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EXTENDED PSEUDO-LINDLEY POISSON DISTRIBUTION: A NEW MODEL FOR COUNT DATA WITH APPLICATION TO VIRULENT TUBERCLE BACILLI DATA

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***Abstract**:*
This paper introduces a new lifetime probability distribution called the Extended Pseudo-Lindley-Poisson Distribution (EPLPD), which generalizes the Pseudo-Lindley Distribution and the Poisson Distribution. The EPLPD is designed to model count data with varying degrees of dispersion, including zero-inflation and over dispersion. We derive the properties of the EPLPD, including its probability mass function, cumulative distribution function, moments, and moment generating function The maximum likelihood estimation method is used to estimate the parameters of the EPLPD. To demonstrate the applicability of the EPLPD, a numerical example was the survival times (in days) of 72 Guinea pigs infected with virulent tubercle bacilli is presented, demonstrating the distribution's superiority over existing models. The results show that the EPLPD provides a better fit to the data compared to the Poisson and Lindley distributions. The EPLPD can be used to model various types of count data in fields such as medicine, biology, and engineering. The EPLPD offers a valuable tool for analyzing count data in various fields, including medicine, biology, and public health.
***Keywords:** Extended Pseudo-Lindley Poisson Distribution, count data, virulent tubercle bacilli, maximum likelihood estimation, goodness-of-fit test.*

Introduction:

Count data are ubiquitous in various fields, including medicine, biology, engineering, and social sciences. Modeling and analyzing count data require specialized statistical distributions that can capture the underlying characteristics of the data. The Poisson distribution is a widely used model for count data, but it assumes that the mean and variance are equal, which is often not the case in real-world applications. The new distribution formed by the addition of parameters are usually termed "generalized distribution", "compounded distribution", "extended distribution" or "modified distribution" (Mudholkar and Srivastava,1998). Azzalini A. 1985, introduced the skew normal distribution by introducing an extra parameter to the normal distribution to add more flexibility to the normal distribution. Marshall and Olkin (1997) defined another method to introduce an additional parameter to any distribution function using the survival functions. Eugene, Lee and Famoye (2002) proposed the beta-generated class of distribution which is a system that allows more families of flexible probability distributions to be generated using the logit of the beta random variable. The Lindley distribution has just a single parameter which makes the distribution maintain just a single right-skewed shape and hence, the Lindley distribution is not very flexible and adaptive to several data types like biological, financial and engineering data which most of the time requires highly flexible models. The two parameter Pseudo- Lindley distribution which was proposed to address these shortcomings unfortunately has the same problems. Again, the complementary risk problem proposed by Poschan (1963) which is a common problem in engineering and survival analysis cannot be handled by both the Lindley and Pseudo Lindley distributions. In view of these problems, a new extension of the Lindley distribution to address these shortcomings is proposed.

Construction of the Pseudo Lindley Poisson distribution

Let $x_1, x_2 \dots \dots \dots x_n$ be independent and identically distributed (iid) random variables of size N from the Pseudo- Lindley (PL) distribution with cumulative distribution function (cdf)

$G_{PL}(x) = 1 - \frac{(\theta+\beta x)e^{-\beta x}}{\theta},\ x > 0, \beta > 0, \theta \geq 1,$ and N be a zero truncated Poisson random variable independent of X's with probability mass function of $by\, P(N = n) = \frac{\lambda^n}{n!(e^{\lambda}-1)}$; $n = 1,2,3 \dots\dots, \lambda > 0.$ Let us define $X_{(1)} = min\{x_1, x_2, \dots\dots x_N\}$, then the conditional random variable cdf of $X_{(1)|N=n}$= k has the cdf $P(X_{(1)|N=n} = K)$= $= 1 - \left(1 - \left[1 - \frac{(\theta+\beta x)e^{-\beta x}}{\theta}\right]^n\right)$. Hence the marginal cumulative distribution function of X can be obtained as

$$\mathrm{F_{plp}(x)} = 1 - \frac{1}{\mathrm{e}^{\lambda} - 1}\left[\mathrm{e}^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x}} - 1\right] \quad ; \quad x > 0, \lambda > 0, \beta > 0, \theta \geq 0$$

Which defines the Extended Pseudo Lindley Poisson Distribution. And a random variable X with cdf given by the equation above is denoted by EPLP distribution. The density function associated to F(X) is

$$fx = \frac{\left(\lambda\beta[\theta - 1 + \beta x]e^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x}} - \beta x\right)}{\theta\left(e^{\lambda} - 1\right)} \quad \text{for } x>0$$

