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A CONFOUNDING PLANS IN A 2^K FACORIAL DESIGN

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Abstract
When the number of treatments is greater than the available blocks in 2^k factorial experiments confounding becomes necessary to reduce the block size as well as reduce experimental errors. It is also unavoidable when the treatments are greater than the block size. 2^k factorial confounding plans were studied with k = 2, 3, 4 and k > 4. Confounding when k = 2 and 3 is not necessary as the treatment combinations are not much. However, from k > 4, confounding becomes necessary as the treatment combinations are many. From our confounding plans, it is only in 2^4 that all the main effects are found in one block when confounded with ABCD. The result is not the same with k > 4. Therefore, when many factors are needed is an experiment, the 2^4 factorial experiment is recommended.

Key words: *confounding, even rule, odd rule, Yates' technique, blocking.*

Introduction

A factorial experiment is a crossed factor design that usually involves several factors and it is such that every possible combination of the factor is included of observed or examine

In 2^k factorial experiments, there are k factors each observed at 2 levels. The levels are 0 and 1 where 0 is the low level while 1 is the high level.

Usually, Roman capital letters are used to denote the factors while the Roman small letters are used to denote the levels of the factors. (1) is used to indicate that all factors involved in the experiment occur at their lowest level. For instance, consider 2^2 factorial experiments. Suppose the factors are designated as A and B. the levels of the factors are (1), a, b, and ab.

In 2^k factorial experiments, there must be available for experimentation a 2^k homogenous blocks to accommodate the treatments. In some cases, it is not possible to have that many homogenous units and the experiment must be performed in more than one incomplete block. When a factorial experiment is performed in more than one incomplete block, such case is called confounding in 2^k factorial experiment. Confounding is a design technique for arranging a complete factorial experiment in blocks, where the block size is smaller the number of treatment combinations in one replicate (Montogomery 2006). Two effects are said to be confounded if their estimates are indistinguishable.

Confounding is done for the following reasons:

(1) To reduce experimental error.
(2) To reduce the block size.
(3) It is unavoidable when the treatment is greater than the block size.

One major setback of confounding is that information about the confounded experimental effect is lost and because of this the main effects are not confounded. Interactions whose effects are considered small or unimportant are chosen. Generally, higher order interactions are advisable to choose. The block which contains the treatment (1) is called the principal block.

Confounding in 2^k factorial experiments have been examined by some authors.

Akra and Edet (2017), confounded 2^5 factorial designs without replication in different block size using higher order interaction method. From the results obtained, it shows that confounding ABCDE (where A is animal manure, B is green manure, C is mineral manure, D is compost manure and E is ash manure) in 2 blocks; 3 and 4 factor interactions were assumed negligible and only four of 2 factor interactions were significant at 5% level of significance. In confounding ABCE & ACDE in 4 blocks and ABCE, ABDE and ACDE in 8 blocks; 3, 4 and 5 factor interactions were assumed negligible. This revealed that only four of 2 factor interactions were significant at 5% level of significance. This means that different organic manure interactions (BD, CD, CE, & DE) results in different number of fruits produced per stand of *carica papaya*. We therefore conclude that factor (A), animal manure does not interact with any of the factor to be insignificant; hence factor (A) is considered to be the best manure that yield more fruits than others manure on different blocks.

Genesis, Adriana and Carsten (2011) studied application of factorial designs to study factors involved in the determination of aldehydes present in beer by on-fiber derivatization in combination with gas chromatography and mass spectrometry. The effect of the temperature, time, and sodium chloride (NaCl) addition on the analytes' derivatization/extraction efficiency was studied through a factorial 2^3 randomized-block design; all of the factors and their interactions were significant at the 95% confidence level for most of the analytes. The effect of temperature and its interactions separated the analytes in two groups. However, a single sampling condition was selected that optimized response for most aldehydes. The resulting method, combining on-fiber derivatization with gas chromatography-mass spectrometry, was validated. Limits of detections were between 0.015 and 1.60 μg/L, and relative standard deviations were between 1.1 and 12.2%. The efficacy of the internal standardization method was confirmed by recovery percentage (73-117%). The method was applied to the determination of aldehydes in fresh beer and after storage at 28 °C.

