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MATHEMATICAL MODEL FOR THE DYNAMICS OF KIDNAPPING IN NIGERIA WITH OPTIMAL CONTROL

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Abstract.
Kidnapping is regarded as one of the most terrifying types of banditry in the world. In Nigeria, it is a volent act use for perpetuating political objectives and interest by politicians. We developed a compartmentalized mathematical model for the dynamics of kidnapping activity. Total human population was sub-divided into S(t), I(t), U(t), R(t), K(t) and D(t), a system non-linear first order ordinary differential equations was developed. Two equilibrium points were established (kidnapper free equilibrium point and equilibrium point with kidnapping activity), stability analysis was carried out and the analytical result shows that the two equilibrium points are local asymptotically stable (LAS). The equilibrium point with kidnapping activity is globally asymptotically stable (GAS) provided that the condition of the Lyapunov function is satisfied. Additionally, the fundamental reproduction number or threshold number was calculated followed by numerical simulations to back up the analytical results.
***Keywords**: Kidnappers, Equilibrium points, Reproduction number, Optimal Control Analysis.*

Introduction

Criminal activities have become of great concern to many nations in the world. Most of the leading criminal cases includes; armed robbery, banditry, cybercrimes, homicide, theft, burglary, kidnapping, etc. The goal of every nation is to curtain (eradicate if possible) the level of criminality in their domain. Different nations have different predominate crime(s) that strife in their domain. Marcelo (2018) revealed that crimes like illegal movement of illicit drugs such as cocaine and marijuana have been predominant in the North America and Colombia. Homicide is another trending crime in the western world today. Research has shown that approximately 490,000 deaths have been attributed to intentional homicide in 2020, (Marieke *et al.*, 2020). In Nigeria, most crimes include; child sexual abuse, domestic violence, human trafficking, money laundering, piracy, kidnapping, terrorism, etc. Nigeria has been considered as 140th position in the ranking of countries with crime especially in terms of banditry, robbery, kidnapping, (Transparency International Report, 2025). Over the year, kidnapping has taken the lead in the Nigerian crime rating due to the fact that politicians are now using it as a tool against their political opponents. Kidnapping initially originated as a novel tactic used by activists in the south-south oil-producing region to air their complaints against foreigners for the failing infrastructure in their neighbourhood, but currently, kidnapping is of great concern to the government and the public due to its frequent occurrences and spreading across the geopolitical regions of the country as well as the large number of deaths and loss of resources that it has incurred.

Kidnappers have motivations and objectives for kidnapping which may vary around the world. These objectives or motives may include; for prostitution, to murder, forceful marriages, to keep them in captivity so as to get ransom, for ritual purposes, etc. depending on the perpetrators and their society. Okoye *et al.,* (2020) Defines it as anarchy inside oneself or within a society which can bring bias, hatred, and violent thoughts about someone or group of people. Meanwhile, Ayuba (2020) investigated the catalysts for the kidnapping in Kaduna state, Nigeria and the result shows that poverty, poor governance leading to high level of corruption and moral degradation in the local communities has been its driving force of the state. Furthermore, Hazen & Horner (2007) noted that kidnapping can be perpetuated due to political bargaining or quest for political positions as well as economic gain and these have crippled many institutions in Nigeria.

Meanwhile, kidnapping has robbed individuals and the country in term of education, politics, transport, etc. Inyang & Ubong (2013) classified the economic effects of kidnapping into two namely; direct and indirect effects. The direct economics effect involves the monetary value that the individual or government may lose to the kidnappers. This may be in terms of ransom to secure the victim and this money most at times affects the economy of a state or society especially when the victim is a civil servant(s). The indirect economic effect of kidnapping is the increase in the national budgetary allocation on security matters, the government of the day will be force to channel more money meant for other economic development of the country. It has been estimated that about 500 dollar are globally paid on annual bases for ransom on kidnapping and Nigeria paid a huge percentage of it (Adebola & Temidayo, 2023). Again, kidnapping has cost a total disruption of academic activities in some places especially in the Northern part of Nigeria where schools have been short down due kidnapping and banditry. Many university lecturers, school proprietors or principals have witness several cases of kidnapping and the greatest among them is the abduction of chibok girls in Maiduguri, Nigeria, abduction of students and staff in St. Mary Catholic School, etc. Patrick *et al.*, (2024) investigated the effects of kidnapping on victim's educational activities in Chikun Local Gov't Area of Kaduna State and the result reveals kidnapping reduce school enrolment and lead to low academic performance by the victims. Furthermore, transportation system by road in Nigeria is recently facing a serious treat, this is owing to the fact that most kidnappers stop and abduct passengers in commercial buses. This is mostly seen in the north central part of Nigeria. Ohinda (2025) reveals that approximately 42 incidents of insecurity on North Central roads in 2021, affecting around 254 lives. In November 2018 alone, there were 71 murders and 183 kidnapped cases. The data further show that 90.7% of the victims of kidnapping occur on interstate transport (especially around North Central axis).

Other effects of kidnapping activities include daily low yield of agricultural food products because farmers (due to fear of the unknown) no longer go to their farm. This is predominantly seen in Benue State of Nigeria. The persistent difficulties in reducing insecurity in developed and emerging nations are highlighted by civil unrest, banditries, pervasive corruption, terror threats, and on-going kidnappings (Adebayo & Adepoju, 2018). .

A question may be asked, what are kidnapper's mode of recruiting members and operation? Peter & Osaat (2021) outlined that the major mode of operation of kidnappers is the grouping method. They pointed out that kidnappers group themselves into different categories and each of these categories has a leader who is answerable to the overall boss. Each group activity ranges from;

i. Group of individuals that are trained to specially task the kidnapped family members or relation pay the ransom without trace (task force group)
ii. Group of individuals that recruit and train susceptible individuals until they are qualified kidnappers and can handle operation (influencers and recruiters)
iii. Group of individuals that give accurate information or personal data about an individual or those that influence people to join kidnapping activities. They are mostly found on the streets or neighbourhood
iv. Group of individuals that carry out the kidnapping using arms (operation team)
v. Group if individuals that monitor the kidnapped (guards)

vi. Group of individuals that track and monitor phone calls and GPRS, internets, mails and social media

