BLOCKING PROBABILITY FOR CUSTOMERS' FLOW IN BANKING SECTOR (A CASE STUDY OF FOUR BANKS IN ANAMBRA STATE)

Etaga, H.O., Ajao, K., Awopeju, A., Etaga, M.C
[1]Department of Statistics, Nnamdi Azikiwe University,
Awka, Anambra State, Nigeria
[2]Department of Educational Foundations,
Nnamdi Azikiwe University, Awka, Anambra State, Nigeria
Email: ho.etaga@unizik.edu.ng

ABSTRACT

Flow of people can be affected by geographical location. In location of industry, these two are taken into consideration to reduce loss and maximize profit. In banking industry, flow of customers in a banking sector can be affected by location. This research is set to investigate possible variation in queuing model of banks with respect to geographical location. For this study, four banks were randomly selected. Primary data were used for the study. The results of the analysis shows that the queue models depend on location of the bank and banks cited in similar location have similar models. In like manner, blocking probability was investigated using Hayward Approximation Estimate, Jagarman Estimate and Recursion Estimate. Blocking Probability computed revealed that irrespective of location, Recursion Estimate is lower than any other method used which implies that the method is highly sensitive to the detection of blocking probability.

Keyword: Blocking Probability, Queuing, Estimates, Heyward Approximation. Recursion,

Introduction.

Queuing theory is the mathematical study of waiting lines of customers in a service system such as fuel stations, supermarket check-out counters, post offices, cafeteria, and banking halls. In queuing theory, a model is constructed so that important queuing characteristics of the service systems can be obtained as a measure of the service performance of the systems. Examples of such characteristics are queue lengths (number of customers waiting to be served), the waiting times involved, etc.

Arrivals at a service system may be drawn from a finite or an infinite population. The distinction is important because the analyses are based on different premises and require different equation for their solution. A finite population refers to the limited size customer pool that will use the service and, at time s, form a line. The reason this finite classification is important is because when a customer leaves its position as a member of the population of users, the size of the user group is therefore reduced by one, which reduces the probability of the next occurrence. Conversely, when a customer is serviced and returns to the user group, the population increases and the probability of a user requiring service also increases. These finite classes of problems require a separate set of formulas from that of the infinite population case. An infinite population is one large enough in relation to the service system so that the changes in the population size caused by subtraction or addition to the population do not significantly affect the system probabilities.

In banking system, the arrival process consists of the arrival rate of customers per unit time and the probability distribution of inter-arrival times between successive customer arrivals. The service process consists of the service discipline, number of servers S =1, 2 …n, the service rate of customers per unit time, the probability distribution of number of customers completely served in a specified time interval, the customer service time or the distribution of inter-service time of successive customers. Blocking occurs when a server is unavailable (unable to serve the customers) caused by limited capacity. Blocking Probability is used to determine the chance of occurrence of event that servers are not serving customers. Most existing queuing models for banks do not estimate blocking probability which is essential in queuing. Therefore, blocking probability in queuing model is a challenge to be solved especially in developing countries.

The study examines flow of customers and the blocking probability in rural and urban banks in Anambra State. The banks covered in the study are First Banks Plc, Abagana (Rural Area); Union Banks Plc, Abagana (Rural Area); First Banks Plc, Ziks Avenue, Awka (Urban Area) and Union Banks Plc, Ziks Avenue, Awka (Urban Area). Therefore, the study is limited to observations in the selected banks in the State.

LITERATURE REVIEW

Robert and Christian (2013) studied queue abandonment using a hospital emergency department, and found that abandonment is not only influenced by waiting time, but also by the queue length and the observable queue flows during the waiting exposure. Also, additional person in the queue or an additional arrival to the queue leads to an increase in abandonment probability equivalent to a fifteen minute or nine-minute increase in waiting time.