Ogbonna et al (2014) studied the inhibition of copper corrosion by acid extract of *Gnetum africana* using weight loss method of monitoring corrosion rate. The inhibition of *Gnetum africana* on copper corrosion was optimized by application of 2^3 factorial design. The interactive effects of temperature, inhibition concentration and reaction time were investigated. The input factors and output response were also optimized. Optimum conditions for inhibition of *Gnetum africana* on copper corrosion were recorded at temperature of 303 K, reaction time of 24 h and inhibition concentration of 0.003 g/L. The result shows that factorial design was adequately applicable in the optimization of process variables and that *Gnetuma africana* sufficiently inhibited the corrosion of copper at the conditions of the experiment.

2^k factorial experiments are also applied in medical and paramedical sciences. In their work, S. Jain et al (2011) studied An Application of Factorial Design To Compare The Relative Effectiveness of Hospital Infection Control Measures. They iteratively applied a full 2^k factorial design on the output of a stochastic, agent-based simulation to compare the effects of the hand hygiene compliance of healthcare workers and the nurse-to-patient ratio on the transmission of methicillin-resistant Staphylococcus aureus (MRSA) in a 20-bed ICU. The results suggest that increasing the nurse-to-patient ratio is more effective at levels below approximately 60% compliance of nurses. However, improving the hand washing compliance of nurses becomes the better strategy at higher baseline compliance levels. In addition, interaction effects between the two infection control measures limit the marginal benefit of improving both factors to high levels.

Adisa et al (2020) studied the use of factorial experiments for optimizing inhibition effect of acid extract of Gnetum Africana on copper corrosion. According to them, the inhibition of copper corrosion by acid extract of Gnetum Africana using weight loss method under various independent variables of time, inhibitor concentration and temperature was studied. Three-level factorial design was employed to fit a model describing the weight loss due to corrosion. Equally, a two-level factorial design and Box-Behnken design were alternatively considered for the design constructions, analysis, and modeling the process of the original data. The optimum factor settings for corrosion inhibition were determined to be 24 h, 0.003 g/L and 303 K for time, inhibitor concentration and temperature respectively. It was concluded that the Box-Behnken design was the most efficient, followed by the two-level factorial design, while the three-level factorial design was the least efficient in predicting the weight loss.

For the use of factorial experiments in Agriculture, Parmar1 et al (2022), used factorial experiments to involve simultaneously more than one factors and each factor is at two or more levels. Several factors affect simultaneously the characteristics under study in factorial experiments and the researcher is interested in the main effects and the interaction effects among different factors. The study will be useful for researchers to know the analysis of factorial experiments for agricultural research using digital tool. Factorial experiment is an experiment whose design consists of two or more factors, each with discrete possible levels. Digital tool (SPSS) is widely useful and user friendly to analyze the data of the factorial experiments for agricultural research like Factorial complete randomized design (CRD) and Factorial randomized block design (RBD) and also for illustration purposes in the classroom teaching as well as for the researchers with interest in factorial experimental designs.

In education, Yisa (2011) applied a 2^3 factorial experiment designed to examine the influence of such factors as teaching method, gender and level of study on students' academic performance. The subjects were tested on effect of teaching method, gender and level of study on their academic performance. The data obtained from the experiment were analyzed using the Analysis of Variance technique of 2k factorial designs devised by Yates (1937), known as Yates Algorithm. In the analysis process, the magnitude and direction of the factor effects were first examined to determine the likely important variables. It was found that each of the Level, Method, and Level-Method interaction effects has large impact on students' academic performance while Gender and all the other interaction effects do not appear to have impact on students' performance. The significance of these effects with large impact was then confirmed by the analysis of variance, which shows that teaching method, level of study, and level-method interaction, have significant effects on students' academic performance, at 5% level of significance, while gender and each of the other interactions have no effect on students' performance.

Methodology

Methods of confounding

There are five methods of confounding in 2^k factorial experiments:

Even and Odd rule

In odd or even method of confounding, the key block or principal block will contain the even number of treatments while the other block will contain odd number of treatments Jaisanka and Pachamuthu (2012). In this method, if the factorial effect whose estimate is desired contains even number of letters in common with it is added and the rest of the treatment subtracted. Conversely, if the factorial effect contains odd number of letters, then all the treatments containing odd number of letters in common with it are added and the rest subtracted.

