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DETERMINATION OF POTENTIOMETRIC SURFACE EQUATION AND HYDRAULIC GRADIENT OF ANAMBRA BASIN USING THREE-POINT SOLUTION TECHNIQUES

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Abstract
The study aimed at determination of potentiometric surface equation and hydraulic gradient of Anambra Basin using three-point solution techniques derived from calculated from vertical electrical sounding (VES) data. The study adopted scalar triple vector techniques in order to model the groundwater piezometric surface within the basin. A three-point estimation method was also used to determine the hydraulic gradient, and other groundwater flow characteristics. Two types of VES curve has been interpreted: A-type and AK-type curves. The result of the hydraulic gradient shows that the formation plane inclined towards the recharge areas on Udi hill outcrop go beyond the basin, which is at a much higher elevation than the discharge area. The result also depicts that the hydraulic head dips 6^0 from the Udi Cuesta towards the southwest region. Hydraulic gradient and hydraulic gradient vector were determined. The magnitude of the hydraulic gradient was found to be .004706 m/m, which depict the hydraulic potential for flow of the study area. The gradient of the direction of flow was 41.53º in the direction of the northeast-southwest (NE-SW). The result also depicts that the kinematic porosity of the study area is 30% and the kinematic seepage velocity value determined across the area is 0.0175m/day. With the result obtained from the kinematic seepage velocity and the gradient of the direction of flow, the study concludes that the groundwater flow at any point in the region can be ascertained.
Keywords: *VES Curve, Groundwater flow, Hydraulic Conductivity, Kinematic Seepage Velocity and Kinematic Porosity.*

Introduction

Water including groundwater is essential to life, and the quality of the water available to a community has a significant impact on that community's health. It is a necessary condition for the extensive socioeconomic development of any country (Gyan-Boakys, 1999, Anakwub a *et al*. 2021, Chinwuko *et al*., 2015). In every part of the world, including the Anambra State, access to and availability of drinkable water is a major concern. Surface water bodies such as streams, rivers, lakes, ponds, and other surface bodies of water are easily polluted (Anakwuba *et al*. 2021). Hence, the need of groundwater is essential considering its little or no pollution means. Every society, including the Anambra State, depends on water resources in various ways, including for domestic, commercial, agricultural, and construction purposes.

Water has historically been a major factor in the spread of disease among humans. Water can be used to spread typhoid fever, cholera, infectious hepatitis, bacillary and amoebic dysenteries, as well as a wide range of gastrointestinal illnesses. However, the seasonal nature of water-borne disease outbreaks emphasizes the ongoing significance of evaluating the quality of both private and public water supplies in any setting.

The only trustworthy source of water for drinking and irrigation purposes is groundwater, which also happens to be the most abundant source of fresh water (Anakwuba *et al*., 2014; Chinwuko *et al*., 2016). It is extremely significant as a source of high-quality, reasonably priced municipal and domestic water supply in developing countries' urban centres. Recent research has revealed that, in order to explore and exploit it, special care and expertise are required. Even though it is a renewable resource, extra care must be taken to decide where to develop it and how much to develop it in order to prevent over-exploitation, which could have negative effects. This geoelectric technique, which is a type of geophysical method, uses vertical electrical sounding in predicting aquifer attributes.

Furthermore, there is therefore need to understand the groundwater productive capacities in different parts of the basin, to be able to address the water demand/supply challenges. Good estimation of groundwater resources of the basin is a prerequisite for any sustainable water management plan in the study area. Most of the Government owned boreholes in parts of the study area are no longer in operation, and there is no knowledge of any source for a regional water supply scheme.

Location and Geology

The study area is located within the central part of the Anambra Basin, south eastern Nigeria. It is found within latitude 06^0 12'N to07^0 05'N and longitudes 07^0 01'E to 07^0 28'E. The total coverage area is approximately $3920km^2$ and has a total population of 2,429,210 million projected from 2006 population census. The basin is bounded on the eastern flank by the Udi Hills, the lower Benue Trough in the north, and by the Awka - Orlu Cuesta in the south, and in the west by a sprawling lowland underlain by shale of Nsukka and Imo Formations (Fig.1). The area is accessibly from the south and southwest by the Onitsha –Enugu expressway and from the northern part of the country through the Oturpko- Nsukka highway.