Ibrahim & Mukhtar (2016), in their investigation have shown that kidnappers are always sensitive in terms of phone tracking and as such, they employ a good ICT analyst to ensure no trance of contact of them is made.
A strong society must prioritize the protection of its people (Nkwatoh & Nathaniel, 2018). The federal government is now largely in charge of the war against insecurity and its response to these ideological menaces or societal difficult circumstances has typically been ineffective, poorly planned, and executed (Umaru *et al.*, 2015). However, Onovo, (2010), has noted that the Nigerian government has created a special joint task force which comprises of the military, police and the vigilante group to reduce the ills of terrorism in Nigeria especially kidnapping. The vigilantes (even semi-armed civilian groups), which are typically found in rural regions, are volunteers and answer directly to the Nigerian security forces. Nearly all states of the federation with high levels of insecurity have voluntary vigilante groups who monitor their neighbourhoods. Members of semi-armed civilian groups in some localities have over time developed an extensive understanding of many forms of crime, firearms, and firearm use. Another form of control measure adopted involve creating public awareness on kidnappers mode of operation and its effect on an individual and the society at large, educating the public on implementation of vital security tips through mass or social media, sensitization of families since it forms the integral part of the society, creating an ICT special squad to monitor the activities of kidnappers such as phone tracking. The dynamics of kidnapping will be more understood using mathematical tools. Mathematical models can be useful in understanding the activities of different crimes when they enter the community and determining under what conditions criminals should be eliminated or prosecuted. Currently, kidnappings are of great interest to researchers, governments and the public due to the high rate of spread and the large number of deaths and loss of resources that have occurred. Many mathematical models have been developed to investigate the dynamics kidnapping in Nigeria and to the best of our knowledge, there is no existing record of a mathematical model for the kidnappers activities that incorporates; those that recruit susceptible individual to kidnaping activities, intelligent unit, effect of public awareness, kidnapping influencers and deserted individuals. For this reason, this study was targeted at predicting the rapidly growing number of kidnapping activities in Nigeria, their mode of operations and recruitment. We shall adopt a simple compartmentalized mathematical model with vital dynamics that combines these factors and control strategies.

Model Formation for kidnapping activities

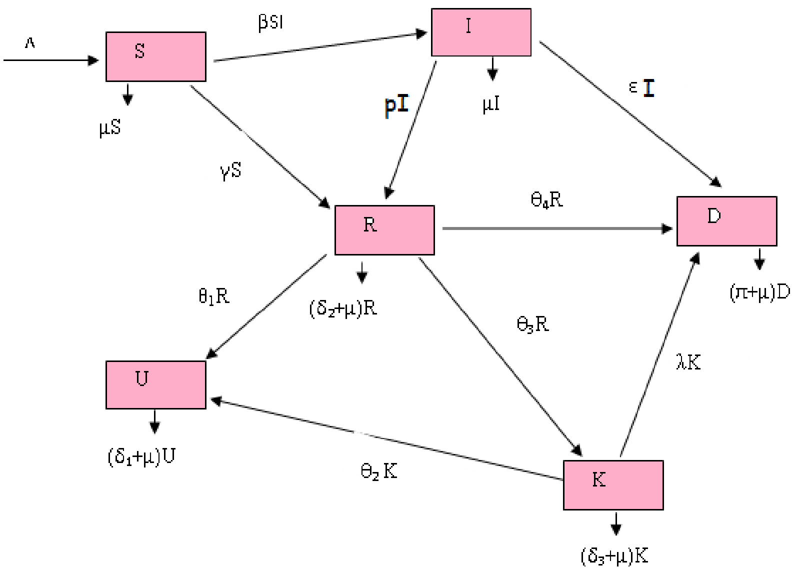
The model divides the total human population into six compartments namely; Susceptible individuals (S), Influencers (I), Recruiters (R), Combatant/Intelligent unit (U), Kidnappers or Hijackers (K) and Recovered (D)

Therefore; $N(t) = S(t) + I(t) + R(t) + U(t) + K(t) + D(t)$

Individuals enter the susceptible compartment by birth at constant rate Λ, moved to influencer compartment at rate β due to interaction and move to recruiters compartment at rate γ. Individuals enter the influencers compartment from susceptible compartment and move to recruiters compartment and recovered class at rates ρ and ε respectively. The recruiter compartment is increased due to conversion of influencers and susceptible individuals and they become kidnappers, recovered and intelligent unit individuals at rates θ_3, θ_4, and θ_1 respectively. Recruiters die due to contact with combatant/intelligent unit. Individuals are recruited into the recovered compartment due to self-realisation of influencers, recruiters and kidnappers or hijackers. They die as a result of their contact with any compartment involved in the crime at rate π. Individual becomes full kidnappers after passing through the recruiters training stage. They leave the kidnappers compartment either by becoming self-realize and helping the combatant/intelligent unit to fetch out others in the crime or become recovered unnoticed at rates θ_2 and λ respectively. Since it is a model with vital dynamics, individuals dies naturally in each compartment at rate μ.

In developing the model system of equation, we assume that individuals are recruited in the model at constant rate. We also assume that susceptible individuals become kidnapper either by interaction with influencers or self-motivated. Again, we assume equal rate of natural death in each compartment of the model, we assume that the conversion rate of Influencers I, Recruiters R and Kidnappers K is directly proportional to the public awareness carried out in the community. Finally, we assume that individuals leave kidnapping activities and become self-realized as a result of the influence of public awareness.

The schematic diagram is given below



From the above flow chart, we formulated the system of non-linear first order ordinary differential equation given below

$$\frac{dS}{dt} = \Lambda - \beta SI - (\mu + \gamma)S \tag{1}$$

$$\frac{dI}{dt} = \beta SI - (\rho + \mu + \varepsilon)I \tag{2}$$

$$\frac{dR}{dt} = \gamma S + \rho I - (\theta_1 + \theta_3 + \theta_4 + \mu + \delta_2)R \tag{3}$$

$$\frac{dU}{dt} = \theta_1 R + \theta_2 K - (\delta_1 + \mu)U \tag{4}$$

$$\frac{dK}{dt} = \theta_3 R - (\theta_2 + \lambda + \mu + \delta_3)K \tag{5}$$

$$\frac{dD}{dt} = \varepsilon I + \theta_4 R + \lambda K - (\pi + \mu)D \tag{6}$$

We note that $\delta_1 > \mu, \delta_3 > \delta_1, \theta_3 > \theta_2, \theta_3 < \gamma, \gamma > \rho, \theta_4 \geq \lambda$ and $\gamma < \beta$

Model Analysis

In this section, we shall carry out the quantitative study of the model which involves positivity and boundedness of the solutions, invariant region of model and existence of the equilibrium point of the model, reproduction number of the model and Stability Analysis.