Galit (2014) investigated the impact of blocking in modeling of queuing system with possibility of revisiting the server(s). The study involves the development and analyzing of a queuing model, which was called Erlang-R, where “R" stands for Re-Entrant customers. The Erlang-R model accommodates customers who return to service several times during their sojourn within the system. According to the researcher, the study was motivated by healthcare systems, in which workloads are time-inhomogeneous and patients often go through a discontinuous service process.

Paul *et al. (2012)* studied effect of blocking probability in queuing system using three server system approaches. Three estimators were used, namely: Maximum likelihood estimator (MLE), a Consistent Asymptotically Normal (CAN) estimator and asymptotic confidence limits for the expected number of entities in the system in a three service point tandem queue with blocking and busy service point. According to the researchers, the estimators adequately capture the variation in the queuing properties of the system studied and made it possible to predict the future occurrence of such event.

Mahima *et al.* (2014) used the knowledge of waiting time, birth and death processes to model the possibility of delay in networking. To quantify capacity improvement, blocking probability of voice traffic was calculated using Erlang B formula. The calculation was based on the assumption that all users require same amount of resources to satisfy their rate requirement. In an OFDMA system, each user requires different number of subcarriers to meet its rate requirement. This resource requirement depends on the Signal to Interference Ratio (SIR) experienced by a user. Therefore, the Erlang B formula cannot be employed to compute blocking probability in an OFDMA network. In the paper, the researchers proposed an analytical expression to compute the blocking probability of relay based cellular OFDMA network. The expression of probability distribution of a user’s resource requirement is based on its experienced SIR. Users were classified into various classes depending on their subcarrier requirement. Considering the system to be a multi-dimensional system with different classes and evaluating the blocking probability of system using the multi-dimensional Erlang loss formulas made it possible for the computation of probability of blocking in the system. The model formulated is useful in performance evaluation, design, planning of resources and call admission control of relay based cellular OFDMA networks like LTE.

Perros and Altoik (1986) further researched the blocking effects in a finite buffer model with tandem flows using the single-node decomposition approach. The arrival process and the service times followed Poisson and exponential distribution.

METHODOLOGY

Non-probability sampling technique (convenient sampling) was used in the selection of banks for the study as purposive sampling was carried out to select rural area with banks whose sub-branch also exist in the urban center of the State. Abagana has First Bank branch Plc and it is also at Awka which is urban centre. In the same locality, Union Bank branch Plc has a branch at Abagana and Awka.

Blocking probability

Blocking occurs when a server is unavailable (unable to serve the customers) due to limited capacity. Blocking Probability is used to determine the chance of occurrence of event that servers are not serving customers and the number of customers is increasing. The methods of estimation of blocking probability that will be used in this work are as obtained in Moshe, 2016:

(1) **Hayward Approximation**

$$P_{blk} = \frac{A_{eq}}{N_{eq}}$$

$$N_{eq} = \frac{A_{eq(M+Z)}}{M+Z-I} - M - 1 \qquad 1$$

Where P_{blk}= blocking probability, M= Average Delay Timei.e. M = $\frac{1}{\mu-\lambda}$, λ = is mean arrival time and μ = is mean service time, $Z = \frac{V}{M}$, $V = M\left(1 - M + \frac{\left(\frac{\lambda}{\mu}\right)}{S+1+M-\left(\frac{\lambda}{\mu}\right)}\right)$

and $A_{eq} = V + 3Z(Z-1)$

(2) **Jagarman**

$$E_m A = 1 - \frac{A}{m} \qquad 2$$

$E_m A$ = blocking probability, $A = \frac{\lambda}{\mu}$ where λ = is mean arrival time and μ = is mean service time, m = $\frac{1}{\mu-\lambda}$

(3) **Recursive**

$$E_{\boldsymbol{m}}A = \frac{\mathrm{A}E_{m-1}(A)}{m + \mathrm{A}E_{m-1}(A)} \qquad 3$$

$A = \dfrac{\lambda}{\mu}$ where λ = is mean arrival time and μ = is mean service time, m = is number of available servers and K = is total number of servers in the system. $E_{\boldsymbol{m}}A$ = blocking probability $I_m(A)$ = is boundary condition such that $I_0(A)$ = 1, where boundary condition is defined as extreme probability of the estimate such that the probability of having blocking in the process when no serve is available is 1.