Two major groups of sediments occur in Anambra Basin namely; Nkporo Shale and the Coal Measures (Nwajide, 2013). The coal measures include; the Mamu, Ajali and Nsukka Formations (Table 1). The Mamu Formation, in the project area, underlies the whole sequence, occurring immediately below the Ajali Formation and overlying the Enugu Shales stretching into Enugu and environs (Nwajide, 2013). The Ajali Formation shows a significant variability in the environment of deposition. The formation is therefore highly permeable and readily recharged in its outcrop area, which forms an extensive bow round the edge of the Anambra Basin, stretching from the northern area of the lower part of the Benue Valley of Oturkpo. The Nsukka Formation consist lithologically, of alternating sequence of sandstones shales, sandy shales and a few coal seams. The Nsukka Formation is essentially an aquiclude, together with the Imo Formation, confining the Ajali aquifers to the west of the study area. The sandstone members as reported above, are aquiferious and have yielded artesian boreholes in some parts of the basin.

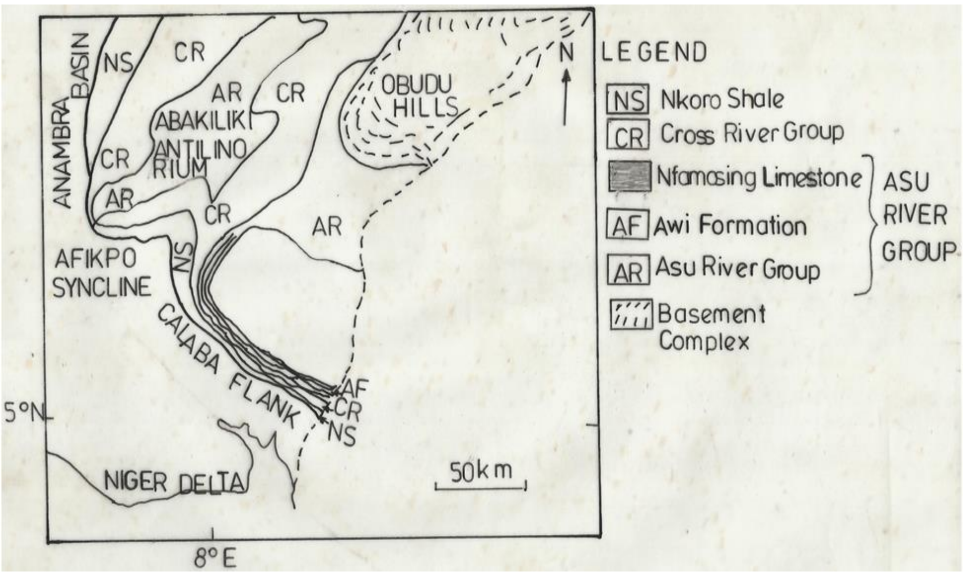

Fig. 1: Geologic map of Southeastern Nigeria

Table 1**:** Correlation Chart for Early Cretaceous Tertiary strata in the Southeastern Nigeria (After Nwajide, 1990).

PICK (m.y)	AGE	ABAKALIKI-ANAMBRA BASIN	AFIKPO BASIN
30	Oligocene	Ogwashi-Asaba Formation	Ogwashi-Asaba Formation
54.9	Eocene	Ameki/Nanka Formation/Nsugbe Sanastone	Ameki Formation
65	Paleocene	Imo Formation Nsukka Formation	Imo Formation Nsukka Formation
73	Maastrichtian	Ajalli Sandstone Mamu Formation	Ajalli Sandstone Mamu Formation
83	Campanian	Nkporo/Owelli Formation/Enugu Shale	Nkporo Shale/Afikpo Shale
87.5	Santonian	Non-deposition	
88.5	Coniacian	Awgu Group (Agbani Sandstone/Awgu Shale)	
93	Turonian	Ezeaku Group	Ezeaku Group (Including Amaseri Sandstone)
100	Cenomanian-Albian	Asu River Group	Asu River Group
119	Aptian Barremian Hauterivian	Unnamed Units	
Precambrian		Basement Complex	