Suppose in 2^3 factorial experiments, we desire to estimate the effects of A, B, AB and ABC, we shall have:

$$A: abc - bc + ac - c + ab - b + a - (1)$$
$$B: abc + bc - ac - c + ab + b - a - (1)$$
$$AB: abc - bc - ac + c + ab - b - a + (1)$$
$$ABC: abc - bc - ac + c - ab + b + a - (1)$$

Sign Table

The method is as follows:
Step 1: Write down the factorial effects as column headings while the treatments are written as row headings.
Step 2: Besides each treatment, write down the corresponding yield.
Step 3: Under column (1) write down the + sign in each row.
Step 4: Under column A write the + sig wherever the letter A occurs in the row heading.
Step 5: Repeat step 4 for each heading.
Step 6: The signs for the interaction are obtained by the product module 2 between the corresponding main effects.
The above method can be shown using 2^3 factorial experiments

Table 1

Sign Table for 2^3 factorial experiments

Yield	**Treatment**	**(1)**	**A**	**B**	**AB**	**C**	**AC**	**BC**	**ABC**
y_1	(1)	+	-	-	+	-	+	+	-
y_2	a	+	+	-	-	-	-	+	+
y_3	b	+	-	+	-	-	+	-	+
y_4	ab	+	+	+	+	-	-	-	-
y_5	c	+	-	-	+	+	-	-	+
y_6	ac	+	+	-	-	+	+	-	-
y_7	bc	+	-	+	-	+	-	+	-
y_8	abc	+	+	+	+	+	+	+	+

Expansion of Product

In this method, the factorial effect whose estimate is desired, takes the value -1 and others take +1. The product gives the effect. The effect of AB in 2^2 factorial experiments will be AB: (a-1) (b-1) = ab-a-b+(1).

Yates Techniques

This was developed by Frank Yates in 1937. It provides an algorithm for calculating the effects of different factors and interaction in an experiment. To explain this method, a 2^3 is used as an example. The treatment combinations are: (1), a, b, ab, c, ac, bc, abc.

Table 2: Yates technique

Yield	*Col* 1	*Col* 2	*Col* 3	SS_{col}
$y_{(1)}$	$y_{(1)} + y_a$	$y_{(1)} + y_a + y_b + y_{ab}$	$y_{(1)} + y_a + y_b + y_{ab} + y_c + y_{ac} + y_{bc} + y_{abc}$	8 x M
y_a	$y_b + y_{ab}$	$y_c + y_{ac} + y_{bc} + y_{abc}$	$y_a - y_{(1)} + y_{ab} - y_b + y_{ac} - y_c + y_{abc} - y_{bc}$	4 x A
y_b	$y_c + y_{ac}$	$y_a - y_{(1)} + y_{ab} - y_b$	$y_b + y_{ab} - y_{(1)} - y_a + y_{bc} + y_{abc} - y_c - y_{ac}$	4 x B
y_{ab}	$y_{bc} + y_{abc}$	$y_{ac} - y_c + y_{abc} - y_{bc}$	$y_{ab} - y_b - y_a + y_{(1)} + y_{abc} - y_{bc} - y_{ac} + y_c$	4 x AB
y_c	$y_a - y_{(1)}$	$y_b + y_{ab} - y_{(1)} - y_a$	$y_c + y_{ac} + y_{bc} + y_{abc} + y_{(1)} - y_a - y_b - y_{ab}$	4 x C
y_{ac}	$y_{ab} - y_b$	$y_{bc} + y_{abc} - y_c - y_{ac}$	$y_{ac} - y_c + y_{abc} - y_{bc} - y_a + y_{(1)} - y_{ab} + y_b$	4 x AC
y_{bc}	$y_{ac} - y_c$	$y_{ab} - y_b - y_a + y_{(1)}$	$y_{bc} + y_{abc} - y_c - y_{ac} - y_b - y_{ab} + y_{(1)} + y_a$	4 x BC
y_{abc}	$y_{abc} - y_{bc}$	$y_{abc} - y_{bc} - y_{ac} + y_c$	$y_{abc} - y_{bc} - y_{ac} + y_c - y_a - y_{ab} + y_a - y_{(1)}$	4 x ABC

Each column in each step is done by pairwise sums or differences from the terms of the previous step. The sum of the first two entries in the yield data become the first entry of column 1, the sum of the fifth and sixth entries in the yield column becomes the third entry of column 1. The next four entries of column 1 are computed from pairwise differences of entries in the yield column, just as the first four entries were computed from pairwise sums. The fifth entry of column 1 is the differences of the second and first entry of the yield and so on. The procedure is repeated for columns 2 and 3.