RESULTS:

At the end of the computations, the values obtained will be summarized in tabular form for easy comparison.

In the research, 2 banks were considered with a branch in rural and a branch in urban location to model the activities of banks in the two possible geographical classifications.

Location 1: Union Bank (Rural Area)

Blocking Probability

By definition, blocking probability is the possibility of denial of service to a customer due to unavailability of server. This could be as a result of intermittent divided attention whereby a server needed to attend to a co-worker within the system either for collection of vital document or cash in the banking system, or even for some personal reasons.

As stated, three blocking probability estimation methods were used. The computation is as follows;

1. **Hayward Approximation Estimate**

$$P_{blk} = \frac{A_{eq}}{N_{eq}}$$

where $N_{eq} = \frac{A_{eq}(M+Z)}{M+Z-I} - M - 1$ and $A_{eq} = V + 3Z(Z-1)$

M= Average Delay Time i.e. M = $\dfrac{1}{\mu-\lambda} = \dfrac{1}{(2.2423-1.8280)} = 2.4137$,

$Z = \dfrac{V}{M}$ where

$$V = M\left(1 - M + \frac{\left(\frac{\lambda}{\mu}\right)}{S+1+M-\left(\frac{\lambda}{\mu}\right)}\right) = 2.4137\left(1 - 2.4137 + \frac{\left(\frac{1.8280}{2.2423}\right)}{3+1+2.4137-\left(\frac{1.8280}{2.2423}\right)}\right)$$

$$V = 2.4137\left(1 - 2.4137 + \frac{0.8152}{5.5985}\right) = 2.4137(1 - 2.4137 + 0.1456) = -3.0608$$

$$Z = \frac{-3.0608}{2.4137} = -1.2681$$

$$A_{eq} = V + 3Z(Z-1) = -3.0608 + 3(-1.2681)(-1.2681 - 1)$$

$$A_{eq} = 5.5677$$

$N_{eq} = \frac{A_{eq(M+Z)}}{M+Z-I} - M - 1 = \frac{5.5677(2.4137-1.2681)}{2.4137-1.2681-I} - 2.4137 - 1$

$N_{eq} = 40.3937$

$$P_{blk} = \frac{A_{eq}}{N_{eq}} = \frac{5.5677}{40.3937} = 0.1378$$

(2) Jagarman Estimate

$$E_{\boldsymbol{m}}A = 1 - \frac{A}{m}$$

where m = $\frac{1}{\mu-\lambda} = \frac{1}{(2.2423-1.8280)} = 2.4137$ and $A = \frac{\lambda}{\mu} = \frac{1.8280}{2.2423} = 0.8152$

Therefore, $E_{\boldsymbol{m}}A = 1 - \frac{0.8152}{2.4137} = 0.6623$

3. Recursive Estimate

$$E_{\boldsymbol{m}}A = \frac{\mathrm{A}E_{m-1}(A)}{m + \mathrm{A}E_{m-1}(A)}$$

For m = 3, we have;

$E_{\boldsymbol{3}}(A) = \frac{\mathrm{A}E_{m-1}(A)}{m + \mathrm{A}E_{m-1}(A)} = \frac{\lambda^3}{6[\mu^2(\mu+\lambda)] + 3\mu\lambda^2 + \lambda^3}$

$= \frac{(1.8280)^3}{6[(2.2423)^2(2.2423+1.8280)] + 3(2.2423)(1.8280)^2 + (1.8280)^3}$

$= \frac{6.1084}{122.7906 + 28.5869} = 0.04035$

Considering the three methods of estimation of blocking probability, the least was achieved using Recursive method and Jagarman method gave the highest probability value.