Methodology

The theory of, and mathematical expressions used for exploration of groundwater by geoelectrical methods are well established (Keller and Frischknecht, 1966 and Keller, 1976). Mathematically, electrical current flow (J) in a conducting medium is governed by Ohm's law and groundwater flow in a porous medium, by Darcy's law, both having similar forms of equation:

$$j = -\sigma \frac{dV}{dh}, \qquad 1$$

$$q = -k \frac{dV}{dh}, \qquad 2$$

Where, J = current density electrical conductivity (Siemens/m = reciprocal resistivity, z

V- electrical potential (volts),q - specific discharge (discharge per unit area),

K - hydraulic conductivity (or permeability; m/s

h = head elevation of the water at any point in the spatial domain (m)

For homogeneous and isotropic medium, electric current and groundwater flow both satisfy the Laplace equation:

For electrical flow.

$$\frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr} = 0 \qquad 3a$$

And for groundwater

$$\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = 0 \qquad 3b$$

Where, V =electrical potential, V, (volts), r = distance r, (metres), T = transmissivity

The analogy between these two macroscopic phenomena is widely accepted (Freeze and Cherry, 1979). Thus, the electrical method provides a powerful analogue and tool for groundwater exploration and modelling, and may be useful e.g. in generating analytic flow nets.

3-Point Analysis Calculation from Resistivity Data

The Hydraulic gradient of planer piezometric surfaces (water table) were calculated from data-set involving the coordinates and depth to water table using electrical resistivity (VES) method. Different earth materials and conductive fluid respond differently during the passage of electric current (Nosal, 1983). The potential therefore varies with bed boundaries and with the nature of the conductive fluid across the boundaries. The analysis of the potential generated would give the apparent resistivity values. The resistance value of each electrode separation was converted to apparent resistivity by multiplying the resistance values by the geometric factor (determined from Schlumberger configuration). The geometric factor, k, function of the geometry of the arrangement is given by,

$$K = \pi\left[\frac{(AB/2)^2 - (MN/2)^2}{MN}\right] \qquad 4$$

Where (AB/2) = Current electrode spacing and (MN/2) = Potential electrode spacing

Then, the apparent resistivity which is measured in ohm meter is (ρ) expressed as;-

$$\rho = \left(\pi\left[\frac{(AB/2)^2 - (MN/2)^2}{MN}\right]\right) * R \qquad 5$$

The apparent resistivity data is plotted against current electrode spacing (AB/2) on a bi-logarithmicgraph in order to produce the geoelectric curve. The VES furnishes information concerning the vertical succession of different conducting zones and their individual thicknesses and resistivities. Calibration resistivity soundings (parametric soundings), were performed at the sites of existing boreholes where pumping tests were carried out for comparative purposes.

3-Point Problem Analysis with respect to Groundwater Potentiomtric

The distribution of hydraulic head and, hence, groundwater flow directions and rates often reveal basic characteristics of the groundwater flow field that control groundwater flow and the transport of any dissolved constituents. The orientation of a potentiometric surface and its gradient were achieved from the water table equation. The coordinates and depths to the water table from three electrical sounding points (VES) result, provided the three points on the plane.

The Hydraulic gradient of planer piezometric surfaces (water table) were calculated from data-set from electrical resistivity (VES) method. The equation of the water table is given thus;-

$$\mathbf{Z = H\ (x,y) = Ax + By + Cz - D = 0}; \quad \text{(Delvin, 2003)} \quad 6$$

The equation coefficients were used to calculate and determine the magnitude and direction of the hydraulic gradient. Where, Z is the hydraulic head.

In many situations where the water table or potentiometric surface can be represented as a plane (Fig. 2), three-point estimation methods was employed to modeling a planer water surface so that the hydraulic gradients and gradient flow direction could be estimated.

