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THE CHOICE OF CONFOUNDING IN A 3^K FACTORIAL DESIGN

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Abstract
When the numbers of treatments are greater than the available blocks in 3^k factorial experiments, confounding becomes very necessary. Confounding is necessary to reduce the block size as well as reduce the experimental errors. 3^2, 3^3 and $3^{k>3}$ factorial experiments were considered. In 3^2 experiments, the AB and AB^2 were recommended for confounding. Similarly, confounding ABC and ABC^2 are recommended since two main effects are found in a block which can be used for experimentation. Confounding ABC and AB^2C is also recommended since two main effects are found in a block which can be used for experimentation. On the contrary confounding ABC^2 and AB^2C^2 are not recommended since only one main effect is found in a block. Furthermore, in 3^3 factorial experiment in 3^2 blocks using ABC, ABC^2, AB^2C and AB^2C^2 as defining contrast, ABC and AB^2C^2 as well as ABC^2 and AB^2C should not be confounded since they aliased with the main effects when used during fractional replication. However, in 3^4 factorial experiments no main effect is aliased with (1)the lowest treatment. In generalized interactions, ABCD and $ABCD^2$ as well as ABCD and ABC^2D are recommended for confounding since three main effects are seen in a block. In contrast, the following pairs of interactions are not recommended for confounding because only two main effects are found in a block: $ABCD^2$ and ABC^2D; $ABCD^2$ and AB^2CD; ABC^2D and AB^2CD.
Keywords: *Confounding, interactions, Aliased, replications.*

Introduction

By 3^k factorial experiment we mean a factorial experiment involving k factors each of which occurs at 3 levels. The factors are represented by Latin letters A, B, C etc and the levels of each factor by 0, 1, 2 where 0 is the low level, 1 is the intermediate level and 2 is the high level.[1]
When the block size is smaller than the number of treatment combinations in one replicate in 3^k factorial experiment, confounding becomes necessary. Confounding is therefore a design technique for arranging a complete factorial experiment in blocks. In this technique, some information usually the higher order interactions are confounded in blocks. This means that information about the confounded interactions is lost. It is because of this that the main effects are not chosen for confounding. A 3^2 factorial design is the simplest design with 9 treatment combinations and 8 degrees of freedom. The higher the value of k, the higher the treatment combinations.Each main effect has two degrees of freedom, and every two-factor interaction has four degrees of freedom, these two factor interactions can be decomposed into four components interaction. While every three-factor interaction has eight degrees of freedom and can be decomposed into four components of interaction with two degrees of freedom each, [1].The 3^k factorial design may be confounded in 3^b balanced incomplete blocks, where b < k. In designing a 3^2 factorial design in 3^1 blocks, we shall have 9 treatment combinations and 3 blocks with 2 degrees of freedom. Similarly, if b = 3 we shall have 27 blocks with 26 degree of freedom and so on. The procedure is to select b independent effects to be confounded with blocks.By independent we mean that no effect chosen is the generalized interaction of the others; as a result, exactly $\frac{3^b-2b-1}{2}$ other effects are automatically confounded. These effects are the generalized interactions of those effects originally chosen, [1].
The choice of confounding for 2^k when k > 2 was considered by [2]. In his findings, the choice of confounding depends on the number of factors, the number of blocks and the block sizes and he recommended partial confounding for the interactions that cannot be confounded.

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Factorial experiment conducted in randomized complete block design was compared by [3], complete confounding, partial confounding and half fractional replication, using mean squares error to evaluate the results of the study. In his conclusion, the result of the analysis of variance showed that the fractional factorial designexhibited is the highest accuracy in estimating the effects and was the best in saving time and cost.

In construction of confounding for mixed factorial experiments[4] developed a method for constructing confounded asymmetrical factorial designs. In his paper, he also presented a single replicate confounded asymmetrical factorial design.
A method for identifying confounded effects in factorial designs was developed by [5], whether symmetrically or asymmetrically arranged in any simple block structure. This method is an extension of those commonly used for symmetrical factorial designs with prime number of levels and a single block category.

In confounding functions for use with binary outcomes [6] illustrated the approach with example. The implementation of the approach was done using strata and R softwares and in conclusion, states that the confounding function approach is a useful method for assessing the impact of unmeasured confounding.