Shape characteristics of pdf

In this section, we show that the probability density function of the Truncated Pseudo-Lindley-Poisson distribution is a proper pdf. Given a random variable x that follows a Truncated Pseudo-Lindley-Poisson distribution, the probability density function (f(x)) is

$$f(x) = \frac{\left(\lambda\beta[\theta - 1 + \beta x]e^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x}} - \beta x\right)}{\theta\left(e^{\lambda} - 1\right)} \quad for\ x > 0$$

, then the integral of the function is

$$\int_0^{\infty} f(x)dx = \int_0^{\infty} \frac{\left(\lambda\beta[\theta - 1 + \beta x]e^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x}} - \beta x\right)}{\theta\left(e^{\lambda} - 1\right)} dx.$$

Find the derivative of with respect to x.

$$\frac{\beta\lambda}{\theta\left(e^{\lambda} - 1\right)}\int_0^{\infty}\left((\theta - 1 + \beta x)e^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x}} - \beta x\right)dx$$

$$= \frac{-1}{e^{\lambda} - 1}\lim_{n\to\infty}\left[e^{\frac{\lambda}{\theta}(\theta+\beta x)e^{-\beta x}}\right]_0^n = \frac{-1}{e^{\lambda} - 1}\lim_{n\to\infty}\left[e^{\frac{\lambda}{\theta}(\theta+\beta n)e^{-\beta n}} - e^{\lambda}\right]$$

$$= \frac{-1}{e^{\lambda} - 1}\left(1 - e^{\lambda}\right) = \frac{e^{\lambda} - 1}{e^{\lambda} - 1} = 1$$

Therefore, the function is a proper pdf for x > 0. The pdf is a decreasing function. This is an indication that the function can only be used to model non-negative events, that is, events that cannot assume negative values such as lifetime events.

Survival and hazard rate function

The survival function of the new TPLP distribution is given by $S_{TPLP}(x) = 1 - F_{TPLP}(x)$

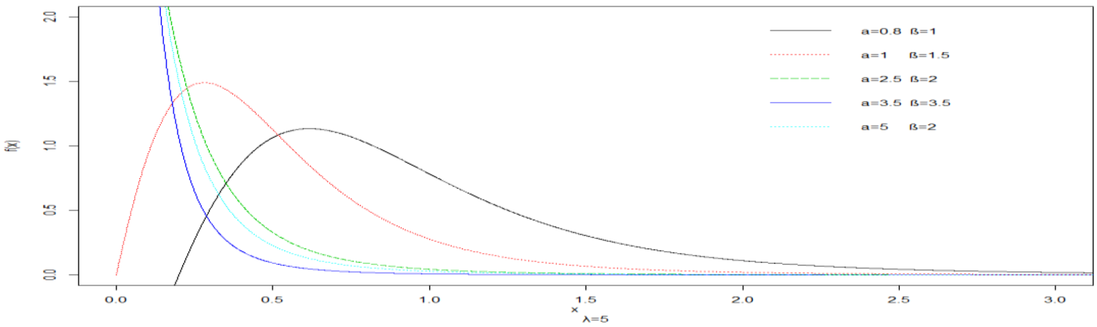
$$= 1 - \left\{1 - \frac{e^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x}} - 1}{e^{\lambda} - 1}\right\} = \frac{e^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x}} - 1}{e^{\lambda} - 1} \qquad x > 0, \lambda > 0, \beta > 0$$

Hazard function of the TPLP distribution is given by

$$h_{TPLP}(x) = \frac{f_{plp}(x)}{S_{plp}(x)} = \frac{f_{plp}(x)}{1 - F_{TPLP}(x)} = \frac{F^{|}(x)}{1 - F(x)} = \frac{\beta\lambda(\theta - 1 + \beta x)e^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x} - \beta x}}{\theta\left(e^{\lambda\left(\frac{\theta+\beta x}{\theta}\right)e^{-\beta x}} - 1\right)} \qquad x > 0, \lambda > 0, \beta > 0, \theta > 0.$$

In figs, 1-4,The plot showing the various shape of the probability density function of the EPLP for different combination of parameter values.

Figure 1. The EPLP density for various parameter values a, β *and* λ.



NOW, in (1), the parameters λ, β *and* θ control the shape of the distribution.

