CANONICAL CORRELATION ANALYSIS OF OVULATION INTERVAL, BODY WEIGHT AND AGE, HEIGHT OF SELECTED REPRODUCTIVE WOMEN

¹Matthew, Chukwuma M.; ²Efeizomor Rita O; ³Ashinze, Nkpuruoma A.; and ⁴Adigwe Marigorretti I.

 ¹⁻²Department of Mathematics and Statistics, Faculty of Science, University of Delta, Agbor
 ³Department of Mathematics and Statistics, School of Applied Sciences, Delta State Polytechnic, Ogwashi-Uku.
 ⁴Department of Statistics, Federal Polytechnic, Kaura Namoda, Zamfara State megawaves4life@yahoo.com; matthew.chukwuma@unidel.edu.ng;

Abstract

It is the wish of most women to be reproductive, possibly bearing children of desired gender, and as well maintain good body weight. Fecundity depends highly on ovulation while body weight, to a high extent depends on height. It is the aim of this study to analyze the Canonical Correlation between Ovulation Interval, Body Weight, on one hand, and Age, Height, on the other, of selected reproductive women classified respectively as the Y and the X variables. The women selected for the study were reproductive (those who are still within the childbearing age and/or experiencing ovulation). Questionnaires were distributed with which their specific information were obtained. The data were analyzed using the R software version Rx64.4.0.0. The canonical variates were obtained and the corresponding canonical correlations were determined. The significance of the canonical correlation were tested and observed to be significant at the 0.05 level of significance. The implication of which is that the first and second canonical variates are statistically correlated, with unit variance and maximum correlation.

Keywords: weight, reproductive, ovulation, childbearing, canonical correlation, multivariate.

Introduction

The stability of marriages in this part of the world (Africa) depends largely on the ability of couples to produce children for the continuity of their lineage, and this invariably depends on body weight and a good prediction of ovulation. Hence, the wish and aspiration of most couples to procreate children of desired gender or to, at least, be fruitful does not come commonly easily because prediction of body weight and ovulation is a very complex task. Couples, therefore indulge in some relevant practices in order to achieve the highly anticipated goal of bearing children of desired gender. The main existing practices are broadly classified into traditional practices and orthodox practices. Some traditional practices include relocating from present place of residence to another place (usually on the advice and guidance of a spiritual leader), changing bed position, changing the position in bed on which copulation takes place, changing the time of copulation, use of charm and traditional medicine, among others (Jonna & Aje, 2016; Joseph & Vicki, 1993; George, 1970; Joseph, 2002) Couples also engage in orthodox methods like artificial insemination, use of pregnancy/conception kits, application of hormone/sperm boosters, surgery, etc (La, 2021; Dauriush et al 2022) in the quest to be fruitful, at least, or to bear children of desired gender. These actually depend on a good acknowledge of the fertile period of the female which is a function of their ovulation interval.

Ovulation is simply the release of an egg from the ovary into the fallopian tube for possible fertilization and conception. A good knowledge of ovulation timing has proved to be a leading factor in the ability of couples to procreate (Ashley, 2022). The effect of body weight on fertility has been studied widely; hence, researchers are commonly of the opinion that extreme body weight affects ability to get pregnant negatively (Stacey, 2023). This implies that women of very low and very high body weights are less likely to conceive than women of average body weight.

Matthew, C. M.; Efeizomor, R. O.; Ashinze, N. A. & Adigwe, M. I.

Age may be seen as the single most important factor contributing to fertility because a woman is born with all the eggs she would have in her lifetime and these eggs decrease in quality and quantity as her age increases (Dunson *et al*, 2004; Leridon, 2005). Height is also a very important factor of conception. It has been found that height affects the number of children ever born and the age at first birth but the relationship is non-linear (Jason, 2022). CCA is a statistical method for the analysis of two sets of data namely, the dependent variable set and the independent variable set, such that the correlation between them is maximized (Onyeagu, 2003).

