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IMPROVING THE FORECASTING ACCURACY OF DYNAMIC CONDITIONAL CORRELATION (DCC) GARCH MODELS FOR EXCHANGE RATES OF EMERGING MARKET CURRENCIES

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Abstract: *This study examines the performance of Dynamic Conditional Correlation (DCC)-GARCH and hybrid DCC-GJR-GARCH models in modelling and forecasting exchange rate volatility in emerging markets, with a focus on Nigeria and South Africa. Daily exchange rate data for USD, GBP, and EUR against the Nigerian Naira (NGN) and South African Rand (ZAR) from January 2020 to May 2025 were analysed. The models were estimated under both multivariate normal and multivariate Student-t distributions to account for the stylised facts of financial returns, including volatility clustering, heavy tails, and potential asymmetry. Empirical results reveal that exchange rate returns exhibit significant leptokurtosis and time-varying volatility. The Student-t distribution consistently outperforms the normal distribution across all models, as evidenced by higher log-likelihood values and lower AIC and BIC criteria, indicating the importance of capturing fat tails in exchange rate modelling. While the DCC-GJR-GARCH model incorporates asymmetric effects, the leverage parameter was largely insignificant across most currency pairs, suggesting weak asymmetry in exchange rate volatility. Forecast evaluation using RMSE, MAE, and the Diebold-Mariano (DM) test shows that the DCC-GJR-GARCH model provides superior predictive accuracy compared to the standard DCC-GARCH model for most exchange rate series. The findings highlight the relevance of combining dynamic correlations with heavy-tailed distributions in improving volatility forecasts, offering valuable insights for risk management and policy formulation in emerging financial markets.*
Keywords: *Dynamic Conditional Correlation (DCC)–GARCH, GJR-GARCH Model, Exchange Rate Volatility, Emerging Market Currencies, Financial Time Series Forecasting*

Introduction

In the dynamic landscape of global finance, exchange rate volatility remains a critical concern for policymakers, investors, and researchers (Makdissi, Youssef, & Mekdessi, 2023; Adegbe and Kummer, 2025; Al Amosh, 2024; Ullah & Nobanee, 2025). Emerging markets, characterised by their evolving financial systems, limited liquidity, and susceptibility to external shocks, often experience heightened exchange rate fluctuations compared to developed economies. These fluctuations can have far-reaching implications, from inflationary pressures and trade imbalances to capital flight and investment uncertainty. As such, accurate modelling and forecasting of exchange rate volatility in these markets is not merely an academic exercise but a practical necessity for economic stability and strategic decision-making (Lestari & Adekunle, 2024; Batiuk & Kuzyk, 2025; Francisca, 2025; Kim & Choi, 2025).

Among the various econometric tools developed to capture the complexities of financial time series, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family of models has emerged as a cornerstone in volatility modelling (Osho & Oloyede, 2024; Rasyid, 2024; Marisetty, 2024; Lee, 2025). Introduced by Bollerslev in 1986 as an extension of Engle's Autoregressive Conditional Heteroskedasticity (ARCH) model, GARCH models allow for time-varying conditional variance, capturing the clustering and persistence of volatility observed in financial data. However, traditional GARCH models are inherently univariate and fail to account for the interdependencies between multiple financial assets or exchange rates

(Araya, Aduda & Berhane, 2024). This limitation gave rise to multivariate GARCH (MGARCH) models, which incorporate the dynamic relationships among multiple time series.

One of the most widely adopted MGARCH frameworks is the Dynamic Conditional Correlation – Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model proposed by Engle in 2002. The DCC-GARCH model offers a parsimonious yet flexible approach to modelling time-varying correlations among asset returns, making it very crucial for analysing exchange rate dynamics (Shiraya, Suzuki, & Yamakami, 2024; Afuecheta et al., 2024; Marisetty, 2024; Fermanian, Poignard, & Xidonas, 2025). It separates the estimation of individual volatilities from the correlation structure, thereby reducing the computational burden associated with full Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH) models such as the Baba–Engle–Kraft–Kroner (BEKK) model or the Vector Error Correction (VEC) model.

