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EVALUATION OF SENSITIVITY OF THREE SELECTED TWO-SAMPLE NONPARAMETRIC TESTS

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Abstract
In this paper, comparison of two sample tests, is motivated by the fact that in the test of significant difference between two independent samples, numerous methods can be adopted, each may lead to significant different results; this implies that wrong choice of test statistic could lead to erroneous conclusion. To prevent misleading information, there is a need for proper investigation of some selected methods for test of significant difference between variables/subjects most especially, independent samples. In this paper, Monte Carlo's Simulation techniques were used in the generation of data of two different distributions and varying sample sizes ranging from 5 to 100 which were repeated 30 times for each sample size. In the simulation, sample sizes 5, 10, 20, 30, 50 and 100 were considered. In the paper, data from a known family of distributions; Gamma (4, 0.3) and Weibull (7, 3) were used. This paper examines the sensitivity/efficiency of Mann-Whitney U test, Modified Mann-Whitney U test and Kolmogorov-Smirnov two-sample test to determine the most powerful test in terms of rejecting the null hypothesis when it is true. From the results, Mann-Whitney U-test was found to be the most powerful test in terms of rejecting the null hypothesis when it is true.
Keywords: Independent Sample; Nonparametric test; two sample tests; Power of test

Introduction
Non-parametric techniques do not rely on data belonging to any particular distribution. These include, among others, distribution free methods, which do not rely on assumptions that the data are drawn from a given probability distribution. As such, it is the opposite of parametric statistics. It includes non-parametric descriptive statistics, statistical models, inference and statistical tests. Non-parametric statistics is defined to be a function on a sample that has no dependency on a parameter whose interpretation does not depend on the population fitting any parameterised distributions[1]. Numerous methods exist for testing statistical hypotheses in various conditions. In some cases, the probability distribution of the population from which samples are drawn are known, for instance, if the population are assumed to be normal, then, the sample size is assumed to be sufficiently large to justify the assumption of normality, otherwise, the test of goodness of fit is carried out to ascertain the distribution of the data. In special cases, if sample sizes of a set of observations is small and the probability distribution of the populations from which samples are drawn are unknown; hence, only distribution free test statistic can be used; non-parametric methods. Thus, in most cases where the assumptions of parametric methods such as normality, homogeneity of variance, independency etc. are violated or not met, the non-parametric methods are usually preferred. This explains the appropriateness of Kruskal-Wallis test in place of One-Way Analysis of Variance in test of significance difference among treatments. These methods require that the populations from which the samples are drawn be continuous so that the probability of obtaining tied observations is at least theoretically zero [2]. Techniques or methods for performing two sample tests abound but the question is "which method(s) perform better and under what conditions do they perform better when dealing with independent samples?" To make an articulate attempt to answer these questions, there is need for proper and adequate comparative study of similar methods that can be used for the purpose of interest. Since the methods perform similar function and are widely used by researchers, there is a need for proper study of their strengths to determine the appropriate condition(s) under which each method performs optimally and which method is relatively more efficient and hence more powerful generally. In the determination of more effective statistical method, not just the null hypothesis should be of paramount interest but also the alternative hypothesis since the power of test plays an important role in the determination of effectiveness of statistical methods. The maximum value of power of

test is 1 and the least is zero which is non-negativity property. The higher the power of test is, the better the method, the lower the value and the less effective the method become.

Material and Methodology

Mann-Whitney u-test is used for determination of the likelihood that two samples/groups emanated from the same population/distribution[3].

The test statistic is:

$$Z = \frac{U - \mu_u}{\sigma_u} \qquad (1)$$

Then:

$$U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \qquad (2)$$

where:

n_1 is the total number of the first group/observation.

n_2 is the total number of the first group/observation.

Then:

$$\mu_U = \frac{n_1 n_2}{2} \qquad (3)$$

is the mean and

$$\sigma_u = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} \qquad (4)$$

is the standard deviation.

This Z-score is, as usual, compared at a given level of significance with an appropriate critical value obtained from a normal distribution table for a rejection or acceptance of the null hypothesis.

Modified Intrinsically Ties Adjusted Mann-Whitney U Test is used to check whether two samples could have been drawn from the same population/distribution [5].

The test statistic is:

$$\chi^2 = \frac{w^2}{var(w)} = \frac{\left(n_{2.}R_{.1} - n_{1.}R_{.2}\right)^2}{\left(n_{2.}R_1^{*2} + n_{1.}R_2^{*2} - 2R_{.1}R_{.2}\right)\left(\pi^+ + \pi^- - (\pi^+ - \pi^-)^2\right)} \qquad (5)$$

Where:

n_1 is the sample size of variable x_1

n_2 is the sample size of variable x_2

R_1 and R_2 are the respective sums of the ranks assigned to observations from populations x_1 and x_2 in the combined ranking of these observations from the two populations.