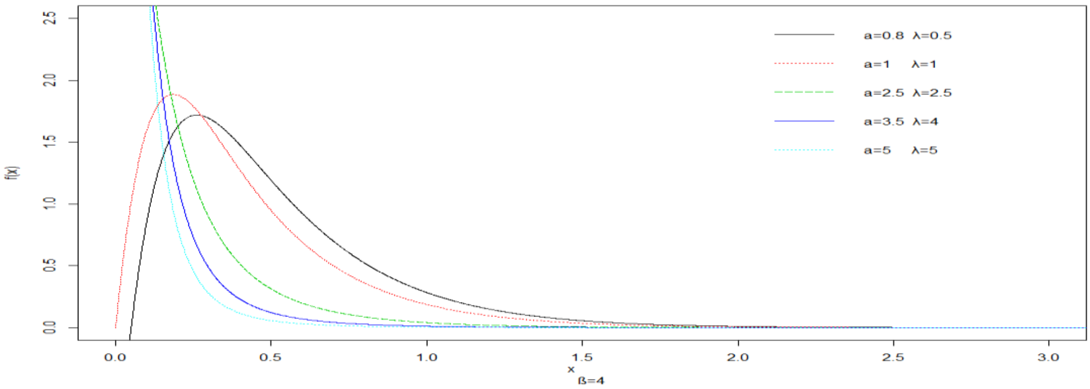


Figure 2: The EPLP density for various parameter values of a, λ *and* β

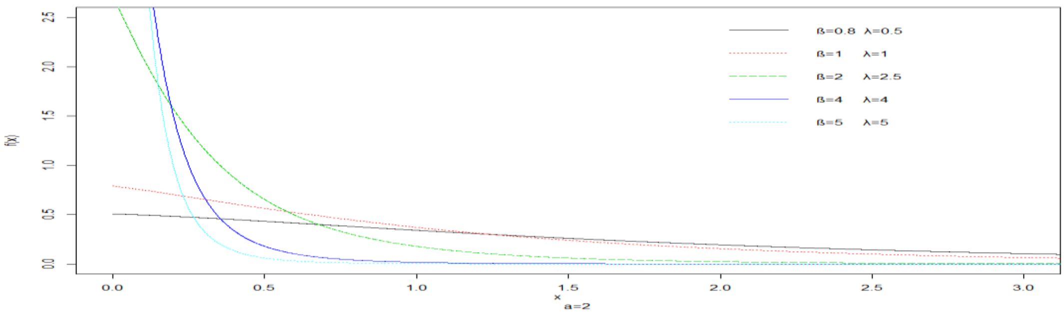


Figure 3: The TPLP density for various parameter values $\lambda,\ \beta\ and$ a Graphs of the Hazard Function

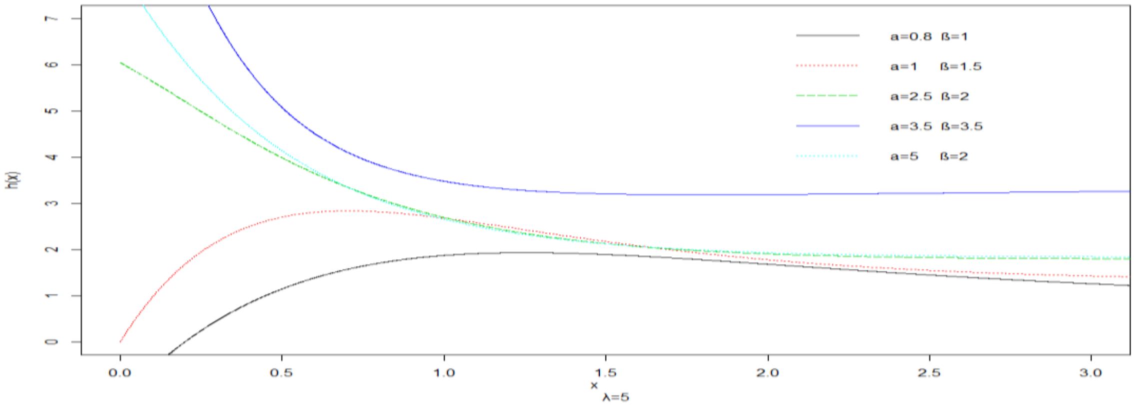


Figure 4, Hazard function of the TPLP distribution for various parameters values 1