The application of a three-factor factorial design to determine if there is a significant difference in the mean yield of maize in Nigeria with respect to the effect of fertilizers, herbicides and water volumes was examined by [7]. The research was done using 3^3 replicated factorial designs with 8 replicates per cell. Their result revealed that among others that there is significant difference in the fertilizers effects on the yield of maize with a significant value of 0.000.

This work is set to (i) confound an appropriate interaction for 3^2 factorial experiment in 3^1 blocks (ii) choose the appropriate generalized interaction in 3^3 factorial experiments in 3^2 blocks and (iii) choose the appropriate generalized interactions when k > 3.

Methodology

In this section, the method in confounding the 3^k factorial design in 3^b blocks of 3^{k-b} observations each where b<k will be discussed.

Albelian Algebra Module 3

We specify the b components of the interactions to be confounded in the block of size 3^{k-b}. An additional $\frac{1}{2}\left(3^b-2-1\right)$ generalized interactions are observed to be confounded in blocks.

The next stage is to construct the defining contrasts using the general expressions

$$\begin{aligned} L_1 &= \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_k x_k \\ L_2 &= \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k \\ &. \\ &. \\ &. \\ L_b &= \xi_1 x_1 + \xi_2 x_2 + \ldots + \xi_k x_k \end{aligned} \qquad (2.1)$$

Where, x_i is the i^{th} factor appearing in a particular treatment combination.

$\alpha_1, \beta_1, \ldots, \xi_1$ are the exponents appearing in the first and second generalized interactions respectively.Wehave $\alpha_i = 0,1,$ or 2; $\beta_i = 0,1,$ or 2; $\ldots\ \xi_i = 0,1,$ or 2 and $x_i = 0,1$. The treatment combinations with the same pair of values $\left(L_1, \ldots, L_b\right)$ are assigned to the same blocks.The treatments are closed with respect to addition module 3. This means that addition of any two treatments in the block gives a treatment already in that block. The content of the other block can also be generated by addition module 3 of the treatment that is not in the principal block to each of the treatment in the principal block.This method is the full factorial experiment for 3^k.

Effects and Interaction for 3^k Experiment

This subsection helps us for all possible confounding generalized interactions. For example, let k = 3 we shall have the following effects/interactions and the corresponding defining equations as shown in Table 2.1

Table: Effects and Interaction for 3^3 experiment

Effects/Interaction	**Defining equation**
A	x_1
B	x_2
$AB\begin{cases} AB \\ AB^2 \end{cases}$	x_1+x_2 x_1+2x_2
C	x_3
$AC\begin{cases} AC \\ AC^2 \end{cases}$	x_1+x_3 x_1+2x_3
$BC\begin{cases} BC \\ BC^2 \end{cases}$	x_2+x_3 x_2+2x_3
$ABC\begin{cases} ABC \\ ABC^2 \\ AB^2C \\ AB^2C^2 \end{cases}$	$x_1+x_2+x_3$ $x_1+x_2+2x_3$ $x_1+2x_2+x_3$ $x_1+2x_2+2x_3$

Depending on the 3^b blocks, the content of 3^{k-b} can be generated from the generalized interactions.

To choose the symbols in 3^k we shall have $\frac{1}{2}\left(3^k-1\right)$ symbols each represents two degrees of freedom.

In general, the 3^k experiment, representing

$A_1, A_2 ... A_k$ the $\frac{1}{2}\left(3^k-1\right)$ symbols can be written as $A_1^{\alpha_1}, A_2^{\alpha_2}, ..., A_k^{\alpha_k}$ where $\alpha_i = 0,1,2\left(i=1,2,...,k\right)$

For example, in 3^2 experiments, there will be 3 symbols and their defining equations will be of the form

$$\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = 0,1,2 \text{ module } 3$$

Fractional Replication of the 3^k Factorial Design

For larger value of k, a large number of runs will be needed. For this type of case, fractional replication of these designs will be needed.

If k = 3,4,5,,,, the larger fraction of the 3^k factorial design is one-third fractions that will have 3^{k-1} runs. The number of treatment combinations that will be needed for this fraction will be 9 for a proper and reasonable fractional replication.

Analysis of 3^k factorial Experiments.