Each process involves 8 sums/differences. The last step is simply the sums of squares for each treatment divided by 2^k x r where r is the replication.

For simplicity:

Step 1: Write the treatments as yield.

Step 2: The first half of column 1 is obtained by adding adjacent pairs of the yield while the last half of the same column is obtained by subtracting the same pairs of the yield.

Step 3: The entries in column 2 are obtained as in step 2 by operating on the entries in column 1 just as we operated in column 1.

In general, in 2^k factorial experiment, there are a total of k columns, each column being obtained by operating on the previous column.

Step 4: The entries in the sum of squares column are obtained for each treatment by squaring the entries in the last column for that treatment and dividing it by 2^k x r, where r is the number of replications.

Use of linear combination

There is another method for constructing these designs. The method uses the linear combination.

$$L = \alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_k x_k \qquad (1)$$

Where x_i is the level of the ith factor appearing in a particular treatment combination and α_i is the exponent appearing on the ith factor in the effect to be confounded. (Montgomery 1991). Equation (1) is called defining contrast.

2.2 Confounding Plans

The 2^k factorial design will be planned in such a way that the block containing at least 75% of the main effects in any block will be recommended for experiment. This is our decision rule for confounding and thereafter, the Yates technique will be employed for analysis.

For an example, consider a 2^3 design with ABC confounded with blocks. Here x_1 corresponds to A, x_2 to B, x_3 to C, and $\alpha_1 = \alpha_2 = \alpha_3 = 1$

Hence the defining contrast that correspond to ABC is

$$L = x_1 + x_2 + x_3 \qquad (2)$$

Equation (1) will be use for the confounding processes.

The plans will be 2^k of block size of 2^{k-b}. b is the number of blocks.

Plan1: k = 3, block size = 4

The treatment combinations are: (1), a, b, ab, c, ac, bc, abc. Using equation (1) and ABC as the interaction confounded with blocks we have:

$$\begin{aligned} L_0 &= x_1 + x_2 + x_3 = 0 \text{ module } 2 \\ L_1 &= x_1 + x_2 + x_3 = 1 \text{ module } 2 \end{aligned} \qquad (3)$$

Equation (3) is simplified as follows:

$$\begin{aligned} (1) &= 0 + 0 + 0 = 0 \\ a &= 1 + 0 + 0 = 1 \\ b &= 0 + 1 + 0 = 1 \\ ab &= 1 + 1 + 0 = 0 \text{ mod } 2 \\ c &= 0 + 0 + 1 = 1 \\ ac &= 1 + 0 + 1 = 0 \text{ mod } 2 \\ bc &= 0 + 1 + 1 = 0 \text{ mod } 2 \\ \text{abc} &= 1 + 1 + 1 = 1 \text{ mod } 2 \end{aligned}$$

Equations with values of 0 are placed in the principal block while the values of 1 are placed in block 2 as shown below in Table 3.

Table 3: 2^3 factorial experiments with ABC confounded

$$\begin{pmatrix} Block1 \\ (1) \\ ab \\ bc \\ ac \end{pmatrix} \quad \begin{pmatrix} Block2 \\ a \\ b \\ c \\ abc \end{pmatrix}$$

Alternatively, when the principal block is generated, other treatments are placed in other blocks by addition module 2 of the treatments not in the principal block.

In a closer look, the principal block contains treatments that are zero and even in common to the confounded interaction ABC.

Since block 2 contains all the main effects a, b and c, block 2 is recommended for experimentation.

Plan2: k = 4, block size = 8

Plan 2 has 16 treatment combinations which may be difficult for an experimenter to manage.