Positivity of the solution

Theorem 1: Let S(0), I(0), R(0), U(0), K(0), and D(0) be non-negative initial conditions, then the solution S(t), I(t), R(t), U(t), K(t) and D(t) of the proposed model equation (1) to (4) are all positive for all t > 0

Proof:

Let $t^* = \sup\{t > 0 : S(t), I(t), R(t), U(t), K(t) and D(t)\}$ then from equation (1)

$\frac{dS}{dt} = \Lambda - \beta SI - (\mu + \gamma)S \geq 0$ that is

$$\frac{dS}{dt} + (\beta I + \mu + \gamma)S = \Lambda \tag{7}$$

Solving equation (7) using integrating factor and getting the integrating factor as; $\ell^{\int(\beta I + \mu + \gamma)dt}$ then

$$\frac{d}{dt}\left(S(t).\ell^{\int \beta I(s)ds + \mu t + \gamma t}\right) \geq \Lambda.\ell^{\int \beta I(s)ds + \mu t + \gamma t} \tag{8}$$

Integrating both side of inequality (8), we get

$$\left(S(t).\ell^{\int \beta I(s)ds + \mu t + \gamma t}\right) \geq \Lambda.\int_0^{t^*} \ell^{\int \beta I(r)dr + \mu t + \gamma t} + C \tag{9}$$

Where C is the integration constant and substituting t = 0 to t = t*, to eliminate the integration constant we get

$$S(t^*) = \Lambda.\ell^{-\int \beta I(r)dr - \mu t - \gamma t}\left(\ell^{\int \beta I(r)dr - \mu t - \gamma t}\right) + S(0)\ell^{-\int \beta I(r)dr - \mu t - \gamma t} \tag{10}$$

It is obvious that each of the term in equation (10) will be positive for all t > 0. Hence model variable can be proven in the same way.

Boundedness of the model solution

Theorem 2: All solutions S(0), I(0), R(0), U(0), K(0), and D(0) of the proposed model equation (1) to (6) are bounded. That is if $\lim_{t\to\infty}\sup N(t) \leq \frac{\Lambda}{\mu}$, then $N(t) = S(t) + I(t) + R(t) + U(t) + K(t) + D(t)$

Proof:

Adding equation (1) to equation (6) we get;

$$\frac{dN(t)}{dt} = \Lambda - \mu N - \delta_1 U - \delta_2 R - \delta_3 K - \pi D \tag{11}$$

The solutions of the proposed model are bounded. Hence, from equation (10), we can get that $\frac{dN(t)}{dt} \leq \Lambda - \mu N$ and this can be written as; $\lim_{t\to\infty}\inf N(t) \leq \lim_{t\to\infty}\sup N(t) \leq \frac{\Lambda}{\mu}$

Invariant region of the Model

Next, we proof that the state variables of the proposed model are feasible in the region Ω

Theorem 3: The region $\Omega \subset R_+^6$ is non-negative invariant for the proposed model in equation (1) to equation (6) with non-negative initial conditions R_+^6

Proof:

Let Ω denote the feasible region of the kidnapping activity in the model equation (1) to (6), then;

$$\Omega = \left\{ (S, I, R, U, K, D) \in R_+^6 : S + I + R + U + K + D \leq \frac{\Lambda}{\mu} \right\} \qquad (12)$$

To verify the positive invariant Ω, the solution in Ω still remains in Ω for all t > 0, this follows that

$\frac{dN(t)}{dt} \leq \Lambda - \mu N$, and by standard comparison theorem, we get

$$N(t) \leq N(0)\ell^{-\mu t} + \frac{\Lambda}{\mu}\left(1 - \ell^{-\mu t}\right) \qquad (13)$$

From the above, it is pertinent that the proposed model is mathematically, biologically and sociologically well posed. Hence, we shall now go ahead to analyse the existence of the equilibrium points of the proposed model.

Existence of Equilibrium Points of the Model

Setting the right hand side of the system (i.e. equation 1 - 6) to zero, with K = 0 and solving for each variable that is;

$$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = \frac{dU}{dt} = \frac{dK}{dt} = \frac{dD}{dt} = 0$$

We get the kidnappers free equilibrium point as $E^0 = \left(S^0, I^0, 0, 0, 0, D^0\right)$, where

$$S^0 = \frac{\varepsilon + \rho + \mu}{\beta} \qquad (14)$$

$$I^0 = \frac{\beta\Lambda - (\mu + \gamma)(\varepsilon + \rho + \mu)}{\beta(\varepsilon + \rho + \mu)} \qquad (15)$$

$$D^0 = \frac{\beta\Lambda - (\mu + \gamma)(\varepsilon + \rho + \mu)}{\beta(\pi + \mu)(\varepsilon + \rho + \mu)} \qquad (16)$$

Again, with $K \neq 0$, we have the equilibrium point with kidnapping activity as $E^* = \left(S^*, I^*, R^*, U^*, K^*, D^*\right)$

$$R^* = \frac{\gamma(\varepsilon + \rho + \mu)^2 + \rho\left(\beta\Lambda - (\gamma + \mu)(\varepsilon + \rho + \mu)\right)}{\beta(\varepsilon + \rho + \mu)(\theta_1 + \theta_3 + \theta_4 + \delta_2 + \mu)} \qquad (17)$$

$$U^* = \frac{\left(\gamma(\varepsilon + \rho + \mu)^2 + \rho\left(\beta\Lambda - (\gamma + \mu)(\varepsilon + \rho + \mu)\right)\right)\left(\theta_1(\theta_2 + \lambda + \delta_3 + \mu) + \theta_2\theta_3\right)}{\beta(\varepsilon + \rho + \mu)(\theta_1 + \theta_3 + \theta_4 + \delta_2 + \mu)(\theta_2 + \lambda + \delta_3 + \mu)(\delta_1 + \mu)} \qquad (18)$$

$$K^* = \frac{\theta_3\left(\gamma(\varepsilon + \rho + \mu)^2 + \rho\left(\beta\Lambda - (\gamma + \mu)(\varepsilon + \rho + \mu)\right)\right)}{\beta(\varepsilon + \rho + \mu)(\theta_1 + \theta_3 + \theta_4 + \delta_2 + \mu)(\theta_2 + \lambda + \delta_3 + \mu)} \qquad (19)$$

$$D^* = \frac{\theta_3\left(\gamma(\varepsilon + \rho + \mu)^2 + \rho\left(\beta\Lambda - (\gamma + \mu)(\varepsilon + \rho + \mu)\right)\right)\left(\theta_1(\theta_2 + \lambda + \delta_3 + \mu) + \theta_2\theta_3\right)}{\beta(\varepsilon + \rho + \mu)(\theta_1 + \theta_3 + \theta_4 + \delta_2 + \mu)(\theta_2 + \lambda + \delta_3 + \mu)} \qquad (20)$$