Location 2: Union Bank (Urban Area)

The bank is situated at Zik's Avenue, Awka. Based on the data collected during field survey, we have;

Blocking Probability

A .Hayward Approximation Estimate

$$P_{blk} = \frac{A_{eq}}{N_{eq}}$$

where $N_{eq} = \frac{A_{eq}(M+Z)}{M+Z-I} - M - 1$ and $A_{eq} = V + 3Z(Z-1)$

M= Average Delay Time i.e. M = $\frac{1}{\mu-\lambda} = \frac{1}{(2.5211-1.8280)} = 1.4428$,

$Z = \frac{V}{M}$ where

$$V = M\left(1 - M + \frac{\left(\frac{\lambda}{\mu}\right)}{S + 1 + M - \left(\frac{\lambda}{\mu}\right)}\right) = 1.4428\left(1 - 1.4428 + \frac{\left(\frac{1.8280}{2.5211}\right)}{3 + 1 + 1.4428 - \left(\frac{1.8280}{2.5211}\right)}\right)$$

$$V = 1.4428\left(1 - 1.4428 + \frac{0.7251}{4.7177}\right) = 1.4428(1 - 1.4428 + 0.1537) = -0.2891$$

$$Z = \frac{-0.2891}{1.4428} = -0.2004$$

$$A_{eq} = V + 3Z(Z - 1) = -0.2004 + 3(-0.2004)(-0.2004 - 1)$$

$$A_{eq} = 0.5213$$

$$N_{eq} = \frac{A_{eq(M+Z)}}{M+Z-I} - M - 1 = \frac{0.5213(1.4428-0.2004)}{1.4428-0,2004-I} - 1.4428 - 1$$

N_{eq} = 1.2292

$$P_{blk} = \frac{A_{eq}}{N_{eq}} = \frac{0.5213}{1.2293} = 0.4241$$

B. Jagarman Estimate

$$E_{\boldsymbol{m}}A = 1 - \frac{A}{m}$$

where m = $\frac{1}{\mu - \lambda} = \frac{1}{(2.5211 - 1.8280)} = 1.4428$ and $A = \frac{\lambda}{\mu} = \frac{1.8280}{2.5211} = 0.7251$

Therefore, $E_{\boldsymbol{m}}A = 1 - \frac{0.7251}{1.4428} = 0.5027$

C. Recursive Estimate

$$E_{\boldsymbol{m}}A = \frac{\mathrm{A}E_{m-1}(A)}{m + \mathrm{A}E_{m-1}(A)}$$

For m = 3, we have;

$$E_{\boldsymbol{3}}(A) = \frac{\mathrm{A}E_{m-1}(A)}{m + \mathrm{A}E_{m-1}(A)} = \frac{\lambda^3}{6\left[\mu^2(\mu + \lambda)\right] + 3\mu\lambda^2 + \lambda^3}$$

$$= \frac{(1.8280)^3}{6\left[(2.5211)^2(2.5211 + 1.8280)\right] + 3(2.5211)(1.8280)^2 + (1.8280)^3}$$

$$= \frac{6.1084}{165.8547 + 28.6150} = 0.03141$$

Considering the three methods of estimation of blocking probability, the least was achieved using Recursive method and Jagarman method gave the highest probability value.

Location 3: First Bank (Rural Area)

A. Hayward Approximation Estimate

$$P_{blk}=\frac{A_{eq}}{N_{eq}}$$

where $N_{eq} = \frac{A_{eq}(M+Z)}{M+Z-I} - M - 1$ and $A_{eq} = V + 3Z(Z-1)$

M= Average Delay Time i.e. M = $\frac{1}{\mu-\lambda}=\frac{1}{(1.7521-1.5374)}=0.2147$,

