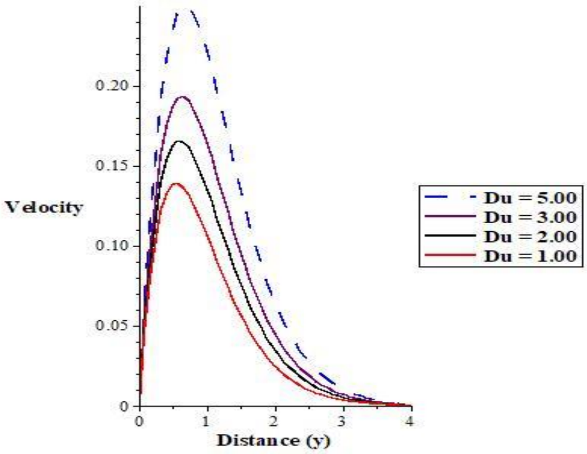

Figure 4. 3: Velocity profiles for different values of Du.

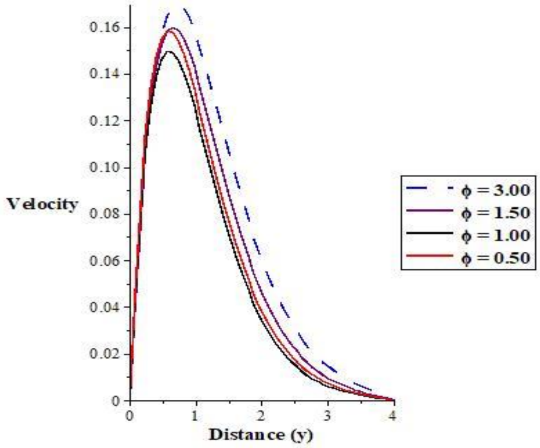

Figure 4. 4: Effect of injection, ϕ on the velocity profiles.