And S^* and I^* remains the same as in equation (14) and (15). Hence the equilibrium point in the presence of kidnappers is $E^* = (S^*, I^*, R^*, U^*, K^*, D^*)$

Reproduction number

Diekmann *et al.*, (1990) defined the basic reproduction number denoted by R_0 as the average number of secondary infections caused by an infectious individual during his or her entire period of infectiousness. Hence it is a vital threshold parameter in epidemiology since it tells whether a disease dies out or persists in a population. Applying this principle in the proposed model, it is necessary to state at this point that;

- If $R_0 < 1$, this in turn implies each kidnapper will influence less than one susceptible individual during his entire period of kidnapping activity. Hence, kidnapping will die out as the kidnappers free equilibrium point is stable.
- If $R_0 > 1$, this in turn implies each kidnapper will influence more than one susceptible individual during his entire period of kidnapping activity. Hence, kidnapping will persist in the population as the kidnappers free equilibrium point is unstable.

Hence we compute the reproduction number of the model using the next generation matrix. Focusing on the variables that involves kidnapping activities (I(t), R(t) and K(t)) and letting the vectors; ***F*** denotes the newly recruited and ***V*** denote the transfer of individuals across compartments. Hence the column matrices and the partial derivatives of F and V, we have square matrices of the form

$$F = \begin{pmatrix} \beta S & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} -(\varepsilon + \rho + \mu) & \rho & 0 \\ 0 & -(\theta_1 + \theta_3 + \theta_4 + \delta_2 + \mu) & \theta_3 \\ 0 & 0 & -(\theta_2 + \gamma + \delta_3 + \mu) \end{pmatrix}$$

The reproduction number is the dominant eigenvalue corresponding to the spectral radius of the matrix (FV^{-1}) which gives

R= -1

Local Stability

Theorem 4:

Let $\psi = E^0 = (S^0, I^0, 0,0,0, D^0)$ be the Kidnappers free equilibrium point. ψ is locally asymptotically stable if $R_e(\mu_i) \leq 0$ (real part) for all $i = 1,2,3,\ldots$ Where μ_i is the eigenvalues of the Jacobian matrix

Proof

Obtaining the Jacobian matrix of system of equation (1) to (6), we have the matrix and linearizing the system of equation near the equilibrium point, we have the matrix;

$$\det\left|J-\mu I\right| = \begin{vmatrix} a & b & 0 & 0 & 0 & 0 \\ c & d & 0 & 0 & 0 & 0 \\ \gamma & \rho & -e & 0 & 0 & 0 \\ 0 & 0 & \theta_1 & -f & \theta_2 & 0 \\ 0 & 0 & \theta_3 & 0 & -g & 0 \\ 0 & \varepsilon & \theta_4 & 0 & \lambda & -h \end{vmatrix} = 0 \quad (21)$$

Where

$$a = \frac{\beta\Lambda - 2(\gamma+\mu)(\varepsilon+\rho+\mu)}{(\varepsilon+\rho+\mu)}$$

$$b = -(\varepsilon+\rho+\mu)$$

$$c = \frac{\beta\Lambda - (\gamma+\mu)(\varepsilon+\rho+\mu)}{(\varepsilon+\rho+\mu)}$$ and d = 0

$$e = (\theta_1+\theta_3+\theta_4+\delta_2+\mu)$$

$$f = (\delta_1+\mu)$$

$$g = (\theta_2+\lambda+\delta_3+\mu)$$

$$h = (\pi+\mu)$$

With eigenvalues as;

$\mu_1 = -(\pi+\mu)$, $\mu_2 = -(\delta_1+\mu)$, $\mu_3 = -(\theta_2+\lambda+\delta_3+\mu)$, $\mu_4 = -(\theta_1+\theta_3+\theta_4+\delta_2+\mu)$

The other two eigenvalues can be obtained from the quadratic equation

$$\mu^2 - (a+b)\mu + ad - bc = 0 \quad (22)$$

Whose roots are;

$$\mu_5 = \frac{-\frac{\left(\beta\Lambda - 2(\gamma+\mu)(\varepsilon+\rho+\mu) - (\varepsilon+\rho+\mu)^2\right)}{(\varepsilon+\rho+\mu)} - \sqrt{\left(\frac{\left(\beta\Lambda - 2(\gamma+\mu)(\varepsilon+\rho+\mu) - (\varepsilon+\rho+\mu)^2\right)}{(\varepsilon+\rho+\mu)}\right)^2 + 4(\beta\Lambda - (\gamma+\mu)(\varepsilon+\rho+\mu))}}{2}$$

or

$$\mu_6 = \frac{-\frac{\left(\beta\Lambda - 2(\gamma+\mu)(\varepsilon+\rho+\mu) - (\varepsilon+\rho+\mu)^2\right)}{(\varepsilon+\rho+\mu)} + \sqrt{\left(\frac{\left(\beta\Lambda - 2(\gamma+\mu)(\varepsilon+\rho+\mu) - (\varepsilon+\rho+\mu)^2\right)}{(\varepsilon+\rho+\mu)}\right)^2 + 4(\beta\Lambda - (\gamma+\mu)(\varepsilon+\rho+\mu))}}{2}$$

Since $R_e(\mu_i) \le 0$, hence the kidnappers free equilibrium point $E^0 = (S^0, I^0, 0,0,0, D^0)$ is locally asymptotically stable.