The factors will be considered as if they are all quantitative and study their linear and quadratic effect and as well as their interactions, then finally regroup this for total effect and linear contrasts where appropriate.

The step by step algorithm for the 3^k factorial experiment is as follows: -

Step I: Column (I) contains the treatment combinations. The treatments are introduced one at a time, each level being combined in turn with every combination of the level above it.

Step II: The observations are put in column 2 against corresponding treatment combinations

Step III: Group the observations in 3s, first third of this column is made up of the sum of each of the three groups of the three observed value in the same group of the three. This estimates the linear components. The last third is the sum of the first and the third minus twice the second value. This estimates the quadratic components.

Step IV: Repeat step III using values in step III.

Step V: Column 5 designates the effects being estimated.

Step VI: Obtain the sum of squares for each effect by squaring corresponding entries in column 4 and dividing by appropriate coefficient contributing to these effects. The general formular for calculating the divisor is $2^p \times 3^q \times n$ where

> P is the number of factors in the interaction.
> q is the number of factors examined in the entire experiment minus the number of linear terms in the interaction.
> n is the number of replications.
> Also note that:
> 0 is the lowest level
> 1 is estimating the linear effects ‘
> 2 is estimating the quadratic effects.

Results and Discussions

Confounding AB and AB^2 in a factorial design of 3^2 in 3^1 blocks.

The linear equation for AB and AB^2 are as follows

$$AB = x_1 + x_2$$
$$AB^2 = x_1 + 2x_2$$

The treatment combination for 3^2 factorial experiment are $00, 01, 02, 10, 11, 12, 20, 21, 22$. From equation (2.1), the linear equations are given below.

$$00: L_1 = 0+0 = 0; \qquad 01: L_1 = 0+1 = 1;$$
$$02: L_1 = 0+2 = 2; \qquad 10: L_1 = 1+0 = 1;$$
$$11: L_1 = 1+1 = 2; \qquad 12: L_1 = 1+2 = 0;$$
$$20: L_1 = 2+0 = 2; \qquad 21: L_1 = 2+1 = 0;$$
$$22: L_1 = 2+2 = 1$$

The contents of the blocks and the corresponding treatment contents are

$$\begin{array}{cccccc} Block1 & Block2 & Block3 & Block1 & Block2 & Block3 \\ \begin{bmatrix} 00 \\ 12 \\ 21 \end{bmatrix} & \begin{bmatrix} 01 \\ 10 \\ 22 \end{bmatrix} & \begin{bmatrix} 02 \\ 11 \\ 20 \end{bmatrix} & \begin{pmatrix} 00 \\ A_L B_Q \\ A_Q B_L \end{pmatrix} & \begin{pmatrix} B_L \\ A_L \\ A_Q B_Q \end{pmatrix} & \begin{pmatrix} B_Q \\ A_L B_L \\ A_Q \end{pmatrix} \end{array}$$

From the treatment in the block, AB interactions can be confounded since in block 2, two main effects of B_L and A_L can be chosen for experimentation as well as block 3 where B_Q and A_Q are found.

Similarly, the block contents for AB^2 are presented below

$$\begin{array}{cccccc} Block1 & Block2 & Block3 & Block1 & Block2 & Block3 \\ \begin{bmatrix} 00 \\ 11 \\ 22 \end{bmatrix} & \begin{bmatrix} 02 \\ 10 \\ 21 \end{bmatrix} & \begin{bmatrix} 01 \\ 12 \\ 20 \end{bmatrix} & \begin{pmatrix} 00 \\ A_L B_l \\ A_Q B_Q \end{pmatrix} & \begin{pmatrix} B_Q \\ A_L \\ A_Q B_L \end{pmatrix} & \begin{pmatrix} B_L \\ A_L B_Q \\ A_Q \end{pmatrix} \end{array}$$

AB^2 can also be confounded since two main effects are found in block 2 and block 3.

Confounding ABC and ABC^2 in 3^2 blocks.