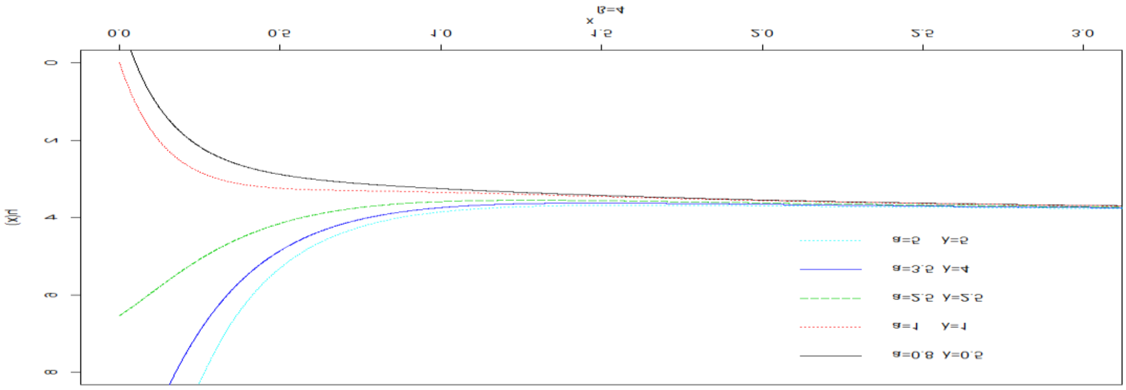


Figure 5: Hazard function of the PLP distribution for various parameters values 2

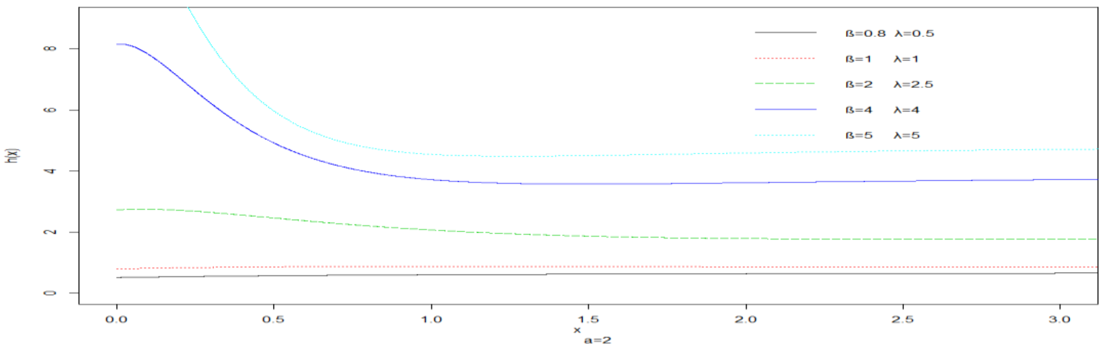


Figure 6: Hazard function of the PLP distribution for various parameters values 3

Flexible Shapes: The hazard function can exhibit various shapes, including increasing, decreasing, constant, bathtub, and upside-down bathtub. This demonstrates the flexibility of the PLP distribution in modeling different types of data, particularly for events with varying hazard rates.

Graph of Survival Function

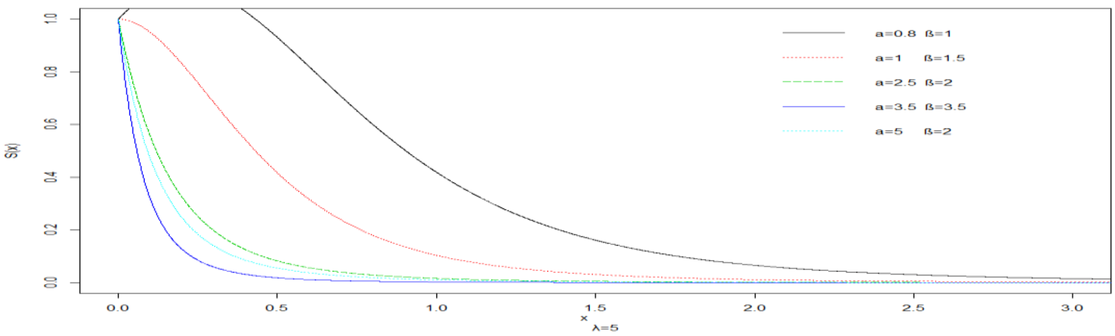


Figure 7: Survival function of the TPLP distribution for various parameters values 1

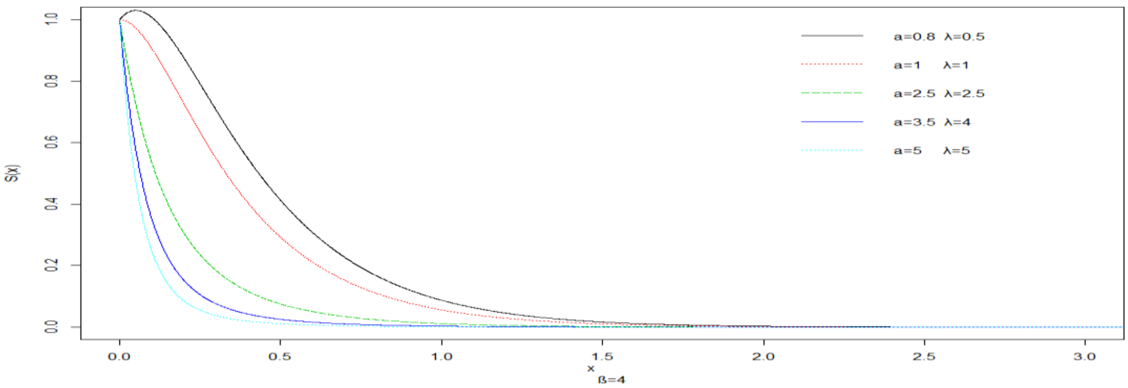


Figure 8: Survival function of the TPLP distribution for various parameters values 2