The treatment combinations are: (1), a, b, ab, c, ac, bc, abc, d, ad, bd, abd, cd, acd, bcd,abcd

Using ABCD as the confounded interaction, the result will be as follows using equation (1).

$$L_0 = x_1 + x_2 + x_3 + x_4 = 0 \text{ module } 2$$
$$L_1 = x_1 + x_2 + x_3 + x_4 = 1 \text{ module } 2$$

$$(1) = 0 + 0 + 0 + 0 = 0$$
$$a = 1 + 0 + 0 + 0 = 1$$
$$b = 0 + 1 + 0 + 0 = 1$$
$$ab = 1 + 1 + 0 + 0 = 0$$
$$c = 0 + 0 + 1 + 0 = 1$$
$$ac = 1 + 0 + 1 + 0 = 0$$
$$bc = 0 + 1 + 1 + 0 = 0$$
$$abc = 1 + 1 + 1 + 0 = 1$$
$$d = 0 + 0 + 0 + 1 = 1$$
$$ad = 1 + 0 + 0 + 1 = 0$$
$$bd = 0 + 1 + 0 + 1 = 0$$
$$abd = 1 + 1 + 0 + 1 = 1$$
$$cd = 0 + 0 + 1 + 1 = 0$$
$$acd = 1 + 0 + 1 + 1 = 1$$
$$bcd = 0 + 1 + 1 + 1 = 1$$
$$abcd = 1 + 1 + 1 + 1 = 0$$

The block contents are:
Table 4
2^4 with ABCD cofounded

$$\begin{pmatrix} Block1 \\ (1) \\ ab \\ ac \\ ad \\ bc \\ bd \\ cd \\ abcd \end{pmatrix} \quad \begin{pmatrix} Block2 \\ a \\ b \\ c \\ d \\ abc \\ abd \\ acd \\ bcd \end{pmatrix}$$

Since block 2 contains all the main effects, it is recommended for experimentation if the experimenter can accommodate the 8 treatment combinations.

1. **Plan 3: k = 5, block size = 16**

The treatment combinations for this design are: (1), a, b, ab, c, ac, bc, abc, d, ad, bd, abd, cd, acd, bcd,abcd, e, ae, be, abe, ce, ace, bce, abce. de, ade, bde, abde, cde, acde, bcde, abcde.

If ABCDE is used as the confounded interaction, the block size will be 16.

The linear combinations are:

$$L_0 = x_1 + x_2 + x_3 + x_4 + x_5 = 0 \text{ module } 2$$
$$L_1 = x_1 + x_2 + x_3 + x_4 + x_5 = 1 \text{ module } 2$$

The block contents are:

Table 5
2^5 with ABCDE confounded

$$\begin{pmatrix} Block1 \\ (1) \\ ab \\ ac \\ bc \\ ad \\ bd \\ cd \\ abcd \\ ae \\ be \\ ce \\ abce \\ de \\ abde \\ de \\ abde \\ acde \\ bcde \end{pmatrix} \qquad \begin{pmatrix} Block2 \\ a \\ b \\ c \\ abc \\ d \\ abd \\ acd \\ bcd \\ e \\ abe \\ ace \\ bce \\ ade \\ bde \\ cde \\ abcde \end{pmatrix}$$

Although, all the main effects are found in Block 2, but the 16 treatments combinations could be difficult for the experimenter to accomplish.

3.1 Plan 3: k = 5, block size = 8

Suppose ABC and CDE are confounded and ABDE is also confounded by multiplication module 2. The linear combination equations are

$$L_0 = x_1 + x_2 + x_3 = 0$$
$$L_0 = x_3 + x_4 + x_5 = 0$$
$$L_0 = x_1 + x_2 + x_4 + x_5 = 0$$

The treatment combinations that will satisfy the above 3 equations are placed on the principal block and the rest block are generated by multiplication module 2 of the treatments not found in the principal block to other blocks. The 4 blocks and its contents are:

Table 6
2^5 with ABC, CDE and ABDE confounded

Block1	Block2	Block3	Block4
(1)	*a*	*c*	*ac*
ab	*b*	*abc*	*bc*
acd	*cd*	*ad*	*d*
bcd	*abcd*	*bd*	*abd*
ace	*ce*	*ae*	*e*
bce	*abce*	*be*	*abe*
de	*ade*	*cde*	*acde*
abde	*bde*	*abcde*	*bcde*

Looking at Table 6, none of the blocks have 75% of the main effects and such it is not recommended.