π^+,π^- are respectively the probabilities that observations or scores by subject from population X_1 is on the average greater than or less than observations or scores by subject from population X_2.

The test hypothesis will be:

$$H_0: \pi^+ - \pi^- = 0$$

vs

$$H_1: \pi^+ - \pi^- \neq 0$$

Reject H_0 at α-level of significance if $\chi^2 \geq \chi^2_{1-\alpha;1}$; otherwise, accept.

Two-sample kolmogorov–Smirnov test

Kolmogorov-Smirnov two-sample test is a test of whether two independent samples have been drawn from the same population (or from populations with the same distribution)[5]. Their test statistic is:

$$D = maximum\left|S_{n_1}(X) - S_{n_2}(X)\right| \quad (6)$$

Where:

$S_{n_1}(X) = the\ observed\ cumulative\ step\ function\ of\ one\ of\ the\ samples.$

$S_{n_2}(X) = the\ observed\ cumulative\ step\ function\ of\ the\ other\ samples.$

$S_{n_1}(X) = {}^{K}/_{n_1}, where\ K = the\ number\ of scores\ equal\ to\ or\ less\ than\ X$

$S_{n_2}(X) = {}^{K}/_{n_2}, where\ K = the\ number\ of scores\ equal\ to\ or\ less\ than\ X$

$n_1\ is\ the\ total\ number\ of\ the\ first\ group.$

$n_2\ is\ the\ total\ number\ of\ the\ second\ group.$

The null hypothesis is rejected when the observed D is equal or larger than the critical value.

Power of a statistical test is the probability of rejecting the null hypothesis when it is in fact false and should be rejected (i.e. the probability of not committing a type II error). Therefore, power of a test is (1-β) which is also known as the sensitivity[6]; where β is the probability of committing type II error = error rate. Error rate is defined as the ratio of number of erroneous decision to number of replicate. That is:

$$\text{E.R} = \frac{Number\ of\ error\ output}{No\ of\ trials\ (Replicate)}$$

In this paper, Monte Carlo's Simulation techniques were used in the generation of data of different distributions and varying sample sizes ranging from 5 to 100 which were repeated 30 times for each sample size. In the simulation, sample sizes 5, 10, 20, 30, 50 and 100 were considered. In the paper, data from a known family of distributions; Gamma (4, 0.3) and Weibull (7, 3) were used.

Algorithm for Monte Carlo Simulation

Monte Carlo (MC) simulation is used to determine the performance of an estimator or test statistic under various scenarios [7]. The structure of a typical Monte Carlo exercise is as follows:

1. Specify the "Data Generation Process".
2. Choose a sample size N for the MC simulation.
3. Choose the number of times to repeat the MC simulation.
4. Generate a random sample of size N based on the Data Generation Process.
5. Using random sample generated in 4 above, calculate the statistic(s).
6. Go back to (4) and repeat (4) and (5) until desirable replicate is achieved.
7. Examine parameter estimates, test statistics, etc.

Result

Algorithm for the Analysis

The data analysis involves the following steps:

1. Pairs of simulate data of sizes n = 5, 10, 20, 30, 50, 100 from the two distributions; Gamma and Weibull.
2. For each sample size, n, replicated 30 times,

i. Calculate the values for test statistics for each of the methods.
ii. Reject/Accept the null hypothesis;
iii. Calculate the error rate = $\frac{\textit{times wrong decision is recorded}}{\textit{number of replicate } (30)}$
iv. Calculate the type I error or power of test = $1 - p(\textit{type II error})$

The simulated data for Gamma and Weibull distribtuions are in the appendix A while for the test statistics and the p-value are in appendix B. From the simulated data using Monte Carlo simulation approach, the following results were obtained:

Table 1: Error Rate of Mann-Whitney, MMWU and Two-sample K-S test Statistics

Family of Data	Test Statistic	5	10	20	30	50	100
Gamma (4,0.3)	Mann-Whitney	0.0000	0.0000	0.0000	0.0000	0.0333	0.0333
	MMWU	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	Two-Sample K-S	0.0000	0.0000	0.0000	0.0000	0.0667	0.0667
Weibull (7,3)	Mann-Whitney	0.0333	0.1333	0.0000	0.0333	0.0000	0.0333
	MMWU	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	Two-Sample K-S	0.0000	0.0000	0.0333	0.1000	0.0000	0.0333

Considering Mann-Whitney U test and MMWU; Mann-Whitney U test statistic is more suitable as the error rates of MMWU are significantly high for all the distributions considered.