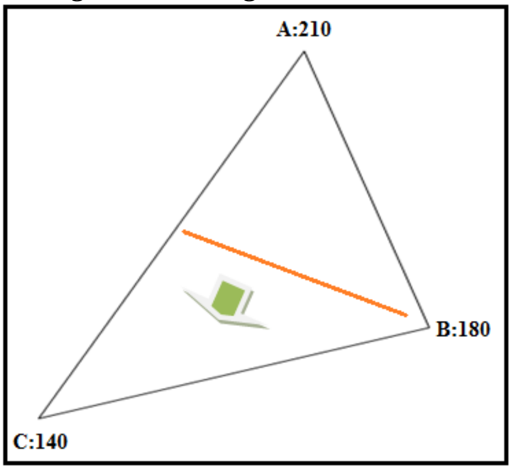

Fig. 2: Modeling of the piezometric surface using the scalar triple product

Result and Discussion

Geoelectric Curves

Three (3) Vertical Electrical Soundings (VES) were carried out at Nsukka. Ohodo, and Aku situated in two different local Government Areas, Nsukka and Udi. The VES curves were generated across the study area (Fig. 3). The VES datainterpretations are represented on Table 1.The VES results revealed that the study area consists of five to seven layers. Two types of VES curve has been interpreted and they are A-type andAK-type curves.