It is the correlation between two canonical variates of the same pair (Heuve, 2018). It is a generalization of many standard statistical techniques such as multiple regression, Analysis of variance, principal component analysis, discriminant analysis (Heuve, 2018). The correlation between latent variables called canonical correlation is maximized by CCA subject to the condition that the canonical variates are of unit variance and the maximization problem is solved with the Lagrange multiplier approach (Onyeagu, 2003; Grittins, 2012) Canonical Correlation Analysis has been applied to infant size and maternal factors (Alamgir *et al*, 2014) in rural Northwest Bangladeshi. The research affirms that CCA simultaneously considers sets of dependent and independent variates and, reduces Type 1 error. CCA has also been applied to the analysis of correlation between nutritional status and sleep quality among centenarians (Guangdang *et al* 2020) and they concluded that the relationship is bidirectional.

It is the aim of this paper to analyze the relationship between Ovulation interval and Body Weight (as the dependent variable set) and; Age and Height (as the independent variable set) of selected reproductive women; hence, it will estimate the canonical correlation between the two sets of variables and; ascertain the effects of the variables of the independent variable set on the variables of the dependent variable set.

Materials and Methods

A sample of six hundred women found in public places like churches, schools, markets, etc. that were able and willing to participate in the study was selected for the purpose of the research. To the selected women, questionnaires were administered and their responses were collated accordingly. Information on their ovulation Interval in days, Body Weight in kilogram, Age in years and Height in meters were carefully extracted to enhance an effective and efficient data analysis. Subsequently, Ovulation Interval and Body Weight were classified as the dependent variable set while Age and Height were classified into the independent variable set.

Computation was done with MINITAB 17 (for data entry and preliminary computations) and R 64×4.3.0 was used to perform the canonical correlation analysis (R Core Team; 2020). The Canonical Correlation Analysis is used in the analysis of two sets of jointly distributed variables Y and X; $U = \frac{Y_p}{X_q} \sim N_{p+q} \left(\mu = {\mu_p \choose \mu_q}, \Sigma = \frac{\Sigma_{11}}{\Sigma_{21}} \right)$

$$Y' = (Y_1, Y_2, Y_3, \dots, Y_p); X' = (X_1, X_2, X_3, \dots, X_q)$$

The Linear transformations:

$$U = a'X \tag{1}$$

$$V = b'Y \tag{2}$$

with maximum correlation and unit variance are the canonical variates.

$$Var(U) = a' \Sigma_{11} a \tag{3}$$

$$Var(V) = b' \Sigma_{22} b \tag{4}$$

$$Cov(U,V) = a'\Sigma_{12}b = b'\Sigma_{21}a$$
(5)

$$Cor(U,V) = \frac{a'\Sigma_{12}b}{\sqrt{a'\Sigma_{12}a}\sqrt{b'\Sigma b}}$$
(6)

Equation 6 is a maximum where

$$a' \Sigma_{11} a = b' \Sigma_{22} b = 1$$
 (7)

Hence the values of a and b such that Equation 6 is maximized, and Equation 7 satisfied, are to be obtained.

The function:

$$F(U,V) = Max_{a,b}(a'\Sigma_{12}b)$$
(8)

is resolved subject to the condition in Equation 7 using the Lagrange Multiplier to obtain

$$\lambda = \lambda_1 = \lambda_2 \tag{9}$$

$$[\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - \lambda^2 \Sigma_{11}]a = 0 \quad (10)$$

Now, λ_1^2 , λ_2^2 , \cdots , λ_p^2 and a_1 , a_2 , \cdots , a_p are the latent roots and latent vectors respectively

of the equation:

$$|\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - \lambda^2 \Sigma_{11}| = 0 \tag{11}$$

$$A = [a_1, a_2, \cdots, a_p]$$
, then, $A' \Sigma_{11} A = I_p$ and

$$A\Sigma_{12}\Sigma_{11}^{-1}A = A_1 \tag{12}$$

 Λ_1 is a diagonal matrix with roots λ_1^2 , λ_2^2 , \cdots , λ_p^2

Similarly, $B = [b_1, b_2, \dots b_q]$ is obtained as the latent vector of the characteristic equation:

$$\begin{aligned} |\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12} - \lambda^2 \Sigma_{22}| &= 0 \end{aligned} (13) \\ B'\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}B &= \Lambda_2 \end{aligned} (14)$$