Theorem 5:

Let ω = $E^* = (S^*, I^*, R^*, U^*, K^*, D^*)$ be the equilibrium point in the presence of kidnappers. ω is locally asymptotically stable if $R_e(\mu_i) \le 0$ (real part) for all $i = 1,2,3,\ldots$ Where μ_i is the eigenvalues of the Jacobian matrix

proof
Linearizing the system of equation near the equilibrium point, we have the matrix;

$$\det\left|J-\mu I\right| = \begin{vmatrix} a & b & 0 & 0 & 0 & 0 \\ c & d & 0 & 0 & 0 & 0 \\ \gamma & \rho & -e & 0 & 0 & 0 \\ 0 & 0 & \theta_1 & -f & \theta_2 & 0 \\ 0 & 0 & \theta_3 & 0 & -g & 0 \\ 0 & \varepsilon & \theta_4 & 0 & \lambda & -h \end{vmatrix} = 0$$

Which is the same as in equation (22), the whole process in the last section remain the same and $R_e(\mu_i) \leq 0$, hence equilibrium point in the presence of kidnappers; $E_1 = \left(S^*, I^*, R^*, I_u^*, K^*, D^*\right)$ is locally asymptotically stable.

3.7.Global stability analysis of the equilibrium point in with kidnapping activity
In this section, we shall carry out global stability analysis of the equilibrium point in the presence of kidnappers using the Lyapunov method. Consider the below definitions

Definition 1: A function $V(y(t))$ (dissipation energy) is called Lyapunov candidate if $V(y(t))$ is locally positive definite, and if the derivative $\overset{*}{V}(y(t))$ along the trajectory of the system $\frac{dy}{dt} = f(y(t))$ is locally negative definite, then $V(y(t))$ is called a Lyapunov function.

Theorem (LaSalle's invariant principle) 6: Consider a system $\frac{dy}{dt} = f(y(t))$ and suppose there exist a Lyapunov function $V(y(t))$ such that

1. $V(y(t))$ is positive definite
2. $\overset{*}{V}(y(t))$ is negative definite in the entire space
3. The only solution of $\frac{dz}{dt} = f(z)$, $\overset{*}{V}(z) = 0$ is z(t) = 0 for all t, where z (t) is the equilibrium point then the system $\frac{dy}{dt} = f(y(t))$ is globally asymptotically stable (GAS).

To proof the global stability of the equilibrium point with kidnapping activity, we adapt the Lyapunov method. Consider a positive definite composite Lyapunov candidate of the form

$$V = \frac{1}{2}m_1\left(S-S^*\right) + \frac{1}{2}m_2\left(I-I^*\right) + \frac{1}{2}m_3\left(R-R^*\right) + m_4\left(U-U^*-U^*\log\frac{U}{U^*}\right)$$
$$+ m_5\left(K-K^*-K^*\log\frac{K}{K^*}\right) + \frac{1}{2}m_6\left(D-D^*\right)$$

(23)

Where $(S-S^*), (I-I^*), (R-R^*), (U-U^*), (K-K^*)$ and $(D-D^*)$ are perturbations from the equilibrium point. It is obvious that V is radially unbounded since $V \to \infty$ as $S(t), I(t), I_u(t), K(t), R(t)$ and $D(t) \to \infty$

Taking the derivative of V with respect to time:

$$\overset{*}{V} = m_1(S-S^*) + m_2(I-I^*) + m_3(R-R^*) + m_4\left(\frac{U-U^*}{U}\right) + m_5\left(\frac{K-K^*}{K}\right) + m_6(D-D^*) \qquad (24)$$

Linearizing the system (1) to (6) around the equilibrium point with kidnapping activity, we get

$$\begin{aligned}\overset{*}{V} &= m_1(\beta I-\mu-\gamma)(S-S^*)^2 + m_2(\beta S-\rho-\mu-\varepsilon)(I-I^*)^2 - m_3(\theta_1+\theta_3+\theta_4+\delta_2+\mu)(R-R^*)^2 \\ &- m_4(\delta_1+\mu)\left(\frac{U-U^*}{U}\right)^2 - m_5(\theta_2+\lambda+\mu+\delta_3)\left(\frac{K-K^*}{K}\right)^2 - m_6(\pi+\lambda)(D-D^*)^2 + m_2\beta I(S-S^*)(I-I^*) \\ &+ m_3\gamma(R-R^*)(S-S^*) - m_1\beta S(I-I^*)(S-S^*) + m_3\rho(I-I^*)(R-R^*) + m_6\varepsilon(D-D^*)(I-I^*) \\ &- m_4\theta_1(R-R^*)\left(\frac{U-U^*}{U}\right) + m_5\theta_3\left(\frac{K-K^*}{K}\right)(R-R^*) + m_4\theta_2\left(\frac{K-K^*}{K}\right)\left(\frac{U-U^*}{U}\right) \\ &+ m_6\theta_4(D-D^*)(R-R^*) + m_6\lambda\left(\frac{K-K^*}{K}\right)(D-D^*)\end{aligned} \qquad (25)$$

Hence E_1 is globally asymptotically stable within the region of attraction provided that the equation (25) is negative definite function for some values of m_1, m_2, m_3 m_4, m_5 and m_6

Optimal Control

We carried out optimal control analysis on the proposed model, in other to minimize kidnapping activity, we incorporate the following control variables;

$u_1(t)$: providing sophisticated technologies such as drone or surveillance gadget to track kidnappers hideout

$u_2(t)$: constant training of combatant/intelligent units personnel to meet up with the kidnapping activity.

Hence, considering the impact of the above mentioned controls, on equation (1) to equation (6), we get a system of equation with control as;

$$\frac{dS}{dt} = \Lambda - \beta SI - (\mu+\gamma)S \qquad (26)$$

$$\frac{dI}{dt} = \beta SI - (\rho+\mu+\varepsilon)I \qquad (27)$$

$$\frac{dR}{dt} = \gamma S + \rho I - (\theta_1+\theta_3+\theta_4+\mu+\delta_2)R \qquad (28)$$

$$\frac{dU}{dt} = \theta_1 R - (1+u_1+u_2)K - \delta_1(1-u_1-u_2)U - \mu U \qquad (29)$$

$$\frac{dK}{dt} = \theta_3 R - (1+u_1+u_2)(\theta_2+\lambda+\delta_3)K - \mu K \qquad (30)$$

$$\frac{dD}{dt} = \varepsilon I + \theta_4 R + \lambda(1+u_1+u_2)K - \pi(1+u_1+u_2)D - \mu D \qquad (31)$$

Description of optimal control

We will minimize the kidnappers/hijackers compartments by using the control $u_1(t)$ and $u_2(t)$ which are all positive. We define the objective function according to Lenhart & Workman (2007) as

$$J = \int_0^{tf} \left(A_1 U(t) + A_2 K(t) + \frac{1}{2}\left(B_1 u_1^2(t) + B_2 u_2^2(t)\right)\right) dt \quad (32)$$

Where A_1 and A_2 are cost associated with providing sophisticated surveillance gadget and training of combatant intelligent units in Nigeria respectively and B_1 and B_2 are weight constants associated with the controls.