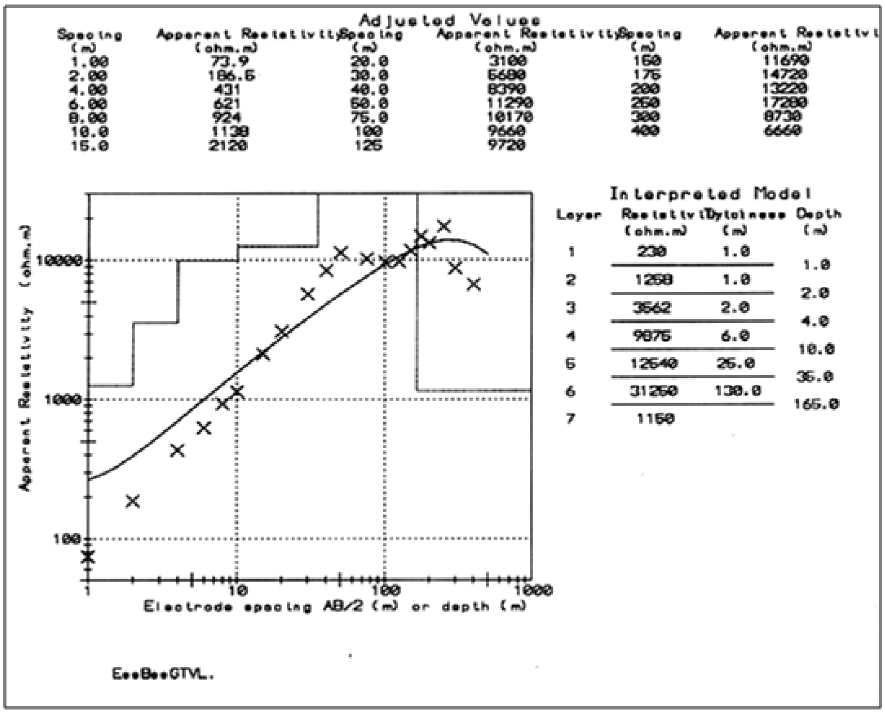

Fig. 3a: Geoelectric curve at Nsukka and Ohodo area

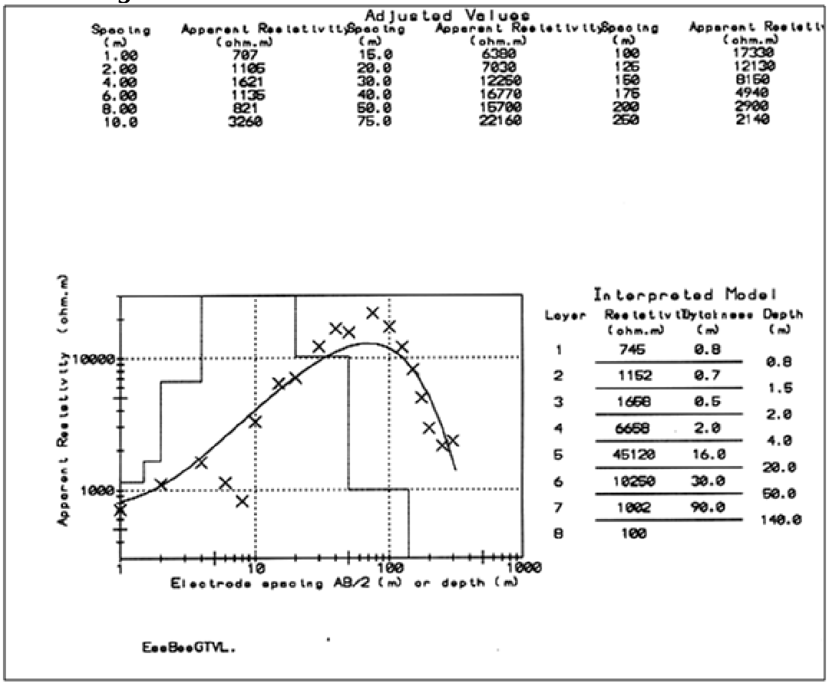

Fig. 3b: Geoelectric curve at Aku area

Table 1: VES Interpretation model

Station Location	Distance To The Location on x-axis	Distance to the Location on y-axis	Depth to Water Table from VES Corrected to mean sea level
Nsukka (A)	31818m	53455m	210m
Ohodo (B)	33727m	41682m	180m
Aku (C)	23227m	40727m	140m

Three-Point Solution Techniques

The orientation of a potentiometric surface and itsgradient were achieved from the water table equation. The coordinates and depths to the water table from three electrical sounding points (VES) result provided the three points on the plane. The plane have the following three data points, Nsukka, P_1(31818, 53455, 210), Ohodo, P_2(33727, 41682, 180), and Aku, P_3(23227, 40727, 140).The equation of the radius vector r, or position vectors P_1, P_2, P_3 and an arbitrary point P(x, y, z) follows;-

$$\vec{r}_1 = 31818i + 53455j + 210k$$

$$\vec{r}_2 = 33727i + 41682j + 180k$$

$$\vec{r}_3 = 23227i + 40727j + 140k$$

$$\vec{r} = xi + yj + zk$$

The principles from vector analysis were applied using the scalar triple products which were coplanar. The scalar triple product was therefore equated to zero and that defined the Hence the equation of the water table becomes parallelepiped which them would be flat and have no volume. This describes the potentiometric surface, a plane and provided the data.

With reference to P_1 we determine the position vectors, PP_1, P_2P_1, P_3P_1, $PP_1 = (r - r_1)$

$Pp_2 = (r - r_2)$

$PP_3 = (r - r_3)$

But

$$(\vec{r} - \vec{r}_1) \cdot (\vec{r}_2 - \vec{r}_1) \times (\vec{r}_3 - \vec{r}_1) = 0$$

Hence the equation of the water table becomes

$$(\vec{r} - \vec{r}) \cdot (\vec{r}_2 - \vec{r}_1) \times (\vec{r}_3 - \vec{r}_1) = \begin{bmatrix} 31818 & 53455 & 210 \\ 1909 & -11773 & -30 \\ -8591 & -12728 & -70 \end{bmatrix}$$

Simplifying

$125439595\mathbf{z} = 442270\mathbf{x} + 391360\mathbf{y} - 8652000000 = 0$

Water table surface equation

$$\vec{Z} = 0.03526x + 0.031199y$$

$$|\nabla z| = (3.526 \times 10^{-3})^2 + (3.1199 \times 10^{-3})^2$$

Magnitude of the hydraulic gradient

$$= 4.706 \times 10^{-3} \text{ m/m}$$

$$\text{Tan } \theta^{-1} = 41.53^0$$

Gradient Vector Of The Function

The gradient of the flow direction

Conclusions

The potentiometric surface orientation of the study area and its gradient have been achieved from the water table equation and the necessary principles from vector analysis were applied using the scalar triple product. The magnitude of hydraulic gradient was calculated to be 4.706 $x10^{-3}$m/m which depict the hydraulic potential for flow of the study area, while the gradient of flow direction was 41.53^0 in the direction of the northeast-southwest (NE-SW). The study concluded that it is possible to ascertain where contaminants in the groundwater may be coming from and in what direction they are flowing, and the time it takes for the contaminant to get to any point.

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