In this case, two interactions will be confounded and thereafter $\frac{1}{2}(3^b - 2b - 1)$ additional interactions will be confounded. The additional interaction is

$$ABC \times ABC^2 = (A^2B^2C^3)^2 = A^4B^4C^6 = AB$$

The linear equations are shown below

$$L_1 = X_1 + X_2 + X_3 = 0; \quad L_2 = X_1 + X_2 + 2X_3 = 1; \quad L_3 = X_1 + X_2 + 0 = 2$$

The standard order for 3^3 is 000, 100, 200, 010, 110, 210, 020, 120, 220, 001, 201, 011, 111, 211, 021, 121, 221, 002, 102, 202, 012, 112, 212, 022, 122, and 222.

Using equation (2.1)the block contents and the corresponding treatment contents are

$$\begin{array}{cccccc} Block1 & Block2 & Block3 & Block1 & Block2 & Block3 \\ \begin{bmatrix} 000 \\ 210 \\ 120 \end{bmatrix} & \begin{bmatrix} 100 \\ 010 \\ 220 \end{bmatrix} & \begin{bmatrix} 200 \\ 110 \\ 020 \end{bmatrix} & \begin{pmatrix} (1) \\ A_Q B_L \\ A_L B_Q \end{pmatrix} & \begin{pmatrix} A_L \\ B_L \\ A_Q B_Q \end{pmatrix} & \begin{pmatrix} A_Q \\ A_L B_L \\ B_Q \end{pmatrix} \end{array}$$

$$\begin{array}{cccccc} Block4 & Block5 & Block6 & Block4 & Block5 & Block6 \\ \begin{bmatrix} 001 \\ 211 \\ 121 \end{bmatrix} & \begin{bmatrix} 101 \\ 011 \\ 221 \end{bmatrix} & \begin{bmatrix} 201 \\ 111 \\ 021 \end{bmatrix} & \begin{pmatrix} C_L \\ A_Q B_L C_L \\ A_L B_Q C_L \end{pmatrix} & \begin{pmatrix} A_L C_L \\ B_L C_L \\ A_Q B_Q C_L \end{pmatrix} & \begin{pmatrix} A_Q C_L \\ A_L B_L C_L \\ B_Q C_L \end{pmatrix} \end{array}$$

$$\begin{array}{cccccc} Block7 & Block8 & Block9 & Block7 & Block8 & Block9 \\ \begin{bmatrix} 002 \\ 212 \\ 122 \end{bmatrix} & \begin{bmatrix} 102 \\ 012 \\ 222 \end{bmatrix} & \begin{bmatrix} 202 \\ 112 \\ 022 \end{bmatrix} & \begin{pmatrix} C_L \\ A_Q B_L C_Q \\ A_L B_Q C_Q \end{pmatrix} & \begin{pmatrix} A_L C_Q \\ B_L C_Q \\ A_Q B_Q C_Q \end{pmatrix} & \begin{pmatrix} A_Q C_Q \\ A_L B_L C_Q \\ B_Q C_Q \end{pmatrix} \end{array}$$

Confounding ABC and ABC^2 is recommended since two main effects of A_L and B_L are found in block 2 as well as A_Q and B_Q found in block 3 which can be used for experimentation.

Confounding ABC and AB^{2}C

Similarly, the block and treatment contents for confounding ABC, AB^2C are shown below

$$\begin{array}{cccccc} Block1 & Block2 & Block3 & Block1 & Block2 & Block3 \\ \begin{bmatrix} 000 \\ 201 \\ 102 \end{bmatrix} & \begin{bmatrix} 100 \\ 001 \\ 202 \end{bmatrix} & \begin{bmatrix} 200 \\ 101 \\ 002 \end{bmatrix} & \begin{pmatrix} (1) \\ A_Q C_L \\ A_L C_Q \end{pmatrix} & \begin{pmatrix} A_L \\ C_L \\ A_Q C_Q \end{pmatrix} & \begin{pmatrix} A_Q \\ A_L C_L \\ C_Q \end{pmatrix} \end{array}$$

$$\begin{array}{cccccc} Block4 & Block5 & Block6 & Block4 & Block5 & Block6 \\ \begin{bmatrix} 010 \\ 211 \\ 112 \end{bmatrix} & \begin{bmatrix} 110 \\ 011 \\ 212 \end{bmatrix} & \begin{bmatrix} 210 \\ 111 \\ 012 \end{bmatrix} & \begin{pmatrix} B_L \\ A_Q B_L C_L \\ A_L B_L C_Q \end{pmatrix} & \begin{pmatrix} A_L B_L \\ B_L C_L \\ A_Q B_L C_Q \end{pmatrix} & \begin{pmatrix} A_Q B_L \\ A_L B_L C_L \\ B_L C_Q \end{pmatrix} \end{array}$$