Moreover, considering Mann-Whitney U test and Two-sample K-S; Mann-Whitney U test is better than Two-Sample K-S since the error rate of it is lower. For Weibull distribution, Two-sample K.S test is better than Mann-Whitney U-test for sample size of 5 and 10 and as the sample size increases Mann-Whitney U-test becomes more efficient than Two-Sample K-S test. Generally, the best statistical tool among the tests considered is Mann-Whitney U test followed by Two-Sample Kolmogorov-Smirnov test.

Table 2 can be computed from Table 1. Power of test is the sensitivity of a test statistic and the greater the value is, the more sensitive the test statistic becomes.

Table 2: Power of Tests

Family of Data	Test Statistic	5	10	20	30	50	100
Gamma (4,0.3)	Mann-Whitney	1	1	1	1	0.9667	0.9667
	MMWU	0	0	0	0	0	0
	Two-Sample K-S	1	1	1	1	0.9333	0.9333
Weibull (7,3)	Mann-Whitney	0.9667	0.8667	1	0.9667	0.9667	0.9667
	MMWU	0	0	0	0	0	0
	Two-Sample K-S	1	1	0.9667	0.9	1	0.9667

From Table 1, it can be observed that Mann-Whitney U-test and Two-sample K.S are more sensitive than MMWU test since it has higher power irrespective of the distribution of the data used.

For better understanding of sensitivity of the three test statistics, line chart of power of test is constructed as shown in figures 1 and 2 below.

Line chart can be used to show position of the strength or power of a test statistic, especially in statistical inference. This shows test statistic with higher power with the maximum power of 1.0.

Graphical Illustration of power of test

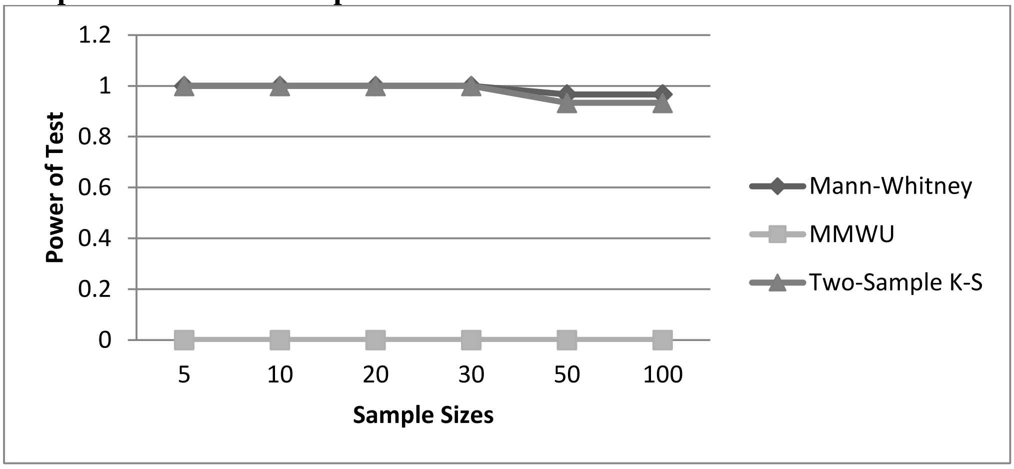

Fig. 1: Power of Tests using Gamma Distribution

As shown in Figure 1, it can be deduced that Mann-Whitney and Kolmogorov-Smirnov two-sample tests have the highest power irrespective of sample size which makes it better than MMWU. The modified method has considerably low power as sample size varies/increases.