Another important stylised fact of financial returns is the presence of asymmetric volatility, often referred to as the leverage effect, where negative shocks tend to increase volatility more than positive shocks of the same magnitude. Traditional GARCH models, including the standard DCC-GARCH, assume that positive and negative shocks have identical effects on future volatility. This assumption can be too restrictive, especially when it comes to emerging markets where political instability, sudden capital flows, or macroeconomic policy changes often lead to asymmetric volatility responses (Dadzie, 2024; Feijo, 2024). The GJR-GARCH model, developed by Glosten, Jagannathan, and Runkle (1993), extends the GARCH framework to incorporate asymmetric effects by adding a leverage term. By integrating GJR-GARCH specifications into the DCC-GARCH framework, researchers can better model the asymmetric nature of exchange rate volatility, improving the accuracy of forecasts.

Empirical evidence indicates that exchange rate returns, especially in emerging markets, exhibit significant deviations from normality, including skewness and leptokurtosis (fat tails) (Ogunnusi et al., 2024; Nsengiyumva, Mung'atu, & Ruranga, 2025). This implies a higher probability of extreme returns than what the normal distribution predicts. Models based on the normal distribution tend to underestimate tail risk, leading to inaccurate forecasts and suboptimal risk management decisions. To address these shortcomings, alternative heavy-tailed distributions, especially the Multivariate Student-t distribution, have been proposed. The Multivariate Student-t distribution provides a more realistic representation of the empirical distribution of exchange rate returns by accounting for extreme observations and heavy tails.

Emerging markets, including Nigeria and South Africa, provide an ideal context for applying and testing these advanced modelling techniques. Both countries' currencies exhibit high volatility and frequent structural breaks due to macroeconomic instability, policy shifts, and external shocks. Accurate modelling of the co-movements among these currencies and major global currencies like the US Dollar (USD), British Pound (GBP), and Euro (EUR) is critical for investors seeking diversification, policymakers aiming to stabilise currency markets, and businesses managing foreign exchange exposure.

Literature Review

The empirical literature on multivariate volatility modelling highlights the growing relevance of Dynamic Conditional Correlation (DCC)-GARCH models in capturing time-varying relationships across financial assets. Several studies, like that of Afuecheta et al. (2024) and Mishra & Dash (2024), confirm that correlations among financial variables, particularly exchange rates and asset returns, are dynamic and influenced by macroeconomic conditions, crises, and market uncertainties. These findings justify the application of DCC-based frameworks in modelling exchange rate dynamics in emerging markets.

Methodological advancements have focused on improving the flexibility and forecasting performance of traditional DCC-GARCH models. Karim and Michael (2025) proposed the DSCC model to overcome structural limitations of the conventional DCC approach. Despite these improvements, the standard DCC-GARCH model is often limited in capturing asymmetric volatility and extreme market behaviour.

Recent studies emphasise the importance of incorporating asymmetry and heavy-tailed distributions in volatility modelling. Adegboyo and Sarwar (2025) and Ullah et al. (2024) show that GJR-GARCH models, particularly under Student-t distributions, provide superior forecasting accuracy by effectively capturing leverage effects and fat tails. Furthermore, hybrid modelling approaches combining GARCH-type models with advanced techniques such as neural networks (Chung et al., 2024). Additionally, evidence from Saijai et al. (2025) and Iglesias-Casal et al. (2025) demonstrates that integrating GJR-GARCH within DCC frameworks improves the modelling of asymmetric correlations and volatility spillovers, especially during crisis periods. However, limited studies have specifically focused on hybrid DCC-GJR-GARCH models for exchange rate forecasting in emerging markets.

The literature suggests that while DCC-GARCH models are effective, their forecasting performance can be significantly enhanced through hybridisation, incorporation of asymmetry, and the use of heavy-tailed distributions. This study builds on these insights by proposing a hybrid DCC-GJR-GARCH model estimated under the multivariate Student-t distribution to improve exchange rate forecasting accuracy in emerging economies.

Methodology

Data and Preprocessing

The data for this study consists of daily exchange rates from 6 different currencies relative to a common base currency, covering the period from January 1, 2020, to May 31, 2025. These datasets were collected from Yahoo Finance, ensuring data integrity and consistency. Figure 1 shows the daily currency pairs of both Nigeria and South Africa, while Figure 2 visualises the log returns of the exchange rates.