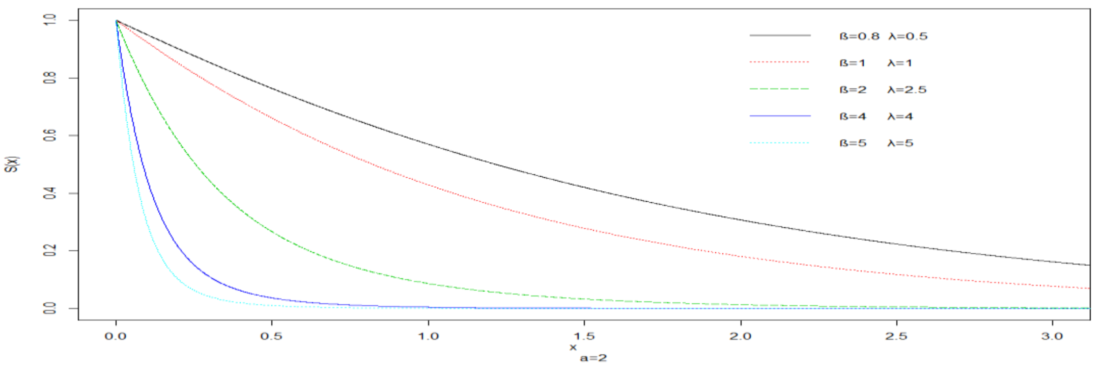


Figure 9: Survival function of the TPLP distribution for various parameters values 3

Survival Function: The survival functions, as shown in Figure above, generally decrease, reflecting the diminishing probability of survival over time. This aligns with typical survival analysis expectations.

In general, the PLP distribution's graphs highlight its right-skewness, unimodal nature, and flexible tail behavior, making it suitable for a wide range of data modeling applications. The hazard and survival function graphs further emphasize its adaptability in survival analysis and reliability studies.

Moments.

Many of the interesting characteristics and features of a distribution can be obtained via its moment generating function and moments.

They can be used to study the most important features and characteristics of a distribution such as skewness and kurtosis. Let X denote a random variable with the probability density function EPLP. By definition of moment generating function of X and using EPLP distribution is defined as The $\boldsymbol{r^{th}}$ non-central moment.

The r^{th}non-central moment of a random variable X from the TPLP distribution say μ_r', is given by $\mu_r' = E(X^r) = \int_0^\infty x^r f_{plp}(x)dx$ given that $F_{plp}(x) = \sum_{n=1}^{\infty} P(N = n)\ G_{X_{(1)|N=n}}(x)$ follows that $f_{plp}(x) = \sum_{n=1}^{\infty} P(N = n)\ g_{X_{(1)|N=n}}(x)$

Where $g_{X_{(1)|N=n}}(x) = \frac{dG_{X_{(1)|N=n}}(x)}{dx}$. Then $G_{X_{(1)|N=n}}(x) = 1 - \left(\frac{\theta+\beta x}{\theta}\right)^n e^{-n\beta x}$

Thus $G_{X_{(1)|N=n}}(x) = \frac{n\beta}{\theta}(\theta - 1 + \beta x)\left(\frac{\theta+\beta x}{\theta}\right)^{n-1} e^{-n\beta x}$

Using 1 we can write

$f_{plp}(x) = \sum_{n=1}^{\infty} \frac{\lambda^n n\beta}{n!(e^\lambda-1)\theta}(\theta - 1 + \beta x)\left(\frac{\theta+\beta x}{\theta}\right)^{n-1} e^{-n\beta x}$ Thus, we can expressed as