$$\begin{array}{cccccc} Block7 & Block8 & Block9 & Block7 & Block8 & Block9 \\ \begin{bmatrix} 020 \\ 221 \\ 122 \end{bmatrix} & \begin{bmatrix} 120 \\ 021 \\ 222 \end{bmatrix} & \begin{bmatrix} 220 \\ 121 \\ 022 \end{bmatrix} & \begin{pmatrix} B_Q \\ A_Q B_Q \\ A_L B_Q C_Q \end{pmatrix} & \begin{pmatrix} A_L B_Q \\ B_Q C_L \\ A_Q B_Q C_Q \end{pmatrix} & \begin{pmatrix} A_Q B_Q \\ A_L B_Q C_L \\ B_Q C_Q \end{pmatrix} \end{array}$$

Confounding ABC and AB^{2}C is recommended since two main effects of A_L and C_L are found in block 2 and A_Q and C_Q found in block 3 which can be used for experimentation.

Confounding ABC^2 and AB^{2}C^2

Also, the block and treatment contents of confounding ABC^2, AB^2C^2 are shown below

$$\begin{array}{cccccc} Block1 & Block2 & Block3 & Block1 & Block2 & Block3 \\ \begin{bmatrix} 000 \\ 101 \\ 202 \end{bmatrix} & \begin{bmatrix} 100 \\ 201 \\ 002 \end{bmatrix} & \begin{bmatrix} 200 \\ 001 \\ 102 \end{bmatrix} & \begin{pmatrix} (1) \\ A_L C_L \\ A_Q C_Q \end{pmatrix} & \begin{pmatrix} A_L \\ A_Q C_L \\ C_Q \end{pmatrix} & \begin{pmatrix} A_Q \\ C_L \\ A_L C_Q \end{pmatrix} \end{array}$$

$$\begin{array}{cccccc} Block4 & Block5 & Block6 & Block4 & Block5 & Block6 \\ \begin{bmatrix} 010 \\ 111 \\ 212 \end{bmatrix} & \begin{bmatrix} 110 \\ 211 \\ 012 \end{bmatrix} & \begin{bmatrix} 210 \\ 011 \\ 112 \end{bmatrix} & \begin{pmatrix} B_L \\ A_L B_L C_L \\ A_Q B_L C_Q \end{pmatrix} & \begin{pmatrix} A_L B_L \\ A_Q B_L C_L \\ B_L C_Q \end{pmatrix} & \begin{pmatrix} A_Q B_L \\ B_L C_L \\ A_L B_L C_Q \end{pmatrix} \end{array}$$

$$\begin{array}{cccccc} Block7 & Block8 & Block9 & Block7 & Block8 & Block9 \\ \begin{bmatrix} 020 \\ 121 \\ 222 \end{bmatrix} & \begin{bmatrix} 120 \\ 221 \\ 022 \end{bmatrix} & \begin{bmatrix} 220 \\ 021 \\ 122 \end{bmatrix} & \begin{pmatrix} B_Q \\ A_L B_Q C_L \\ A_Q B_Q C_Q \end{pmatrix} & \begin{pmatrix} A_L B_Q \\ A_Q B_Q C_L \\ B_Q C_Q \end{pmatrix} & \begin{pmatrix} A_Q B_Q \\ B_Q C_L \\ A_L B_Q C_Q \end{pmatrix} \end{array}$$

Confounding ABC^2 and AB^{2}C^2 is not recommended since only one main effect is found in a block which cannot be used for experimentation.