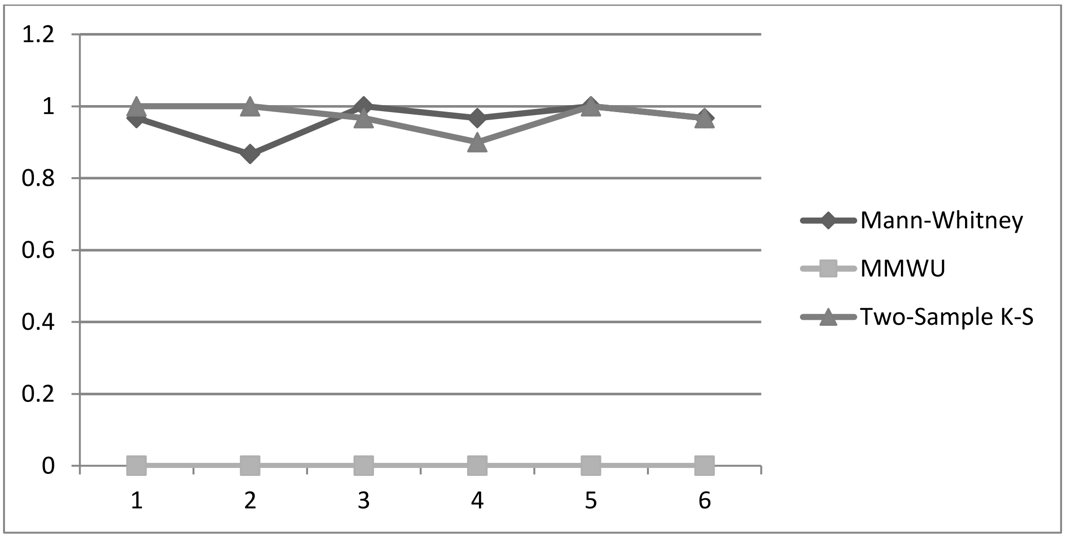

Fig. 2: Power of Tests using Weibull Distribution

Figure 2 revealed that Mann-Whitney and two-sample Kolmogorov-Smirnov have the same power at sample size 100 while from sample size 20 through 30 the Mann-Whitney U-test was found to be more powerful. It was equally observed that the two-sample Kolmogorov-Smirnov test has better power than the Mann-Whitney U-test at sample points 5, 10 and 50.

Summary and Conclusion

We have in this paper presented a nonparametric statistical method for the analysis of two sample tests. Based on the result of the analysis used, it is observed that Mann-Whitney U test and Two-sample Kolmogorov-Smirnov test are more sensitive than Modified Mann-Whitney U-test (MMWU) since it has higher power.

References

1. Murphy, Kevin (2012). Machine Learning: A Probabilistic Perspective. MIT. p. 16. ISBN 978- 0262018029.
2. Gibbon, J.D.(1992): Nonparametric Statistics: An Introduction, Quantitative Applications in Social Sciences, Sage Publications, New York.
3. Landers, J. (1981): "Qualification in History, Topic 4, Hypothesis Testing II-Differing Central Tendency", Oxford, All souls College.
4. Oyeka I.C.A and Okeh U.M (2013):"Modified Intrinsically Ties Adjusted Mann-Whitney U Test". *IOSR* Journal of Mathematics. ISSN: 2278-5728, Vol.7, pp 52-56.
5. Siegel, S (1988):"Nonparametric Statistics for the Sciences", McGraw-Hill, Kogakusha Ltd, Tokyo, 399. International student Edition
6. Gupta, S.C. (2011): Fundamentals of Statistics.Sixth Revised and Enlarged Edition. Himilaza Publishing House PVT Ltd. Mumbai-400 004. Pp 16.28-16.31.
7. Schaffer, M. (2010): "Procedure for Monte Carlo Simulation". SGPE QM Lab 3. Monte Carlos Mark version of 4.10.2010.

APPENDIX A

Simulated Data of Weibull (7,3) for sample size of 5

1	Family A	Family B	6	Family A	Family B	11	Family A	Family B	16	Family A	Family B
	2.0549	2.14784		2.96402	3.51324		2.94196	2.07892		3.90147	2.44272
	1.85224	2.75316		2.91013	1.81929		3.29378	3.2387		2.10908	3.49174
	3.08328	2.80008		3.45238	2.77946		3.04343	2.58707		1.94705	2.84907
	2.90837	2.35402		3.27096	3.50724		2.90073	2.99723		3.2844	3.06377
	3.14121	2.47486		2.89771	3.23779		2.137	3.23329		3.32377	3.30414
2	2.88806	3.13144	7	2.93772	3.0863	12	2.90932	2.87299	17	3.63141	3.52207
	2.77326	2.87969		2.40444	2.46448		3.13589	2.93109		2.6874	2.33098
	3.19739	2.35186		2.45472	2.52569		2.17833	2.6693		3.20825	1.22832
	2.07176	2.60132		3.11643	2.74658		2.21586	2.99436		2.84293	2.87588
	1.93168	3.07957		2.18975	2.70344		2.82251	2.72761		2.8962	3.54094
3	2.3961	2.76598	8	3.42744	3.04821	13	2.8056	3.24986	18	2.27389	2.90326
	3.19833	3.31849		3.18529	3.55529		2.6185	3.19856		2.94375	3.20292
	2.30491	2.60751		2.8518	3.36748		3.08287	2.38237		2.8712	2.70119
	2.57209	3.05334		2.64509	2.85469		3.19403	3.01101		2.74751	3.12434
	2.53297	2.9933		2.40766	3.00503		2.38355	2.7196		1.98065	2.3821

4	2.7355	2.2393	9	2.82601	3.02524	14	2.9674	3.09445		.	.
	3.33363	3.41242		2.6148	1.71342		2.35219	3.26889		.	.
	2.45461	2.87357		3.43739	3.43751		2.36213	3.96679		.	.
	2.5648	3.06979		3.30584	2.36777		3.08225	2.71729		.	.
	2.83251	2.34117		1.99436	2.74342		3.1595	2.87068		.	.
5	2.57779	2.65905	10	2.53308	2.07577	15	3.18469	2.17382	30	2.78797	1.79462
	3.33877	2.48479		1.97652	2.99061		3.43874	3.21076		1.21927	1.96139
	3.11277	3.11663		3.03554	2.96309		3.10281	1.85871		2.54097	3.56915
	3.68913	2.94917		2.63108	2.2902		3.4404	2.65302		2.0308	3.43897
	3.38483	3.15906		2.68229	3.05492		2.90121	3.00366		2.88939	2.36357