$\mu_r' = \int_0^\infty x^r \sum_{n=1}^{\infty} \frac{\lambda^n n\beta}{n!(e^\lambda-1)\theta}\ (\theta - 1 + \beta x)\left(\frac{\theta+\beta x}{\theta}\right)^{n-1} e^{-n\beta x}\,dx$

$$= \sum_{n=1}^{\infty} \frac{\lambda^n n\beta}{n!\,(e^\lambda - 1)\theta}\int_0^\infty x^r(\theta - 1 + \beta x)\left(\frac{\theta + \beta x}{\theta}\right)^{n-1} e^{-n\beta x}\,dx$$

$$= \sum_{n=1}^{\infty} \frac{\lambda^n n\beta}{n!\,(e^\lambda - 1)\theta^n}\int_0^\infty x^r(\theta - 1 + \beta x)(\theta + \beta x)^{n-1}e^{-n\beta x}\,dx$$

$$= \sum_{n=1}^{\infty} \frac{\lambda^n n\beta}{n!\,(e^\lambda - 1)\theta^n}\int_0^\infty x^r(\theta - 1 + \beta x)(\theta + \beta x)^{n-1}e^{-n\beta x}\,dx$$

We can write

$$\mu_r' = \sum_{n=1}^{\infty} \frac{\lambda^n n\beta}{n!\,(e^\lambda - 1)\theta^n}\int_0^\infty x^r[(\theta - 1) + \beta x)][1 + (\theta - 1) + \beta x]^{n-1}e^{-n\beta x}\,dx$$

Preposition 2

Let $L_1\,(n, \beta, \theta, r) = \int_0^\infty x^r[(\theta - 1) + \beta x)][1 + (\theta - 1) + \beta x]^{n-1}e^{-n\beta x}\,dx$. Then

$$L_1\,(n, \beta, \theta, r) = \sum_{i=0}^{n-1}\sum_{j=0}^{i+1}\binom{n-1}{i}\binom{i+1}{j}\frac{(\theta - 1)^{i-j+1}}{n^{r+j+1}\beta^{r+j}}\,\Gamma((r + j + 1)$$

The above integral can rewritten by using the complete gamma function.

$\Gamma(m) = \int_0^\infty t^{m-1}e^{-t}\ dx$ $\quad t > 0$, is the complete gamma function.

Proof: Consider the integral $\int_0^\infty x^r[(\theta - 1) + \beta x)]\ [1 + (\theta - 1) + \beta x]^{n-1}e^{-n\beta x}\,dx$

Using the binomial expansion formula, we have the following result; $[1 + (\theta - 1) + \beta x]^{n-1} = \sum_{i=0}^{n-1}\binom{n-1}{i}\,[(\theta - 1) + \beta x]^i$

The integral becomes $\sum_{i=1}^{n-1}\binom{n-1}{i}\int_0^\infty x^r[(\theta-1)+\beta x)]^{i+1}e^{-n\beta x}\,dx$ also $[(\theta-1)+\beta x)]^{i+1}=\sum_{j=1}^{i+1}\binom{i+1}{j}(\theta-1)^{i-j+1}\beta^j x^j$

Thus, we have the integral as $=\sum_{i=1}^{n-1}\sum_{j=1}^{i+1}\binom{n-1}{i}\binom{i+1}{j}\beta^j(\theta-1)^{i-j+1}\int_0^\infty x^{r+j}e^{-n\beta x}dx$ Now, consider the integral $\int_0^\infty x^{r+j}e^{-n\beta x}dx$ Let $v=n\beta x$ $implies\ that$ $x=\frac{v}{n\beta}$

again when $x=0,\ v=0, when\ x=\infty, v=\infty$.

We have that $\int_0^\infty x^{r+j}e^{-n\beta x}dx = \int_0^\infty\left(\frac{v}{n\beta}\right)^{r+j}e^{-v}\frac{dv}{n\beta}$

$$=\left(\frac{1}{n\beta}\right)^{r+j}\frac{1}{n\beta}\int_0^\infty v^{r+j}e^{-v}dv$$

But $\Gamma(M)=\int_0^\infty v^{m-1}e^{-v}dv,\quad v>0$

where $\Gamma(.)$ is the complete gamma function it follows that $m-1=r+j\ implies\ that\ m=r+j+1$. Thus, $\int_0^\infty x^{r+j}e^{-n\beta x}dx = \frac{1}{n^{r+j}\beta^{r+j}}\Gamma(r+j+1)$

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\binom{n-1}{i}\binom{i+1}{j}\beta^j\frac{(\theta-1)^{i-j+1}}{n^{r+j+1}\beta^{r+j}}\Gamma(r+j+1)$$

Hence the proof is established.