The non-zero positive square roots λ_i of λ_i^2 are the canonical correlations between the canonical variates $U_i = a'_i X$ and $V = b'_i Y$; $\forall i = 1, 2, \dots, p \leq q$

Also, the relationship between a_i and b_i is

$$b_i = \frac{\sum_{22}^{-1} \sum_{21} a_i}{\lambda_i} \tag{15}$$

 U_i and V_i are clearly uncorrelated and

Similarly, $B = [b_1, b_2, \dots b_q]$ is obtained as the latent vector of the characteristic equation:

$$\begin{aligned} |\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12} - \lambda^2 \Sigma_{22}| &= 0 \end{aligned} (13) \\ B'\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}B &= \Lambda_2 \end{aligned} (14)$$

The non-zero positive Square roots λ_i of λ_i^2 are the canonical correlations between the canonical variates $U_i = a'_i X$ and $V = b'_i Y$; $\forall i = 1, 2, \dots, p \leq q$

Also, the relationship between a_i and b_i is

$$b_i = \frac{\Sigma_{22}^{-1} \Sigma_{21} a_i}{\lambda_i} \tag{15}$$

 U_i and V_i are clearly uncorrelated and

$$Cov(U_i, U_j) = Cov(V_i, V_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$
(16)
$$Cov(V_i, U_i) = \begin{cases} \lambda_i & i = 1, 2, \cdots, p \\ 0, & otherwise \end{cases}$$
(17)
$$\rho = \begin{bmatrix} I_p & \Delta \\ \Delta & I_q \end{bmatrix}$$
(18)

Where
$$\Delta$$
 is a $p \times q$ matrix containing the first canonical correlations between U_i and V_i .
Where the sample covariance or sample correlation matrix is used, a test of significance of canonical correlation is given by

$$\Lambda = \prod_{i=1}^{n} (1 - r_i^2)$$
(19)

 r_i^2 are the sample estimates of λ_i^2 ; that is,

$$H_0: \Sigma_{12} = 0 \ vs \ H_1: \Sigma_{12} \neq 0$$

$$\chi_\beta^2 = -\left[(N-1) - \frac{1}{2} (p+q+1) \right] \log \Lambda \qquad (20)$$

$$H_0 \ \text{is rejected if}$$

$$\chi_\beta^2 > \chi_{\alpha(pq)}^2 \qquad (21)$$

Data Analysis

$$\begin{split} \lambda_1 &= 0.03412483 \,\lambda_2 = 0.007914432 \\ \therefore \lambda &= \begin{pmatrix} 1 & 0 & 0.03412483 & 0 \\ 0 & 1 & 0 & 0.007914432 \\ 0.03412483 & 0 & 1 & 0 \\ 0 & 0.007914432 & 0 & 1 \end{pmatrix} \\ U &= \begin{pmatrix} -0.1744417 & -0.9489457 \\ -0.9846675 & 0.3154393 \end{pmatrix} \\ V &= \begin{pmatrix} 0.999944400 & 0.99895553 \\ -0.01058306 & 0.04569297 \end{pmatrix} \\ U_1 &= -0.1744417(Age) - 0.9489457(Height) \\ U_2 &= -9.846675(Age) + 0.3154393(Height) \\ V_1 &= 0.999944400(Ovu. Int.) + 0.99895553(Weight) \\ V_2 &= -0.01058306(Ovu. Int.) + 0.04569297(Weight) \\ \chi_{\beta}^2 &= 1.01972 \\ \chi_{\alpha,pq}^2 &= \chi_{0.05,4}^2 = 9.48773 \end{split}$$