Simulated Data of G(4,0.3) for sample size of 5

1	Family A	Family B	6	Family A	Family B	11	Family A	Family B	16	Family A	Family B
	0.540693	1.132631		0.358768	2.823391		0.936434	0.536449		0.850696	1.043107
	1.099999	0.252437		0.784115	0.5173		1.426788	0.606461		1.762179	2.177001
	1.257383	0.530388		2.281035	1.112356		0.772563	2.050076		0.286225	3.211577
	1.350839	0.666464		1.511695	0.187971		1.855367	0.84629		0.440526	1.015095
	0.822252	1.973966		1.730911	1.321935		1.224994	0.936048		1.282424	1.298899
2	0.388482	1.164518	7	0.930303	0.387684	12	1.607881	0.809028	17	0.54221	0.451363
	3.367172	2.664101		2.490846	0.779647		1.655428	0.979611		2.76167	1.723086
	1.615149	1.2452		2.677828	0.920208		0.890015	0.887636		1.067012	2.417414
	1.652795	2.030195		1.745883	2.201069		0.986908	1.319078		0.477154	1.742978
	0.987986	0.780936		1.840938	0.61363		1.124742	1.157486		0.744717	1.153614
3	2.032877	1.637004	8	0.431434	2.036976	13	1.513215	0.774746	18	1.307757	2.038412
	0.743287	1.946726		0.414789	1.874285		0.395753	0.468649		0.79132	1.727142
	1.741365	1.070626		2.36039	0.751185		0.819668	0.781594		0.797857	1.449444
	0.617396	0.666522		0.559442	1.497564		1.561971	1.220831		1.096657	1.06419
	0.756069	1.882293		1.706953	1.129871		0.909478	0.224561		1.411843	0.643461
4	0.58093	0.35945	9	1.333053	1.22878	14	0.669428	0.87758		.	.
	1.286098	0.97924		0.65221	2.670646		0.59603	1.572339		.	.
	0.943381	2.477932		1.62474	1.541398		1.402757	1.547851		.	.
	1.287931	1.295778		2.186741	0.828445		0.932162	0.955364		.	.
	1.821823	1.25112		0.742813	2.59682		1.19045	2.021954		.	.
5	1.447551	0.588709	10	0.75928	0.630048	15	0.350237	1.420993	30	1.346355	1.04301
	0.772286	1.272155		1.065302	2.526689		0.31228	1.365376		1.211608	1.109491
	1.424602	1.242772		1.035784	0.737097		1.617787	1.18782		0.624364	1.601369
	0.966316	0.347502		0.655443	0.679389		0.64783	0.432679		0.954253	0.819451
	1.202835	1.173304		0.635915	0.570286		1.690238	0.430992		1.289154	0.319329

Simulated Data of G(4,0.3) for sample size of 10

1	Family A	Family B	4	Family A	Family B	7	Family A	Family B	10	Family A	Family B
	1.47499	2.16983		1.88868	2.08685		3.22103	1.83574		2.32448	2.0299
	1.42161	2.39375		2.18657	3.36969		1.95863	2.58577		3.64263	3.64746
	2.48126	1.25254		1.90092	2.17343		2.44478	1.56001		1.81066	1.61465
	3.21448	3.0012		2.62925	2.64181		1.85193	1.73784		1.68763	1.54577
	2.18517	1.18478		3.73508	2.57196		2.63755	2.43675		2.84849	2.0025
	2.92477	1.72976		2.05133	3.0852		1.33498	2.15427		2.1945	2.61388
	1.45767	2.26609		2.13	1.8455		2.17335	1.59713		1.46567	1.92751
	2.59934	1.11223		1.84952	2.77504		1.17087	1.4318		2.31214	2.4049
	1.88754	2.22914		2.4633	2.67339		1.42922	3.73844		1.72302	3.44591
	1.46279	2.76588		1.67666	1.44516		2.63589	2.12359		1.31912	2.79673
2	1.94833	2.122	5	2.34784	1.67795	8	2.92256	1.48792		.	.
	1.72293	1.19955		1.28589	1.78727		2.31797	2.19734		.	.
	2.12779	2.02204		2.03819	2.16127		1.91942	1.37619		.	.
	1.9467	1.46287		3.01982	1.50966		2.91263	1.91007		.	.
	2.47781	2.48328		2.75024	1.47947		1.79161	2.2411		.	.
	3.24088	1.66386		2.10093	2.27131		1.51698	1.38885		.	.
	1.65636	4.05627		1.46282	1.81715		2.48521	2.39244		.	.
	2.01588	2.38656		2.94184	1.53951		3.22881	1.52286		.	.
	1.82542	3.30058		1.70085	1.73702		1.54559	2.7478		.	.
	1.798	2.40882		2.11124	1.64646		2.29959	1.6585		.	.
3	1.33483	3.20387	6	2.35779	1.82536	9	2.07225	1.63325	30	1.66275	2.71397
	2.78723	2.1343		2.24233	2.24667		2.25963	1.77296		1.1359	1.84029
	2.02814	2.32298		2.11648	1.65336		1.56957	1.8824		3.34375	2.46886
	2.11915	1.55359		1.6537	1.69109		2.95141	1.49451		2.74775	2.49164
	1.82279	2.56755		1.83126	1.62992		1.8214	4.30432		2.12868	3.28533
	2.04075	2.20246		2.03558	2.18532		2.83388	2.29046		3.12887	2.29621
	2.30211	4.10863		1.76504	2.28915		2.18556	1.85333		1.33987	2.81134
	1.7381	1.87024		2.29264	2.29546		1.65249	1.53898		3.01046	2.22307
	1.91443	1.36457		2.49943	2.92259		2.41737	2.1812		2.8458	3.17786
	4.27942	2.10988		2.13913	2.38827		1.84901	1.60819		1.51609	1.97865