The aims is to find the optimal control $u_1^x(t)$ and $u_2^x(t)$ such that;

$$J(u_1^*, u_2^*) = \min\{(u_1, u_2) : u_1, u_2 \in \Omega\} \quad (33)$$

Where $\Omega = \{u_i : 0 \le u_i(t) \le 1\}$

Applying the pontryagin's maximum principle according to (Peter *et al.*, 2020), (Ayoade *et al.*, 2019) and (Zamir *et al.*, 2021), and converting equation (26) to equation (31) to minimize point wise lagrange, L with respect to the control variables, we get

$$L = A_1 U(t) + A_2 K(t) + \frac{1}{2}\left(B_1 u_1^2(t) + B_2 u_2^2(t)\right) \quad (34)$$

We define the Hamiltonian for the control problem as,

$$H = L + \lambda_s \frac{dS}{dt} + \lambda_i \frac{dI}{dt} + \lambda_r \frac{dR}{dt} + \lambda_u \frac{dU}{dt} + \lambda_k \frac{dK}{dt} + \lambda_d \frac{dD}{dt} \quad (35)$$

Equation (35) can be converted to

$$\begin{aligned}
& A_1 U(t) + A_2 K(t) + \frac{1}{2}\left(B_1 u_1^2(t) + B_2 u_2^2(t)\right) \\
& + \lambda_s \left[\Lambda - \beta SI - (\mu + \delta)S\right] \\
& + \lambda_i \left[\beta SI - (\rho + \varepsilon + \mu)I\right] \\
& + \lambda_r \left[\gamma S + \rho I - (\theta_1 + \theta_3 + \theta_4 + \delta_2 + \mu)\right] \\
& + \lambda_u \left[\theta_1 R - \theta_2 (1 + u_1 + u_2)K - \delta_1 (1 - `u_1 - u_2)U - \mu U\right] \\
& + \lambda_k \left[\theta_3 R - \left[(\theta_2 + \lambda + \delta_3)(1 + u_1 + u_2)\right]K - \mu K\right] \\
& + \lambda_d \left[\varepsilon I + \theta_4 R + \lambda (1 + u_1 + u_2)K - \pi (1 + u_1 + u_2)D - \mu D\right]
\end{aligned} \quad (36)$$

Where $\lambda_s, \lambda_i, \lambda_r, \lambda_u, \lambda_k$ and λ_d are adjoint variables

Theorem 7: consider the optimal control $u^* = (u_1^*, u_2^*) \in \Omega$ there exist an initial conditions at t = 0 of the control model equation (26) to equation (31), such that $J(u_1^*, u_2^*) = \min\{(u_1, u_2) : u_1, u_2 \in \Omega\}$.

Proof:

The state variables and control equation (26) to equation (31) are positive values and the control set Ω is close and convex. Hence the integrand objective function is convex. If equation (26) to equation (31) is bounded, Lipchitz property of the state system is satisfied. Therefore, the state variables are bounded and the existence of the optimal control solution of the model is concluded.

From theorem 7, and according to Lenhart & Workman (2007), we can derive that if $(x(t), u^*)$ is the optimal pair solution equation (26) to equation (31), then there exist a non-trivial solution of the adjoint function that satisfies the following condition;

$$\frac{dx}{dt}=\frac{\partial H(t,x,u,\lambda)}{\partial\lambda}$$
$$\frac{\partial H(t,x,u,\lambda)}{\partial u}=0 \qquad (37)$$
$$\frac{d\lambda}{dt}=\frac{\partial H(t,x,u,\lambda)}{dx}$$

Where H is the Hamiltonian

Theorem 8: Let S, I, R, U, K, and D be the state variable associated with the optimal control (u_1^*, u_2^*) for the model equation (26) to equation (31) and equation (32), there exist a co-state $\lambda_s, \lambda_i, \lambda_r, \lambda_u, \lambda_k$ and λ_d which verifies $\frac{d\lambda}{dt}=\frac{\partial H}{\partial x}$ with the transversality condition $\lambda(tf)=0$

Proof:

$$\frac{d\lambda_s}{dt}=-\frac{\partial H}{\partial S}=\lambda_s(\beta I+\mu+\delta)-\lambda_i\beta I-\lambda_r\gamma$$
$$\frac{d\lambda_i}{dt}=-\frac{\partial H}{\partial I}=\lambda_s\beta S+\lambda_i(\rho+\varepsilon+\mu-\beta S)+\lambda_r\rho+\lambda_d\varepsilon$$
$$\frac{d\lambda_r}{dt}=-\frac{\partial H}{\partial R}=\lambda_r(\theta_1+\theta_3+\theta_4+\delta_2+\mu)-\lambda_u\theta_1-\lambda_k\theta_3-\lambda_d\theta_4 \qquad (38)$$
$$\frac{d\lambda_u}{dt}=-\frac{\partial H}{\partial U}=-A_1+\delta_1\lambda_u(1-u_1-u_2)-\lambda_u\mu$$
$$\frac{d\lambda_k}{dt}=-\frac{\partial H}{\partial K}=-A_2+\theta_2(1+u_1+u_2)\lambda_u+\lambda_k[(\theta_2+\lambda+\delta_3)(1+u_1+u_2)-\mu]-\lambda(1+u_1+u_2)\lambda_d$$
$$\frac{d\lambda_d}{dt}=-\frac{\partial H}{\partial D}=\lambda_d[\pi(1+u_1+u_2)+\mu]$$

With the transversality conditions

$$\lambda_s(tf)=\lambda_i(tf)=\lambda_r(tf)=\lambda_u(tf)=\lambda_k(tf)=\lambda_d(tf)=0 \qquad (39)$$

Next, we differentiate the Hamiltonian with respect to the control variables, that is $\frac{\partial H}{\partial u_i}=0$, for i = 1, 2 we get;

$$u_1^*=\frac{\lambda_u(\theta_2K+\delta_1U)+\lambda_k(\theta_2+\lambda+\delta_3)+\lambda_d(\pi D-\lambda K)}{B_1} \qquad (40)$$
$$u_2^*=\frac{\lambda_u(\theta_2K+\delta_1U)+\lambda_k(\theta_2+\lambda+\delta_3)+\lambda_d(\pi D-\lambda K)}{B_2} \qquad (41)$$