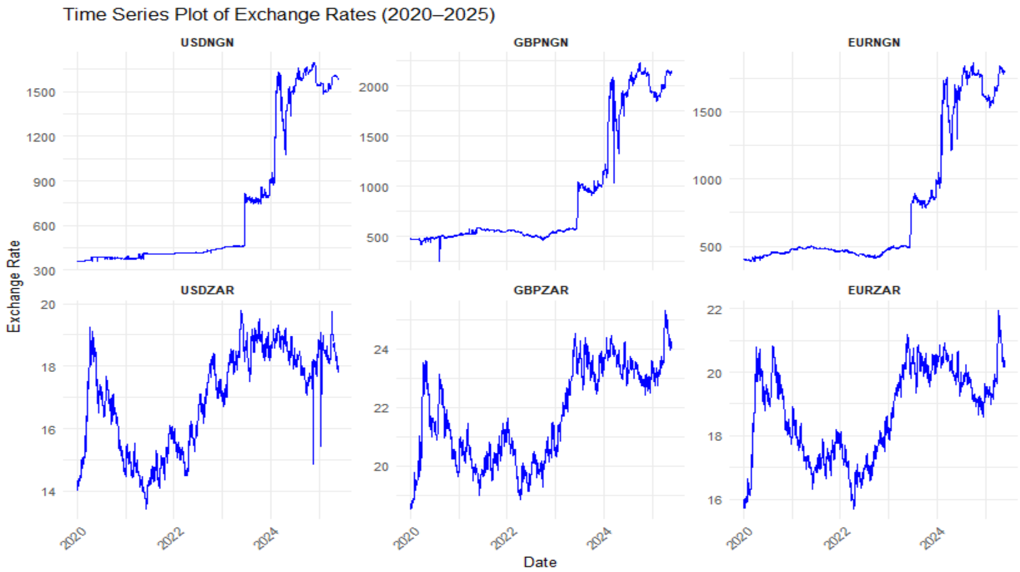

Figure 1. Daily Closing Exchange Rates from January 1, 2020, to May 31, 2025

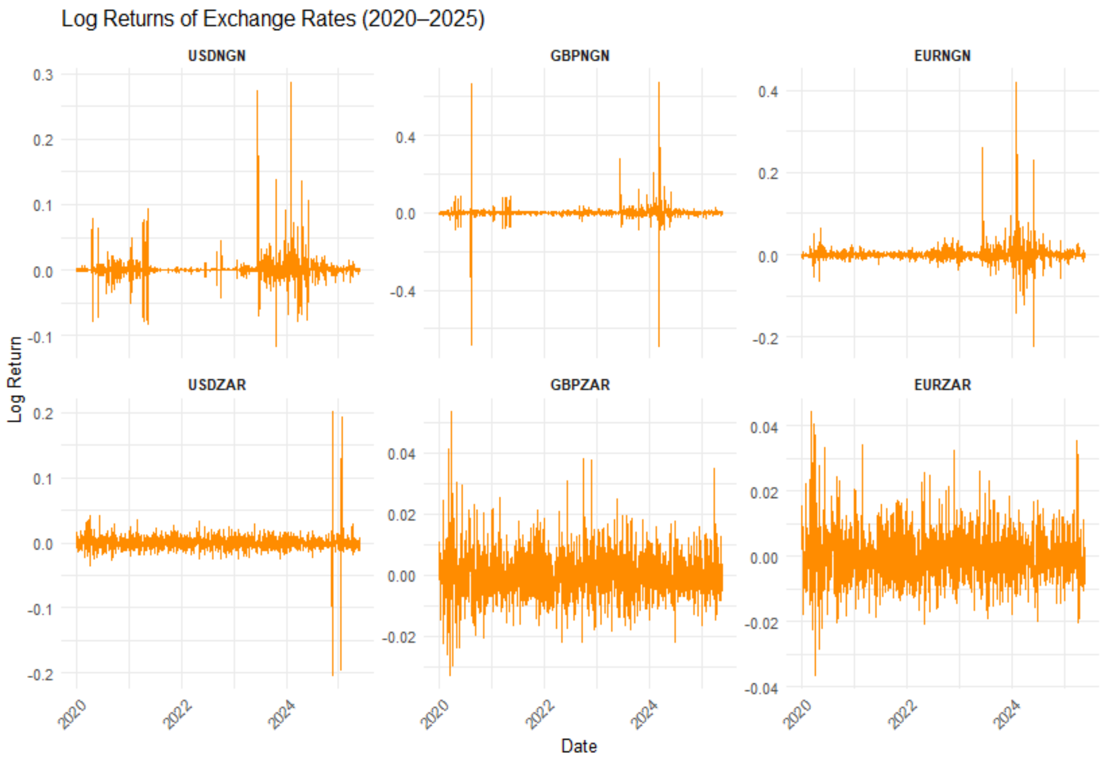

Figure 4.2: Log Returns of Exchange Rates

The missing values were observed and settled using linear interpolation. Linear interpolation estimates a missing value x_t at time t using the surrounding known values $x_{t\text{-}1}$ and x_{t+1} as follows:

$$x_t = x_{t\text{-}1} + \left(\frac{x_{t+1} + x_{t\text{-}1}}{t_{+1} \text{-} t_{\text{-}1}}\right)(t \text{-} t_{\text{-}1}) \qquad (1)$$