Appendix B

Table A: Test Statistic and P-values of Weibull Distribution using Mann-Whitney U-Test for Sample size 5

S/N	Chi-Square	P-Value	Remark	Accuracy of Decision
1	0.522	0.6015	Accept Ho	Correct Decision
2	0.522	0.6015	Accept Ho	Correct Decision
3	1.776	0.0758	Accept Ho	Correct Decision
4	0.104	0.9168	Accept Ho	Correct Decision
5	1.358	0.1745	Accept Ho	Correct Decision
6	0.104	0.9168	Accept Ho	Correct Decision
7	0.9168	0.4647	Accept Ho	Correct Decision
8	1.149	0.2506	Accept Ho	Correct Decision
9	0.313	0.754	Accept Ho	Correct Decision

10	0.522	0.6015	Accept Ho	Correct Decision
11	0.104	0.9168	Accept Ho	Correct Decision
12	0.522	0.6015	Accept Ho	Correct Decision
13	0.522	0.6015	Accept Ho	Correct Decision
14	1.149	0.2506	Accept Ho	Correct Decision
15	1.776	0.0758	Accept Ho	Correct Decision
16	0.104	0.9168	Accept Ho	Correct Decision
17	0.522	0.6015	Accept Ho	Correct Decision
18	1.149	0.2506	Accept Ho	Correct Decision
19	0.104	0.9168	Accept Ho	Correct Decision
20	0.94	0.3472	Accept Ho	Correct Decision
21	0.313	0.754	Accept Ho	Correct Decision
22	0.522	0.6015	Accept Ho	Correct Decision
23	1.776	0.0758	Accept Ho	Correct Decision
24	1.776	0.0758	Accept Ho	Correct Decision
25	0.94	0.3472	Accept Ho	Correct Decision
26	0.522	0.6015	Accept Ho	Correct Decision
27	1.984	0.0472	Reject Ho	Correct Decision
28	1.358	0.1745	Accept Ho	Correct Decision
29	0.731	0.4647	Accept Ho	Correct Decision
30	0.313	0.754	Accept Ho	Correct Decision

Table b: Test Statistic and P-values of Weibull Distribution using MMWU Test for size 5