Thus, we have;

$$u_1^*=\max\left\{0,\min\left(\frac{\lambda_u(\theta_2K+\delta_1U)+\lambda_k(\theta_2+\lambda+\delta_3)+\lambda_d(\pi D-\lambda K)}{B_1}\right)\right\} \qquad (42)$$
$$u_2^*=\max\left\{0,\min\left(\frac{\lambda_u(\theta_2K+\delta_1U)+\lambda_k(\theta_2+\lambda+\delta_3)+\lambda_d(\pi D-\lambda K)}{B_2}\right)\right\} \qquad (43)$$

Numerical Solution of the Optimal Control

We carried out numerical simulation of the optimality system using MATLAB R2018a Version. The weight costs, A_1, A_2, B_1 and B_2 are 0.75, 0.5, 0.1 and 0.5 respectively. The initial conditions are S(0) = 50, I(0) = 5, R(0) = 5, U(0) = 3, K(0) = 4 and D(0) = 2 and the values of the parameters and variables in the table below are assumed theoretical data for solving the state and adjoint variable.

Parameter(s)	Description	Value
Λ	Constant recruitment rate	50
μ	Natural death rate	0.002
λ	Recovery rate of kidnappers or hijackers	0.006
β	Interaction rate of susceptible and influencers	0.008
γ	Rate of becoming a recruiter during training	0.007
ε	Recovery rate of influencers due to public awareness, self-realization or family/community influence	0.3
ρ	Rate at which influencer becoming recruiters	0.005
θ_1	Conversion of recruiters to combatant/intelligent unit	0.004
θ_2	Conversion rate of kidnapper/Hijackers to combatant/intelligent unit	0.002
θ_3	Rate of becoming a full skilled kidnapper	0.05
θ_4	Recovery rate of recruiters due to public awareness, self-realization or family/community influence	0.006
δ_3	Death rate of rehabilitated kidnappers/Hijackers due to contact with kidnapper/ Hijackers	0.09
δ_2	Death rate of combatant/intelligent individuals due to contact with combatant/intelligent unit	0.3
π	Death rate kidnapper/Hijackers due to contact with combatant/intelligent	0.002
δ_1	Death rate of recruiter due to contact with combatant/intelligent	0.04

Graphical solution of the Optimal Control

The below are the simulation of the optimal control of kidnaping activity in Nigeria

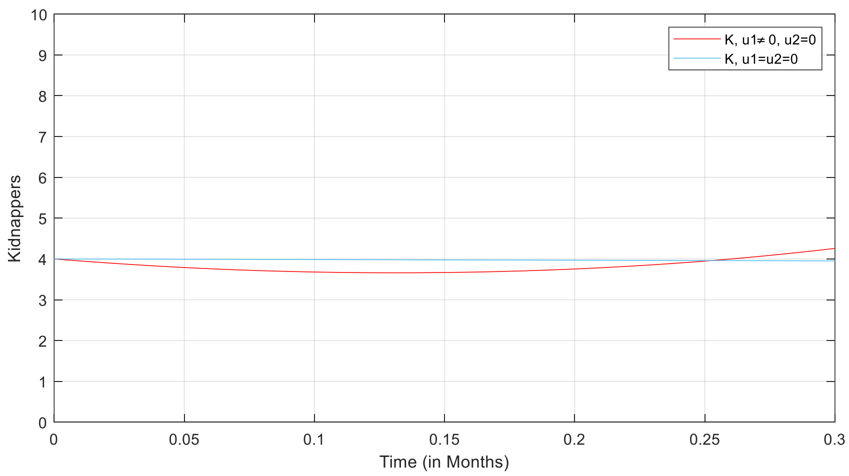


Figure 2: Population of kidnappers when $u_1(t)$ is implemented

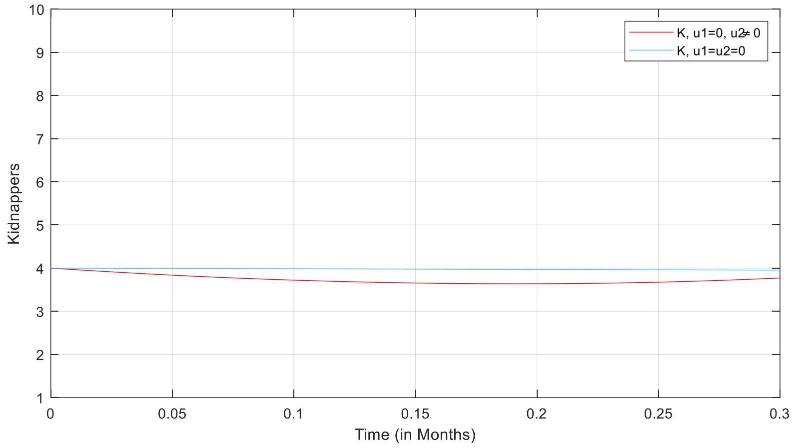


Figure 3: Population of kidnappers when only $u_2(t)$ is implemented

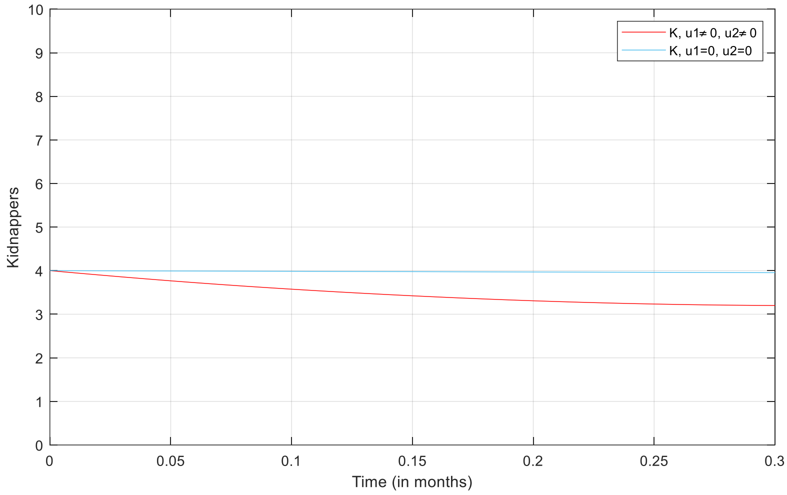


Figure 4: Population of kidnappers when both $u_1(t)$ and $u_2(t)$ are implemented