$Z=\frac{V}{M}$ where

$$V=M\left(1-M+\frac{\left(\frac{\lambda}{\mu}\right)}{S+1+M-\left(\frac{\lambda}{\mu}\right)}\right)=0.2147\left(1-0.2147+\frac{\left(\frac{1.5374}{1.7521}\right)}{3+1+0.2147-\left(\frac{1.5374}{1.7521}\right)}\right)$$

$$V=0.2147\left(1-0.2147+\frac{0.8775}{3.3372}\right)=0.2147(1-0.2147+0.2629)=0.2250$$

$$Z=\frac{0.2250}{0.2147}=1.4797$$

$$A_{eq}=V+3Z(Z-1)=0.2250+3(1.4797)(1.4797-1)$$

$$A_{eq}=2.1294$$

$$N_{eq} = \frac{A_{eq(M+Z)}}{M+Z-I} - M - 1 = \frac{2.1294(0.2147+1.4797)}{0.2147+1.4797-I} - 0.2147 - 1$$

$$N_{eq} = 4.2008$$

$$P_{blk}=\frac{A_{eq}}{N_{eq}}=\frac{2.1294}{4.2008}=0.5069$$

B. Jagarman Estimate

$$E_{\boldsymbol{m}}A = 1-\frac{A}{m}$$

where m = $\frac{1}{\mu-\lambda}=\frac{1}{(1.7521-1.5374)}=4.6577$ and $A=\frac{\lambda}{\mu}=\frac{1.5374}{1.7521}=0.8775$

Therefore, $E_{\boldsymbol{m}}A = 1-\frac{0.8775}{4.6577}=0.8116$

C. Recursive Estimate

$$E_{\boldsymbol{m}}A=\frac{\mathrm{A}E_{m-1}(A)}{m+\mathrm{A}E_{m-1}(A)}$$

For m = 3, we have;

$$E_3(A)=\frac{AE_{m-1}(A)}{m+AE_{m-1}(A)}=\frac{\lambda^3}{6\left[\mu^2(\mu+\lambda)\right]+3\mu\lambda^2+\lambda^3}$$

$$=\frac{(1.5374)^3}{6\left[(1.7521)^2(1.7521+1.5374)\right]+3(1.7521)(1.5374)^2+(1.5374)^3}$$

$$=\frac{3.6338}{60.5897+16.0573}=0.0474$$

Considering the three methods of estimation of blocking probability, the least was achieved using Recursive method and Jagarman method gave the highest probability value.

Location 4: First Bank (Urban Area)
The bank is also situated at Ziks Avenue, Awka. The bank has 4 servers.

A. Hayward Approximation Estimate

$$P_{blk}=\frac{A_{eq}}{N_{eq}}$$

where $N_{eq}=\frac{A_{eq}(M+Z)}{M+Z-I}-M-1$ and $A_{eq}=V+3Z(Z-1)$

M= Average Delay Time i.e. M = $\frac{1}{\mu-\lambda}=\frac{1}{(4.6813-4.4956)}=0.1857$,

$Z=\frac{V}{M}$ where

$$V=M\left(1-M+\frac{\left(\frac{\lambda}{\mu}\right)}{S+1+M-\left(\frac{\lambda}{\mu}\right)}\right)=0.1857\left(1-0.1857+\frac{\left(\frac{4.4956}{4.6813}\right)}{4+1+0.1857-\left(\frac{4.4956}{4.6813}\right)}\right)$$

$$V=0.1857\left(1-0.1857+\frac{0.9603}{4.2254}\right)=0.1857(1-0.1857+0.2273)=0.1934$$

$$Z=\frac{0.1934}{0.1857}=1.0415$$

$$A_{eq}=V+3Z(Z-1)=0.1934+3(1.0415)(1.0415-1)$$

$$A_{eq}=0.3231$$

$$N_{eq}=\frac{A_{eq(M+Z)}}{M+Z-I}-M-1=\frac{0.3231(0.1857+1.0415)}{0.1857+1.0415-I}-0.2147-1$$

$$N_{eq}=0.5305$$

$$P_{blk}=\frac{A_{eq}}{N_{eq}}=\frac{0.3231}{0.5305}=0.6090$$

B. Jagarman Estimate

$$E_mA=1-\frac{A}{m}$$

where m = $\dfrac{1}{\mu-\lambda} = \dfrac{1}{(4.6813-4.4956)} = 5.3850$ and $A = \dfrac{\lambda}{\mu} = \dfrac{4.4956}{4.6813} = 0.9603$