S/N	Chi-Square	P-Value	Remark	Accuracy of Decision
1	5.20833333	0.022479	Reject H_o	Incorrect Decision
2	9.300595238	0.002291	Reject H_o	Incorrect Decision
3	5.00801282	0.02523	Reject H_o	Incorrect Decision
4	6.853070175	0.008849	Reject H_o	Incorrect Decision
5	5.00801282	0.02523	Reject H_o	Incorrect Decision
6	5.42534722	0.019846	Reject H_o	Incorrect Decision
7	6.20039683	0.012772	Reject H_o	Incorrect Decision
8	5.07305195	0.024301	Reject H_o	Incorrect Decision
9	5.20833333	0.022479	Reject H_o	Incorrect Decision
10	5.00801282	0.02523	Reject H_o	Incorrect Decision
11	5.20833333	0.022479	Reject H_o	Incorrect Decision
12	5.20833333	0.022479	Reject H_o	Incorrect Decision
13	6.20039683	0.012772	Reject H_o	Incorrect Decision
14	9.300595238	0.002291	Reject H_o	Incorrect Decision
15	5.00801282	0.02523	Reject H_o	Incorrect Decision
16	6.20039683	0.012772	Reject H_o	Incorrect Decision
17	5.00801282	0.02523	Reject H_o	Incorrect Decision
18	5.7444853	0.016541	Reject H_o	Incorrect Decision
19	5.07305195	0.024301	Reject H_o	Incorrect Decision
20	5.20833333	0.022479	Reject H_o	Incorrect Decision
21	9.300595238	0.002291	Reject H_o	Incorrect Decision
22	5.7444853	0.016541	Reject H_o	Incorrect Decision
23	6.853070175	0.008849	Reject H_o	Incorrect Decision
24	1.18E+01	5.81E-04	Reject H_o	Incorrect Decision
25	5.42534722	0.019846	Reject H_o	Incorrect Decision
26	5.07305195	0.024301	Reject H_o	Incorrect Decision
27	9.300595238	0.002291	Reject H_o	Incorrect Decision
28	5.20833333	0.022479	Reject H_o	Incorrect Decision
29	5.20833333	0.022479	Reject H_o	Incorrect Decision
30	5.20833333	0.022479	Reject H_o	Incorrect Decision

Table c: Test Statistic and P-values of Weibull Distribution using Two K.S Test for size 5

S/N	Chi-Square	P-Value	Remark	Accuracy of Decision
1	0.6	0.251	Accept H_o	Correct Decision
2	0.4	0.752	Accept H_o	Correct Decision
3	0.8	0.052	Accept H_o	Correct Decision
4	0.4	0.752	Accept H_o	Correct Decision
5	0.6	0.251	Accept H_o	Correct Decision
6	0.4	0.752	Accept H_o	Correct Decision
7	0.6	0.251	Accept H_o	Correct Decision
8	0.6	0.251	Accept H_o	Correct Decision
9	0.6	0.251	Accept H_o	Correct Decision
10	0.2	1	Accept H_o	Correct Decision
11	0.4	0.752	Accept H_o	Correct Decision
12	0.2	1	Accept H_o	Correct Decision
13	0.4	0.752	Accept H_o	Correct Decision
14	0.4	0.752	Accept H_o	Correct Decision
15	0.4	0.752	Accept H_o	Correct Decision
16	0.6	0.251	Accept H_o	Correct Decision
17	0.4	0.752	Accept H_o	Correct Decision
18	0.4	0.752	Accept H_o	Correct Decision
19	0.4	0.752	Accept H_o	Correct Decision
20	0.6	0.251	Accept H_o	Correct Decision
21	0.4	0.752	Accept H_o	Correct Decision
22	0.4	0.752	Accept H_o	Correct Decision
23	0.6	0.251	Accept H_o	Correct Decision
24	0.6	0.251	Accept H_o	Correct Decision
25	0.6	0.251	Accept H_o	Correct Decision
26	0.4	0.752	Accept H_o	Correct Decision
27	0.6	0.251	Accept H_o	Correct Decision
28	0.6	0.251	Accept H_o	Correct Decision
29	0.4	0.752	Accept H_o	Correct Decision
30	0.4	0.752	Accept H_o	Correct Decision

Table d: Test Statistic and P-values of Gamma Distribution using K.S for Sample 10

S/N	Chi-square	P-value	Remark	Decision
1	0.3	0.238	Accept Ho	Correct Decision
2	0.3	0.238	Accept Ho	Correct Decision
3	0.25	0.452	Accept Ho	Correct Decision
4	0.15	0.959	Accept Ho	Correct Decision
5	0.25	0.452	Accept Ho	Correct Decision
6	0.15	0.959	Accept Ho	Correct Decision
7	0.2	0.739	Accept Ho	Correct Decision
8	0.3	0.238	Accept Ho	Correct Decision
9	0.3	0.238	Accept Ho	Correct Decision
10	0.25	0.452	Accept Ho	Correct Decision
11	0.2	0.739	Accept Ho	Correct Decision
12	0.2	0.739	Accept Ho	Correct Decision
13	0.25	0.452	Accept Ho	Correct Decision
14	0.2	0.739	Accept Ho	Correct Decision
15	0.25	0.452	Accept Ho	Correct Decision
16	0.15	0.959	Accept Ho	Correct Decision

17	0.1	1	Accept Ho	Correct Decision
18	0.2	0.739	Accept Ho	Correct Decision
19	0.15	0.959	Accept Ho	Correct Decision
20	0.25	0.452	Accept Ho	Correct Decision
21	0.25	0.452	Accept Ho	Correct Decision
22	0.3	0.238	Accept Ho	Correct Decision
23	0.15	0.959	Accept Ho	Correct Decision
24	0.3	0.238	Accept Ho	Correct Decision
25	0.2	0.739	Accept Ho	Correct Decision
26	0.35	0.112	Accept Ho	Correct Decision
27	0.25	0.452	Accept Ho	Correct Decision
28	0.2	0.739	Accept Ho	Correct Decision
29	0.2	0.739	Accept Ho	Correct Decision
30	0.3	0.238	Accept Ho	Correct Decision