The r^{th} non central moment of the TPLP distribution can be expressed as $\mu_r'=\sum_{n=1}^{\infty}\frac{\lambda^n n\beta}{n!(e^\lambda-1)\theta^n}L_1(n,\beta,\theta,r)$ The first four moments of the TPLP distribution can be expressed as $\mu_1'=\mu=\sum_{n=1}^{\infty}\frac{\lambda^n n\beta}{n!(e^\lambda-1)\theta^n}L_1(n,\beta,\theta,1)$, $\mu_2'=\sum_{n=1}^{\infty}\frac{\lambda^n n\beta}{n!(e^\lambda-1)\theta^n}L_1(n,\beta,\theta,2)$ $\mu_3'=\sum_{n=1}^{\infty}\frac{\lambda^n n\beta}{n!(e^\lambda-1)\theta^n}L_1(n,\beta,\theta,3)$, $\mu_4'=\sum_{n=1}^{\infty}\frac{\lambda^n n\beta}{n!(e^\lambda-1)\theta^n}L_1(n,\beta,\theta,4)$

The first moment $(\mu_1'=\mu)$ is the mean of the distribution. Then mean (μ), variance (δ^2), coefficient of variation (CV), coefficient of skewness (CS) and coefficient of kurtosis (CK) are given respectively by $\mu=\mu_1=E(X)$,

$$\delta^2=\mu_2'-\mu^2, CV=\frac{\delta}{\mu}=\frac{\sqrt{\mu_2'}-\mu^{2'}}{\mu}=\sqrt{\frac{\mu_2'}{\mu^2}}-1$$

$$CS=\frac{E[(X-\mu)^3]}{E[(X-\mu)^2]^{3/2}}=\frac{\mu_3'-3\mu\mu_2'+2\mu^3}{(\mu_2'-\mu^2)^{3/2}}$$

$$Ck=\frac{E[(X-\mu)^4]}{E[(X-\mu)^2]^2}=\frac{\mu_4'-4\mu\mu_3'+6\mu^2\mu_2'-3\mu^4}{(\mu_2'-\mu^2)^2}$$

Moment Generating Function

The moment generating function of the TPLP distribution is defined as

$\mathrm{M_x(t)=E(e^{tx})}$, $\mathrm{E(e^{tx})}=\sum_{k=0}^{\infty}\frac{t^k E(X^k)}{K!}$

where

$E(X^k)$ *is the* k^{th} *non central moment of the TPLP distribution given in (3.19). It follows that the moment generating function of the TPLP distribution can be expressed as*

$$E(e^{tx}) = \sum_{k=0}^{\infty}\sum_{n=1}^{\infty}\frac{\lambda^n n\beta t^k}{n!\,k!\,(e^{\lambda}-1)\theta^n}\ L_1\,(n,\beta,\theta,k)$$

$M_x(t) = E(e^{tx}) = E(e^{tx}) = \sum_{k=0}^{\infty}\sum_{n=1}^{\infty}\frac{\lambda^n n\beta t^k}{n!k!(e^{\lambda}-1)\theta^n}\ L_1\,(n,\beta,\theta,k)$

: Maximum Likelihood Estimation of the parameters of the Extended Pseudo- Lindley-Poisson Distribution

For a random independent sample $x_1, x_2 \ldots\ldots x_n$ *of size n form the EPLP distribution, the maximum likelihood estimation of the parameters of the EPLP distribution involve the maximization of the log likelihood function defined by*

$$L = \sum_{n=1}^{n}\log f_{plp}(x_i)$$

$$= \sum_{e=1}^{n}\log\left[\frac{\beta\lambda(\theta-1+\beta x_i)e^{\lambda\left(\frac{\theta+\beta x_i}{\theta}\right)e^{-\beta x_i}} - \beta x_i}{\theta(e^{\lambda}-1)}\right]$$

$$= \log\beta + \log\lambda - \log\theta - \log\left(e^{\lambda}-1\right) + \log(\theta-1+\beta x_i) + \lambda\left(\frac{\theta+\beta x_i}{\theta}\right) - \beta x_i - \log\beta x_i$$

$$= \sum_{i=1}^{n}\left\{\log\beta + \log\lambda - \log\theta - \log\left(e^{\lambda}-1\right) + \log(\theta-1+\beta x_i) + \lambda\left(\frac{\theta+\beta x_i}{\theta}\right)e^{-\beta x_i} - \beta x_i\right\}$$

$= n\log\beta + n\log\lambda - n\log\theta - n\log\left(e^{\lambda}-1\right) + \sum_{i=1}^{n}\log(\theta-1+\beta x_i) +$
$\lambda\sum_{i=1}^{n}\left(\frac{\theta+\beta x_i}{\theta}\right)e^{-\beta x_i} - \beta\sum_{i=1}^{n}x_i$