Confounding AB²C and AB² C²

The block and treatment contents for confounding AB^2C, AB^2C^2 are

$$\begin{array}{cccccc} Block1 & Block2 & Block3 & Block1 & Block2 & Block3 \\ \begin{bmatrix} 000 \\ 110 \\ 220 \end{bmatrix} & \begin{bmatrix} 100 \\ 210 \\ 020 \end{bmatrix} & \begin{bmatrix} 200 \\ 010 \\ 120 \end{bmatrix} & \begin{pmatrix} (1) \\ A_L B_L \\ A_Q B_Q \end{pmatrix} & \begin{pmatrix} A_L \\ A_Q B_L \\ B_Q \end{pmatrix} & \begin{pmatrix} A_Q \\ B_L \\ A_L B_Q \end{pmatrix} \\ Block4 & Block5 & Block6 & Block4 & Block5 & Block6 \\ \begin{bmatrix} 001 \\ 111 \\ 221 \end{bmatrix} & \begin{bmatrix} 101 \\ 211 \\ 021 \end{bmatrix} & \begin{bmatrix} 201 \\ 121 \\ 011 \end{bmatrix} & \begin{pmatrix} C_L \\ A_L B_L C_L \\ A_Q B_Q C_L \end{pmatrix} & \begin{pmatrix} A_L C_L \\ A_Q B_L C_L \\ B_Q C_L \end{pmatrix} & \begin{pmatrix} A_Q C_L \\ A_L B_Q C_L \\ B_L C_L \end{pmatrix} \\ Block7 & Block8 & Block9 & Block7 & Block8 & Block9 \\ \begin{bmatrix} 002 \\ 112 \\ 222 \end{bmatrix} & \begin{bmatrix} 102 \\ 212 \\ 022 \end{bmatrix} & \begin{bmatrix} 202 \\ 012 \\ 122 \end{bmatrix} & \begin{pmatrix} C_Q \\ A_L B_L C_Q \\ A_Q B_Q C_Q \end{pmatrix} & \begin{pmatrix} A_L C_Q \\ A_Q B_L C_Q \\ B_Q C_Q \end{pmatrix} & \begin{pmatrix} A_Q C_Q \\ B_L C_Q \\ A_L B_Q C_Q \end{pmatrix} \end{array}$$

Confounding AB^2C and AB^2C^2 is not recommended since only one main effect of A_L A_Q and C_L and C_Q are found in blocks 2, 3, 4 and 7 which cannot be used for experimentation.

Fractional Replication

When k>3, the experimental runs will be too large hence the need to replicate the experiment. If k = 3,4,5,,,, the larger fraction of the 3^k factorial design is one-third fractions that will have 3^{k-1} runs.

To construct this design, select a two degree of freedom component interaction preferably interactions with higher order and partition the 3^k design into three blocks.
We shall begin with 3^3 in 3^2 blocks using ABC, ABC^2, AB^2C and AB^2C^2. To select the interactions to be confounded, we shall aliase the two interactions with the lowest treatment contribution denoted by (1) as seen below.

$$(1) = ABC \times ABC^2 = (A^2B^2C^3)^2 = A^4B^4C^6 = AB$$
$$(1) = ABC \times AB^2C = (A^2B^3C^2)^2 = A^4B^6C^4 = AC$$
$$(1) = ABC \times AB^2C^2 = (A^2B^3C^3)^2 = A^4B^6C^6 = A$$
$$(1) = ABC^2 \times AB^2C = (A^2B^3C^3)^2 = A^4B^6C^6 = A$$
$$(1) = ABC^2 \times AB^2C^2 = (A^2B^3C^4)^2 = A^4B^6C^8 = AC$$
$$(1) = AB^2C \times AB^2C^2 = (A^2B^4C^3)^2 = A^4B^8C^6 = AB$$

ABC and AB^2C^2 as well as ABC^2 and AB^2C should not be confounded since they aliased with the main effects.

The required number of block contents for a good replication is 9. The design is $3^{-r}x3^k$. As k increases the value of r decreases until the required divisor gives the required number of 9 treatment combination.

A 3^3 design has 27treatment combinations per replication, a 3^4 design has 81, a 3^5 design has 243 and so on.
If k= 3, the 3^{k-1} runs will be 3^{-1}x 3^3 = 9 as required
If k=4, the 3^{k-1} runs will be

$$3^{-1} \times 3^4 = 81$$

$$3^{-2} \times 3^4 = 9$$ as such 3^{-2} x 3^4 is used for experiment.

If k=5

$$3^{-1} \times 3^5 = 243$$

$$3^{-2} \times 3^5 = 27$$

$$3^{-3} \times 3^5 = 9$$ which is used for experimentation.