Table e: Test Statistic and P-values of Weibull Distribution using Mann-Whitney U-Test

S/N	Chi-square	P-value	Remark	Decision
1	2.495	0.0126	Reject Ho	Incorrect Decision
2	0.756	0.4497	Accept Ho	Correct Decision
3	2.343	0.0191	Reject Ho	Incorrect Decision
4	1.436	0.1509	Accept Ho	Correct Decision
5	0.983	0.3258	Accept Ho	Correct Decision
6	2.041	0.0413	Reject Ho	Incorrect Decision
7	0.227	0.8206	Accept Ho	Correct Decision
8	1.058	0.2899	Accept Ho	Correct Decision
9	0.68	0.4963	Accept Ho	Correct Decision
10	0.529	0.5967	Accept Ho	Correct Decision
11	0.151	0.8798	Accept Ho	Correct Decision
12	0.076	0.9397	Accept Ho	Correct Decision
13	1.058	0.2899	Accept Ho	Correct Decision
14	0.907	0.3643	Accept Ho	Correct Decision
15	0.076	0.9397	Accept Ho	Correct Decision
16	0.907	0.3643	Accept Ho	Correct Decision
17	1.285	0.1988	Accept Ho	Correct Decision
18	0.907	0.3643	Accept Ho	Correct Decision
19	0.151	0.8798	Accept Ho	Correct Decision
20	0.832	0.4057	Accept Ho	Correct Decision
21	0.302	0.7624	Accept Ho	Correct Decision
22	0.454	0.6501	Accept Ho	Correct Decision
23	0.68	0.4963	Accept Ho	Correct Decision
24	2.343	0.0191	Reject Ho	Incorrect Decision
25	1.361	0.1736	Accept Ho	Correct Decision
26	0.076	0.3643	Accept Ho	Correct Decision

27	0.907	0.3643	Accept Ho	Correct Decision
28	0.076	0.9397	Accept Ho	Correct Decision
29	0.227	0.8206	Accept Ho	Correct Decision
30	1.058	0.2899	Accept Ho	Correct Decision

Table f: Test Statistic and P-values of Weibull Distribution using MMWU

S/N	Chi-square	P-value	Remark	Decision
1	1.77E+01	2.56E-05	Reject Ho	Incorrect Decision
2	10.41666667	0.001248831	Reject Ho	Incorrect Decision
3	1.62E+01	5.57E-05	Reject Ho	Incorrect Decision
4	1.17E+01	6.29E-04	Reject Ho	Incorrect Decision
5	10.72501073	0.001056971	Reject Ho	Incorrect Decision
6	1.41E+01	1.72E-04	Reject Ho	Incorrect Decision
7	10.03613007	0.001534993	Reject Ho	Incorrect Decision
8	1.09E+01	9.88E-04	Reject Ho	Incorrect Decision
9	10.33484911	0.001305418	Reject Ho	Incorrect Decision
10	10.1999184	0.001404469	Reject Ho	Incorrect Decision
11	10.01602564	0.00155184	Reject Ho	Incorrect Decision
12	10.0040016	0.001562004	Reject Ho	Incorrect Decision
13	1.09E+01	9.88E-04	Reject Ho	Incorrect Decision
14	10.61120543	0.001124044	Reject Ho	Incorrect Decision
15	10.0040016	0.001562004	Reject Ho	Incorrect Decision
16	10.61120543	0.001124044	Reject Ho	Incorrect Decision
17	1.13E+01	7.72E-04	Reject Ho	Incorrect Decision
18	10.61120543	0.001124044	Reject Ho	Incorrect Decision
19	10.01602564	0.00155184	Reject Ho	Incorrect Decision
20	10.50861707	0.001188191	Reject Ho	Incorrect Decision
21	10.06441224	0.001511608	Reject Ho	Incorrect Decision
22	10.1461039	0.001446065	Reject Ho	Incorrect Decision
23	10.33484911	0.001305418	Reject Ho	Incorrect Decision
24	1.62E+01	5.57E-05	Reject Ho	Incorrect Decision
25	11.48897059	0.000700104	Reject Ho	Incorrect Decision
26	10.0040016	0.001562004	Reject Ho	Incorrect Decision
27	10.61120543	0.001124044	Reject Ho	Incorrect Decision
28	10.61120543	0.001124044	Reject Ho	Incorrect Decision
29	10.03613007	0.001534993	Reject Ho	Incorrect Decision
30	1.09E+01	9.88E-04	Reject Ho	Incorrect Decision