Therefore, $E_{\boldsymbol{m}}A = 1 - \frac{0.9603}{5.3850} = 0.8217$

C. Recursive Estimate

$$E_{\boldsymbol{m}}A = \frac{\mathrm{A}E_{m-1}(A)}{m + \mathrm{A}E_{m-1}(A)}$$

For m = 4, we have;

$$E_{\boldsymbol{4}}(A) = \frac{\mathrm{A}E_{m-1}(A)}{m + \mathrm{A}E_{m-1}(A)} = \frac{\lambda^4}{24\left[\mu^3(\mu+\lambda)\right] + 12\mu^2\lambda^2 + 4\mu\lambda^3 + \lambda^3}$$

$$= \frac{(4.4956)^4}{24\left[(4.6813)^3(4.6813+4.4956)\right] + 12(4.6813)^2(4.4956)^2 + 4(4.6813)(4.4956)^3 + (4.4956)^{4^4}}$$

$$= \frac{408.4611}{22594.7043+5314.8266+1701.1334+408.4511} = \frac{408.4611}{30019.1154} = 0.0136$$

Considering the three methods of estimation of blocking probability, the least was achieved using Recursive method and Jagarman method gave the highest probability value.

Estimated Values for Blocking Probability

For each of the locations considered, based on the available data, the Table below shows summary of the values;

Table 1: Blocking Probabilities

	Location	**Hayward Method**	**Jagarman Estimation**	**Recursive Method**
Union Bank	Rural Centre	0.1378	0.6628	0.0404
	Urban Centre	0.4241	0.5027	0.0314
First Bank	Rural Centre	0.5069	0.8116	0.0474
	Urban Centre	0.6090	0.8217	0.0136

From Table 1, probability of blocking of rural centre of First Bank is higher than that of Union Bank irrespective of estimation method. Also, First bank has higher blocking probability for urban centre than Union Bank. This implies First Bank is more prone to blocking within the system than Union Bank.

In the determination of the blocking probability especially for banking sector, Recursive method has consistent ability when compared with Hayward method and Jagarman estimation. Therefore, Recursive method is best used for blocking probability of the banking sector.

Conclusion

Blocking probability of urban centre is higher than that of rural centre which implies there is higher chance of delay in urban centre than rural centre. Among the three methods of blocking probability estimation, Recursive Method gave consistent result which shows its superiority over other methods considered.

.

References

Bhavin P and Pravin B. (2012) Case Study for Bank ATM Queuing Model. International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622. Vol. 2, Issue 5, Pp.1278-1284

Galit Y. (2014): Erlang-R: A Time-Varying Queue with Reentrant Customers, in Support of Healthcare Staffing. MSOM, Vol.16, No. 2, Pp. 283-299.

Mahima M., Ranjan B.J and Abhay K. (2014): Application of Queuing in Flow of Information. Special Report. Information Network Lab. Department of Electrical Engineering Indian Institute of Technology Mumbai 400076 India.

Melamed B. (1979): Characterizations of Poisson traffic streams in Jackson queuing networks. Advances in Applied Probability, 11(2):422–438.

Moshe Z. (2016): Introduction to Queuing Theory and Stochastic Teletraffic Model. Electrical Electronics Department, City University of Hong Kong, China.

Perros H.G. and Altiok T. (1986): Approximate analysis of open networks of queues with blocking tandem configurations. IEEE, Transactions on Network Engineering. SE-12, 450-461.

Robert J. B. and Christian T. (2013): Waiting Patiently: An Empirical Study of Queue Abandonment in an Emergency Department. M.Sc Thesis. Department of Biostatistics, Wharton School, University of Pennsylvania, Philadelphia, PA 19104.