To find out which interaction to confound in the generalized interaction of ABCD, which are ABCD, $ABCD^2$, ABC^2D, and AB^2CD we aliase them with (1)to have

$$(1) = ABCD \times ABCD^2 = (A^2B^2C^2D^3)^2 = A^4B^4C^4D^6 = ABC$$
$$(1) = ABCD \times ABC^2D = (A^2B^2C^3D^2)^2 = A^4B^4C^6D^4 = ABD$$
$$(1) = ABCD \times AB^2CD = (A^2B^3C^2D^2)^2 = A^4B^6C^4D^4 = ACD$$
$$(1) = ABCD \times ABCD^2 = (A^2B^2C^2D^3)^2 = A^4B^4C^4D^6 = ABC$$
$$(1) = ABCD^2 \times ABC^2D = (A^2B^2C^3D^3)^2 = A^4B^4C^6D^6 = AB$$
$$(1) = ABCD^2 \times AB^2CD = (A^2B^3C^2D^3)^2 = A^4B^6C^4D^6 = AC$$
$$(1) = ABC^2D \times AB^2CD = (A^2B^3C^3D^2)^2 = A^4B^6C^6D^4 = AD$$

FOR k =5,
Using ABCDE, the interactions are ABCDE, $ABCDE^2$, $ABCD^2E$, ABC^2DE, AB^2CDE

$$(1) = ABCDE \times ABCDE^2 = (A^2B^2C^2D^2E^3)^2 = A^4B^4C^4D^4E^6 = ABCD$$
$$(1) = ABCDE \times ABCD^2E = (A^2B^2C^2D^3E^2)^2 = A^4B^4C^4D^6E^4 = ABCE$$
$$(1) = ABCDE \times ABC^2DE = (A^2B^2C^3D^2E^2)^2 = A^4B^4C^6D^4E^4 = ABDE$$
$$(1) = ABCDE \times AB^2CDE = (A^2B^3C^2D^2E^2)^2 = A^4B^6C^4D^4E^4 = ACDE$$
$$(1) = ABCDE^2 \times ABCD^2E = (A^2B^2C^2D^3E^3)^2 = A^4B^4C^4D^6E^6 = ABC$$
$$(1) = ABCDE^2 \times ABC^2DE = (A^2B^2C^3D^2E^3)^2 = A^4B^4C^6D^4E^6 = ABD$$
$$(1) = ABCDE^2 \times AB^2CDE = (A^2B^3C^2D^2E^3)^2 = A^4B^6C^4D^4E^6 = ACD$$
$$(1) = ABCD^2E \times ABC^2DE = (A^2B^2C^3D^3E^2)^2 = A^4B^4C^6D^6E^4 = ABE$$
$$(1) = ABCD^2E \times AB^2CDE = (A^2B^3C^2D^3E^2)^2 = A^4B^6C^4D^6E^4 = ACE$$
$$(1) = ABC^2DE \times AB^2CDE = (A^2B^3C^3D^2E^2)^2 = A^4B^6C^6D^4E^4 = ADE$$

It is easy to see that as k>3, no main effect is aliased with (1).

Conclusion

In 3^2 experiments, the AB and AB^2 were recommended for confounding while in 3^3 factorial experiments, confounding ABC and ABC^2 is recommended since two main effects are found in a block which can be used for experimentation. Similarly, confounding ABC and AB^2C is recommended since two main effects also in a block which can be used for experimentation. On the contrary confounding ABC^2 and AB^2C^2 is not recommended since only one main effect is found in a blocks which cannot be used for experimentation.

In 3^3 factorial experiment in 3^2 blocks using ABC, ABC^2, AB^2C and AB^2C^2 as defining contrast, ABC and AB^2C^2 as well as ABC^2 and AB^2C should not be confounded since they aliased with the main effects during when fractional replication is needed.

However, in 3^4 factorial experiments no main effect is aliased with (1). In generalized interactions, ABCD and $ABCD^2$ as well as ABCD and ABC^2D are recommended for confounding since three main effects are seen in a block. In contrast, the following pairs of interactions are not recommended for confounding because only two main effects are found in a block: $ABCD^2$ and ABC^2D; $ABCD^2$ and AB^2CD; ABC^2D and AB^2CD.

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