Discussion of Results

A 0.1744417 unit decrease in Age leads to a unit increase in U_1 where Height is kept constant; a 0.9489457 unit decrease in Height leads to a unit increase in U_1 where Age is constant; a 9.846675 unit decrease in Age leads to a unit increase in U_2 where Height is constant; a 0.3154393 unit increase in Height leads to a unit increase in U_2 where Age is constant; a 0.99994400 increase in Ovulation Interval leads to a unit increase in V_1 where Weight is held constant; a 0.99895553 unit increase in Weight leads to a unit increase in V_1 where Weight is held constant; a 0.99895553 unit increase in Weight leads to a unit increase in V_1 where I where Ovulation Interval is held constant; a 0.01058306 unit decrease in Ovulation Interval leads to a unit increase in V_2 where Weight is held constant; a 0.04569297 unit increase in Weight leads to a unit increase in V_2 where Weight leads to a unit increase in U where Ovulation Interval is held constant; a 0.04569297 unit increase in Weight leads to a unit increase in V_2 where Weight leads to a unit increase in U where Ovulation Interval is held constant; a 0.04569297 unit increase in Weight leads to a unit increase in V_2 where Ovulation Interval is held constant; while, a calculated Chi-square value of 1.01972 and a tabulated value of 9.48773 led to the acceptance of the null hypothesis.

Matthew, C. M.; Efeizomor, R. O.; Ashinze, N. A. & Adigwe, M. I.

Conclusion

This study has successfully fitted a Canonical Correlation model for Ovulation Interval, Weight, and Age, Height. The model was fully discussed. A decrease in Age leads to an increase in U_1 where Height is kept constant; a decrease in Height leads to an increase in U_1 where Age is constant; a decrease in Age leads to an increase in U_2 where Height is constant; an increase in Height leads to an increase in U_2 where Age is constant; an increase in Ovulation Interval leads to a unit increase in V_1 where Weight is held constant; an increase in Weight leads to a unit increase in V_1 where Ovulation Interval is held constant; a decrease in Ovulation Interval leads to a unit increase in V_2 where Weight is held constant; an increase in Weight leads to an increase in V_2 where Ovulation Interval is held constant; an increase in Weight leads to an increase in V_2 where Ovulation Interval is held constant; an increase in Weight leads to an increase in V_2 where Ovulation Interval is held constant. However, its statistical significance could not be established because, the calculated Chi-square value and its tabulated counterpart led to the acceptance of the null hypothesis.

References

- Alamgir, K.; Rebecca, D. M.; Abu, A. S.; Rolf, D. W. K; Alain, B. L.; Paul, C.; Keith, P. W Jr & Mohammed, N. (2014): Correlation of Infants, Size at Birth and Maternal Factors: A study in Rural Northwest Bangladesh. *Plos ONE* 9(4) e94243. doi 10.1371/journal.pone.0094243.
- Dauriush, F.; Tahereh, M.; Tayebeh, M. G; Nasrin, A \& Zahra, K. (2022): Using Noninvasive Methods to choose Gender, Sex Selection with Diet and Determination of Ovulation Time in Iran. *Iran Journal of Public Health*. 5(8), 1886 - 1892.
- George, J. J.; (1970): Cultural Factors Affecting Human Fertility. *Journal of The American Scientific Affiliation (JASA)* 22, 52 - 59.
- Jonna, A.; Aje, C. (2016): Culture and Religious beliefs in relation to health. *Best Practice and Research Clinical Obstetrics and Gynecology*. 32, April 2016. Pp 97 87
- Joseph, G. S. (2002): Gender Selection: Cultural and Religious Perspectives. *Journal of* Assisted Reproduction and Genetics 2002, Sept: 19(9): 400 410.
- Joseph, G. S. & Vicki, R. (1993): Family Planning: Cultural and Religious Perspectives. *Human Reproduction*, 8 (6), 1 June 1993. Pp 969 - 976. https://doi.org/ 10.1093/ oxfordjournls.humrep.a138176
- La, W. (2021): The Best Method for Gender Selection: *Infertility and Reproductive Endocrinology*. Accessed through www.google.com on 4th November 2023, 12:58pm.
- R Core Team (2020): R. A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org