Let $\theta = (\theta,\beta,\lambda)$ *be the unknown parameter vector .The associated score function is given by*

$$U(\theta) = \left(\frac{\partial L}{\partial\theta}\frac{\partial L}{\partial\beta}\frac{\partial L}{\partial\lambda}\right).$$

where $\frac{\partial L}{\partial\theta}, \frac{\partial L}{\partial\beta}$ *and* $\frac{\partial L}{\partial\lambda}$ *are the partial derivatives of the log-likelihood function w.r.t to each parameter given by*

$$\frac{\partial L}{\partial\theta} = \frac{-n}{\theta} + \sum_{i=1}^{n}\frac{1}{(\theta-1+\beta x_i)} - \lambda\sum_{i=1}^{n}\frac{\beta x_i}{\theta^2}e^{-\beta x_i}$$

$$\frac{\partial L}{\partial\beta} = \frac{n}{\theta} + \sum_{i=1}^{n}\frac{x_i}{(\theta-1+\beta x_i)} + \lambda\sum_{i=1}^{n}\frac{x_i}{\theta}e^{-\beta x_i}(1-\theta-\beta x_i) - \sum_{i=1}^{n}x_i$$

$$\frac{\partial L}{\partial\lambda} = \frac{n}{\lambda} - \frac{ne^{\lambda}}{e^{\lambda}-1} + \sum_{i=1}^{n}\left(\frac{\theta+\beta x_i}{\theta}\right)e^{-\beta x_i}$$

The maximum likelihood estimate of $\theta = (\theta,\beta,\lambda)$ *can be obtained by solving the non-linear system of equation,* $U(\theta) = 0$. *Since the equation s are not in closed form, the solutions can be found numerically using some specialized numerical optimization method.*

Analysis of Data

In this section, the newly derived distribution was compared with similar distributions that can be used in modeling lifetime datasets. This is necessary in order to show its superiority over some of the existing distributions. In the comparison, AIC and BIC were used. Recall that a model with the least Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC) value is better than distributions with higher values of AIC or BIC. This implies a lower

AIC or BIC indicates a better fit. The two measures of better fit were used because the AIC does not penalize the number of parameters as strongly as BIC.

Table1: The survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli

0.10, 0.33, 0.44 ,0.56, 0.59, 0.72 ,0.74 ,0.77 ,0.92, 0.93, 0.96, 1.00, 1.00, 1.02 , 1.05,1.07, 1.07, 1.08, 1.08, 1.08, 1.09, 1.12, 1.13, 1.15, 1.16, 1.20, 1.21, 1.22, 1.22, 1.24,1.30, 1.34, 1.36, 1.39, 1.44, 1.46, 1.53, 1.59, 1.60, 1.63, 1.63, 1.68, 1.71 ,1.72, 1.76,1.83, 1.95, 1.96, 1.97, 2.02, 2.13, 2.15, 2.16, 2.22, 2.30, 2.31, 2.40, 2.45, 2.51, 2.53 , 2.54, 2.54, 2.78, 2.93, 3.27, 3.42, 3.47, 3.61, 4.02, 4.32, 4.58, 5.55

Source: Ghitany et al. (2008)

In Table 1, the results of the maximum likelihood fit of the PLP, PL, Lindley and exponential distributions to the data set are presented alongside the AIC, K-S statistic (and its corresponding p-value) and log likelihood values of the respective distributions.

Table 2: Maximum likelihood fit of the survival times data

Distributions	**PLPD**	**PLD**	**Lindley**	**Exponential**
Parameter Estimates (s)	$\hat{\beta} = 0.2158$ $\hat{\lambda} = 15.9685$ $\hat{\theta} = 0.9896$	$\hat{\beta} = 1.1835$ $\hat{\theta} = 0.9152$	$\hat{\beta} = 0.8683$	$\hat{c} = 0.5655$
loglikelihood	-94.63	-96.53	-106.93	-113.04
AIC	195.27	197.07	215.86	228.07
BIC	194.83	196.77	215.72	227.94
K-S	0.1090	0.1518	0.2467	0.2946
K-S (p-value)	0.3349	0.0650	0.0002	$4.961\text{e}-06$

(Standard error of estimates in parenthesis)

Results in Table 2 clearly show that all the fitted distributions to the data were out-performed by the TPLPD. This is because, the proposed TPLPD is observed to possess the lowest AIC value and the highest p-value of the K-S statistic.

Discussion of Results

For the data set, The TPLPD proved to be the best model for the data set. It has the highest p-values of the K-S statistic and the lowest AIC value.

Conclusion

In this work, we have studied the development of a new probability distribution that can be used in the modeling of lifetime data analysis. The new distribution EPLP compounds the Pseudo Lindley distribution and the Poisson distribution. Several structural properties of the new distribution have been studied. These properties include moments, hazard, survival and the method of maximum likelihood was used to estimate the model parameters. The hazard function of the TPLP distribution has different shapes including bath up shapes, upside down bathtub shape. The new distribution was further applied to two real data sets in order to illustrate the applicability and usefulness of the distribution.

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