Further confounding 2^5 with other combinations of interactions will not give us the desired result as no 75% of the main effect will be seen in any of the blocks.

Further confounding with k > 5 will not give us the desired results.

Data Presentation and Analysis

An experiment arose in the finishing of metal strips in a metallurgical process. The measured response for smoothness of the surface where a small value was desirable and a large number indicated roughness was needed. The factors (two levels) of each of which were included for this experiment were as follows:

Factor		Levels	
Solution Temperature	(T)	High	Low
Solution Concentration	(C)	High	Low
Roll Size	(R)	2	1
Roll Tension	(F)	High	Low

Assuming that 2^4 treatments are completely randomized in each replication of the experiment and that all the assumptions of analysis of variance are satisfied, test for the significance of all the main effects and the interactions. Take $\alpha = 0.05$

Table 7
Metallurgical Experiments

	Rep 1				Rep 11			Total Yield		
HH2L	4				4			8		
	HH2H	18				13			31	
		LL1L	18				16			34
			LL2L	10				12		
22				LL1H	10				12	
	22				LH2L	8				10
		18				HL1L	12			
16			28				LH1L	9		
	7			16				HL2L	15	
		15			30				LL2H	8
			19			27				HL1H
21				14			35			
LH1H	9				7			16		
	HH1L	17				15			32	
		LH2H	14				10			24
			HL2H	21				21		
42				HH1H	24				18	
	42									

Source: Fundamentals of Design and Analysis of Experiments by F.C. Eze (2003)

The coding will be arranged for the treatments to be in a definite order using binary addition and thereafter convert them to the factors of TCRF representing solution temperature, solution concentration, roll size and roll tension respectively with their corresponding levels and total yield.

Table 8

Arranged Treatment Combinations of the Metallurgical Experiments

TCRF	Treatment combinations	Total yields
0000	LL1L	34
0001	LL1H	22
0010	LL2L	22
0011	LL2H	27
0100	LH1L	16
0101	LH1H	16
0110	LH2L	18
0111	LH2H	24
1000	HL1L	28
1001	HL1H	35
1010	HL2L	30
1011	HL2H	42
1100	HH1L	32
1101	HH1H	42
1110	HH2L	8
1111	HH2H	31

The Yates techniques is shown in Table 9:

Table 9

Yates Technique

Treatments	Yields	Col 1	Col. 2	Col. 3	Col. 4	SS col.	Effects
$LL1L$	34	56	105	179	427	–	(1)
$LL1H$	22	49	74	248	51	81.281	F
$LL2L$	22	32	135	–1	–23	16.531	R
$LL2H$	27	42	113	52	41	52.531	RF
$LH1L$	16	63	–7	3	–55	94.531	C
$LH1H$	16	72	6	–26	17	9.031	CF
$LH2L$	18	74	19	23	–27	22.781	CR
$LH2H$	24	39	33	18	–3	0.281	CRF
$HL1L$	28	–12	–7	–31	69	148.781	T
$HL1H$	35	5	10	–22	53	87.781	TF
$HL2L$	30	0	9	13	–29	26.281	TR
$HL2H$	42	6	–35	14	–5	0.781	TRF
$HH1L$	32	7	17	17	9	2.531	TC
$HH1H$	42	12	6	–44	1	0.031	TCF
$HH2L$	8	10	5	–11	–61	116.281	TCR
$HH2H$	31	23	13	8	19	11.281	$TCRF$

The sums of squares due to total $\left(SS_T\right)$ is

$$\sum_{ijk} X_{ijk}^2 - \frac{T_{...}^2}{2^k * r}$$

$$= 4^2 + 18^2 + ... + 18^2 - \frac{427^2}{16*2} = 6521 - 5697 = 823.22$$

Sums of squares due to treatment are:

$$SS_t = 81.28 + 16.531 + ... + 11.281 = 670.715$$

$$\therefore SS_e = SS_T - SS_t = 823.22 - 670.715 = 152.505$$

The degree of freedom for total sum of squares is

$$2^4 * 2 = 32$$

The degree of freedom for error sum of squares 32 – 16 = 16.
The analysis of variance Table is shown in Table 10.