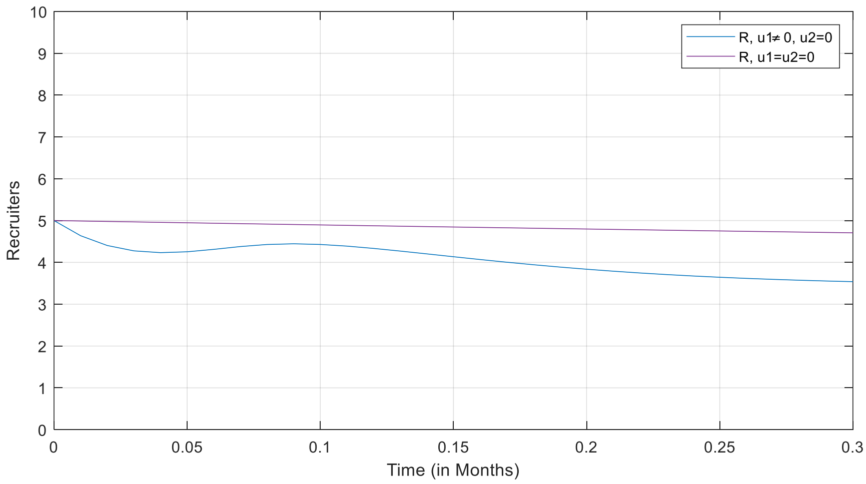


Figure 5: Population of Recruiters when $u_1(t)$ is implemented

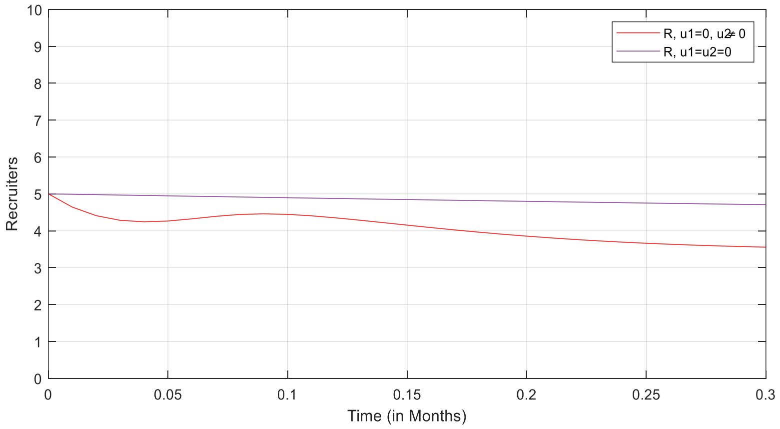


Figure 6: Population of Recruiters when only $u_2(t)$ is implemented

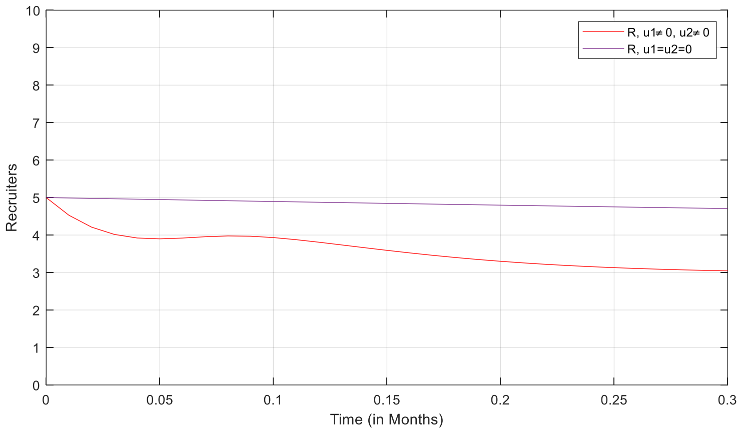


Figure 7: Population of Recruiters when both $u_1(t)$ and $u_2(t)$ are implemented

Discussion

The numerical results are the comparison of the strategies to obtain the best control to adopt to minimize kidnapping activities in Nigeria with minimal cost effect. Thus, Figure 2 shows that the population of kidnappers decreases more when sophisticated drones or surveillance gadgets are used to track their hideouts. The result also shows that the population of kidnappers will drop in the first two months and then start to increase after 3 months this may be as a result of re-strategizing of kidnappers or technical failure on the path of using the equipment. Figure 3 show a better result (when we compare it with the result in figure 2), the pollution of kidnappers continue to decreases within the first 4 to 5 months, this is because of the intense recruiting, carrying out of workshops and training of combatant/intelligent unit personnel to combat kidnapping. Figure 4 show a drastic reduction of kidnapping activity, the population decline within 2 to 3 months. This is due to the simultaneous combination of the use of technologies and training of security personnel.

Recruiters are individuals that train susceptible individual to become professional kidnappers. They are sometimes seen as influencers and kidnappers at the same time. Hence we also considered the optimal control of the recruiters owning to the fact that they are the heart-beat of kidnapping activity. The result shows generally that the population of recruiters continue to decline with time. Figure 5 shows that the population of recruiters will decrease more when drones or surveillances are used to monitor their activity within the populace susceptible individuals. Figure 6 shows no much difference when combatant/intelligent unit personnel are constantly train and updated about kidnapping activity and disseminated into the society. Finally, the result in figure 7 shows that the population of recruiters will decline faster when sophisticated drone or surveillance gadget as well as constant training of combatant/intelligent units personnel to meet up with the kidnapping activity.

Conclusion

According to Sanchi, *et al.*, (2022) rural banditry has damaged Nigeria's entire educational system, raising the possibility of kidnapping. According to the article, the problem of rural banditry is becoming more and more concerning because of factors like high recessions, a poor security system, economic hardship, Nigeria's uncontrolled migration, the proliferation of firearms, and the existence of areas with few regulations that serve as bandit hideouts. Hence, the above results has shown that kidnapping activities in Nigeria can be reduced within four months if control strategies like providing sophisticated technologies such as drone or surveillance gadget to track kidnappers hideout and constant training of combatant/intelligent units personnel to meet up with the kidnapping activity are considered with topmost priority. Other strategies may include; effective arresting and prosecuting of individuals that are involved in the crime, involving recovered individuals in the intelligent/combat unit to detect kidnapper mode of operations and hideouts and communal awareness to sensitize the populace especially among lower class individuals in the society.

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