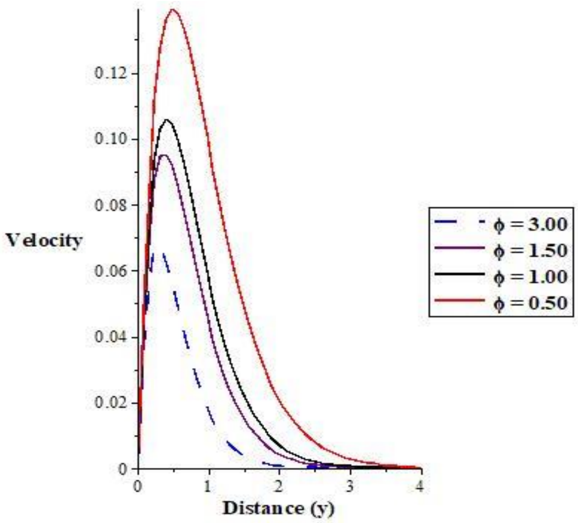

Figure 4. 5: Effect of suction, ϕ on the velocity profiles

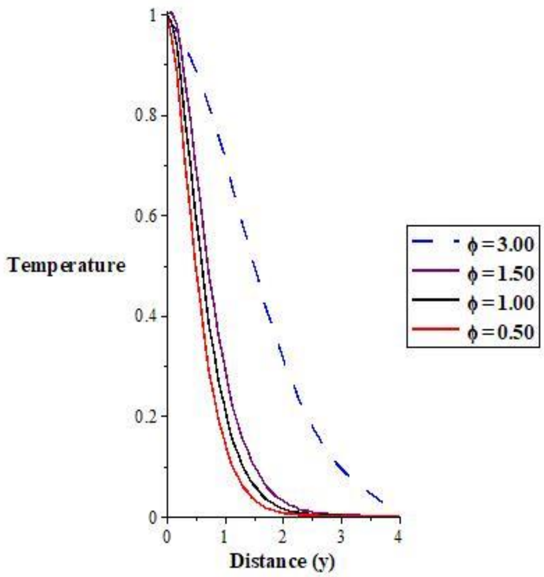

Figure 4.6: Effect of injection, ϕ on the temperature profiles

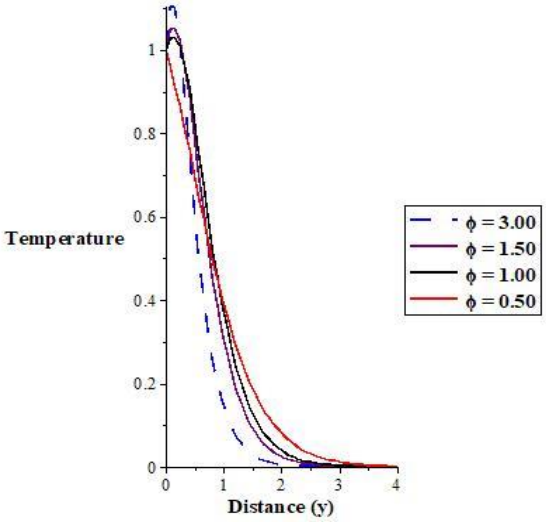

Figure 4.7: Effect of suction, ϕ on the temperature profiles

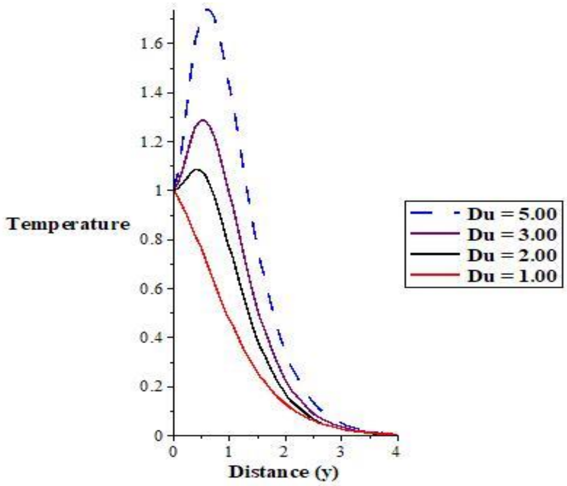

Figure 4.8: Effect of Du on the temperature profiles

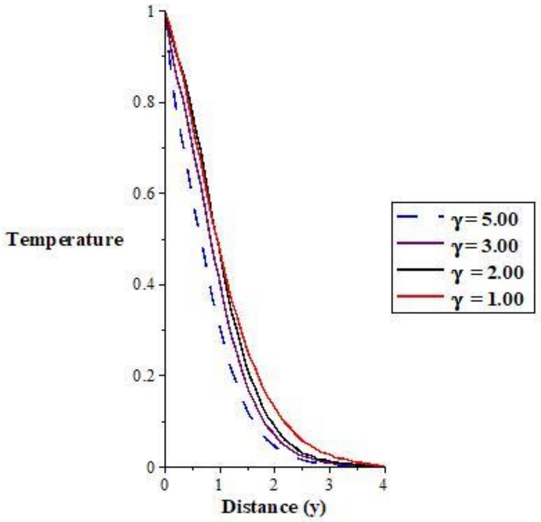

Figure 4.9: Effect of γ on the temperature profiles

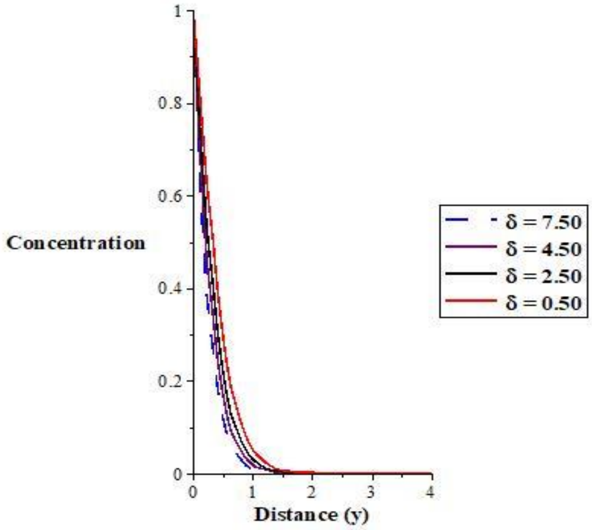

Figure 4.10: Effect of δ on the concentration profiles

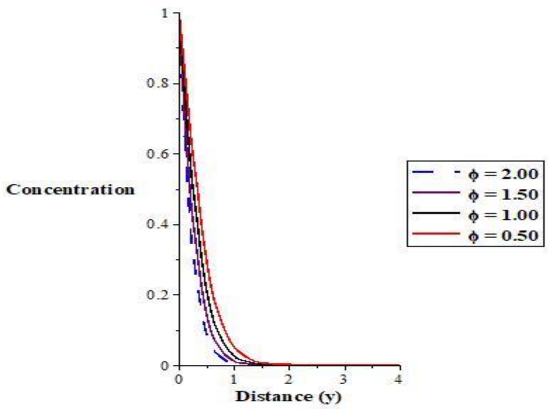

Figure 4.11: Effect of suction, ϕ on the concentration profiles

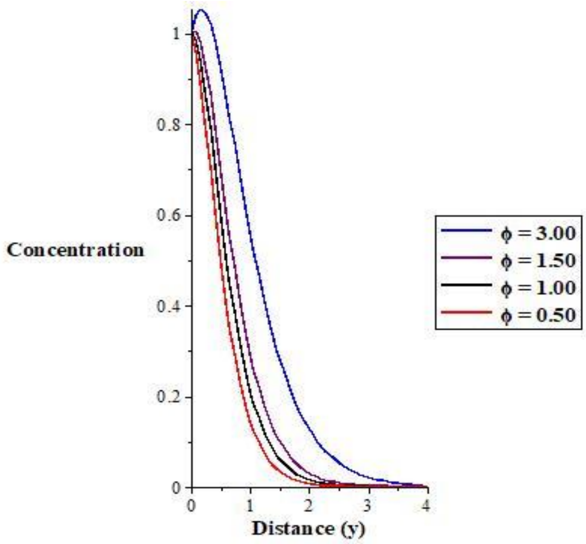

Figure 4.12: Effect of injection, ϕ on the concentration profiles

Conclusion

In this present study, an analytical investigation has been carried out to study MHD convective flow through a porous medium with Dufour and suction/ injection effects through a porous medium by using FDM. Numerical experiment is performed using MAPLE. Results were shown as Figures and discussed for physical parameters associated with the flow of the fluid. The following conclusion were drawn from the study:

1. Increase in Dufour parameter, mass and thermal Grashof numbers lead to rise in the Velocity of the fluid.
2. The momentum, energy and mass boundary layers become higher as injection is significant while an opposite trend is seen for increased values of suction parameter. In case of suction, the fluid at ambient conditions is brought closer to the surface and reduces the thermal boundary layer thickness. The same principle operates but in reverse direction in case of injection.
3. The temperature of the fluid falls for higher values of γ while a reverse tendency is noticed when Dufour parameter is increased.
4. It is observed that rise in the chemical reaction causes lowering of the mass boundary layer of the fluid.

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"TETF/DR&S/CE/UNIV/MAFARA/IBR/2023/VOL.I".

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