Table 10
ANOVA Table for 2^4 metallurgical Experiments

S.V	**d.f**	**SS**	**MS**	**F-ratio**
(1)	-	-	-	-
F	1	81.281	81.281	8.53
R	1	16.531	16.531	1.73
RF	1	52.531	52.531	5.51
C	1	94.531	94.531	9.92
CF	1	9.031	9.031	0.95
CR	1	22.781	22.781	2.39
CRF	1	0.281	0.281	0.0294
T	1	148.781	148.781	15.61
TF	1	87.781	87.781	9.21
TR	1	26.281	26.281	2.76
TRF	1	0.781	0.781	0.082
TC	1	2.531	2.531	0.27
TCF	1	0.031	0.031	0.00325
TCRF	1	11.281	11.281	1.183
Error	16	152.505	9.532	
Total	32	825.22		

From F-distribution Table of $F_{1,16(0.05)}$ we have 4.49. comparing the F-calculated with the F-tabulated, the conclusion is the effects on the roll size (R) is non-significant while the effects on the main effects on roll tension (T), solution concentration (C), solution temperature (T) are significant.

From the example above, the experiments involve an engineering process which may be difficult to finish in a day hence the need for confounding. Therefore, the need for Plan 2 with k = 4 and block size of 8 becomes necessary.

Table 11

2^4 confounded with b = 8

$$\begin{pmatrix} Block1 \\ (1) \\ ab \\ ac \\ ad \\ bc \\ bd \\ cd \\ abcd \end{pmatrix} \qquad \begin{pmatrix} Block2 \\ a \\ b \\ c \\ d \\ abc \\ abd \\ acd \\ bcd \end{pmatrix}$$

Block 2 will be used since it contains all the main effects.

Table 12

2^4 confounded metallurgical Experiments

$$\begin{pmatrix} Block2 \\ a = F = 22 \\ b = R = 22 \\ c = C = 16 \\ d = T = 28 \\ abc = CRF = 24 \\ abd = CRT = 8 \\ acd = TCF = 42 \\ bcd = TCR = 8 \end{pmatrix}$$

Since the 2^4 has been confounded in blocks, the number of treatments now reduces to 8 which is 2^3.

The Yates technique is shown in Table 13.

Table 13

2^3 confounded metallurgical Experiments

Treatment	Yields	Col. 1	Col. 2	Col. 3	SS. col
F	22	44	88	170	–
R	22	44	82	–28	49
C	16	32	12	18	2.25
T	28	50	–40	–16	16
CRF	24	0	0	–6	2.25
CRT	8	12	18	–52	169
TCF	42	–6	12	18	20.25
TCR	8	–34	–28	–40	100

The treatment T is now the lowest treatment since the total treatment after the Yates technique corresponded to 170.

$$SS_T = \sum_{ijk} X_{ijk}^2 - \frac{T_{...}^2}{2^k * r} = 22^2 + 22^2 + ... + 8^2 - \frac{170^2}{16} = 4476 - 1806.25 = 2669.75$$

$SS_t = 49 + 20.25 + ... + 100 = 358.75$

$\therefore SS_e = 2669.75 - 358.75 = 2311$

The ANOVA Table is shown in Table 14

Table 14

ANOVA Table for 2^3 confounded metallurgical Experiments

S.V	d.f	SS	MS	F-ratio
F	-	-	-	-
R	1	49	49	0.19
C	1	2.25	2.25	0.000875
T	1	16	16	0.062
CRF	1	2.25	2.25	0.00087
CRT	1	169	169	0.66
TCF	1	20.25	20.25	0.079
TCR	1	100	100	0.389
Error	9	2311	256.78	
Total	16	2669		

From F-distribution $F_{1,9,(0.05)}$ we have 5.12. All the main effects and interactions are non-significant.

Conclusion

The study comprises confounding plans for 2^2, 2^3, 2^4 and 2^5. The study extended to k > 5. It was observed that 2^2, 2^3 factorial designs do not need any confounding as the treatment combinations are small and suitable for experimentation. However, 2^4 factorial experiments need confounding as the treatment combination is 16 which may be difficult for the experimenter. When the 2^4 factorial experiment is confounded, all the 4 main effects are confounded in one block. No other factorial design from k > 4 can confound all or 75% of the main effects in one block when the treatment combination is manageable. Therefore, for large design, 2^4 factorial design is recommended.

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