To achieve stationarity, the exchange rate series was transformed into log returns using the following formula:

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \qquad (2)$$

where $r_{i,t}$ represents the logarithmic return for currency i at time t, $P_{i,t}$ and $P_{i,t-1}$ are the exchange rates.

3.2 GARCH Model

The GARCH model, developed by Bollerslev, extends the ARCH model by incorporating lagged conditional variances and including both past squared shocks and past forecasted variances, thus providing a more flexible and parsimonious representation of volatility.

Let r_t be the return series at time t, modeled as:

$$r_t = \mu + \varepsilon_t \qquad (3$$

with $\epsilon_t \sim N(0, \sigma_t^2)$, where σ_t^2 is the conditional variance.

A $GARCH(p, q)$ model specifies the conditional variance σ_t^2 as:

$$\sigma_t^2 = \alpha_o + \sum_{i=1}^{q} \sigma_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (4)$$

where $\alpha_o > 0$ ensures a positive baseline variance, $\alpha_i \geq 0$ captures the effect of past shocks (ARCH terms), $\beta_j \geq 0$ captures the persistence of past volatility (GARCH terms), p is the number of lagged conditional variances, and q is the number of lagged squared residuals.

The most commonly used specification is GARCH(1,1), which is written as:

$$\sigma_t^2 = \alpha_o + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (5)$$

where α_1 measures the impact of recent shocks (news about volatility from the last period), and β_1 measures the persistence of volatility from the previous period.

Glosten–Jagannathan–Runkle GARCH (GJR-GARCH) Model

The GJR-GARCH model, proposed by Glosten, Jagannathan, and Runkle (1993), is an extension of the standard GARCH model. It is designed to capture asymmetric effects, specifically, the phenomenon where negative shocks (bad news) tend to increase volatility more than positive shocks (good news) of the same magnitude. The conditional variance h_t of the error term ϵ_t in the GJR-GARCH(p, q) model is specified as:

$$h_t = \omega + \sum_{i=1}^{q} (\alpha_i \epsilon_{t-i}^2 + \gamma_i \epsilon_{t-i}^2 I_{\{\epsilon_t < 0\}}) + \sum_{t=j}^{p} \beta_j h_{t-j} \quad (6)$$

where $\omega > 0$ is a constant, $\alpha_i \geq 0$ are the ARCH terms capturing the effects of past squared shocks, $\gamma_i \geq 0$ are the leverage terms capturing the asymmetric effect of negative shocks, $\beta_i \geq 0$ are the GARCH terms capturing the persistence of past volatility, and $I_{\{\epsilon_{t-i} < 0\}}$ is an indicator function that takes the value 1 if $\epsilon_{t-i} < 0$ (a negative shock) and 0 otherwise.

For the commonly used GJR-GARCH(1,1) model, the conditional variance equation simplifies to:

$$h_t = \omega + \alpha_1 \epsilon_{t-1}^2 + \gamma_1 \epsilon_{t-1}^2 I_{\{\epsilon_{t-1} < 0\}} + \beta_1 h_{t-1} \quad (7)$$

Dynamic Conditional Correlation GARCH (DCC-GARCH) Model

The Dynamic Conditional Correlation (DCC)-GARCH model, developed by Engle (2002), is a multivariate GARCH framework designed to capture time-varying correlations between multiple financial assets. Given that the traditional multivariate GARCH models, such as the Constant Conditional Correlation (CCC) model assumes static correlations over time, the DCC model improves this by allowing the correlation matrix to evolve dynamically, thereby providing a more flexible and realistic representation of interdependencies in financial markets.

Let $r_t = (r_{1t}, r_{2t}, ..., r_{Nt})'$ be a dimensional vector of asset returns at time t, and let the conditional covariance matrix of r_t be denoted by H_t. In the DCC-GARCH(1,1) model, the conditional covariance matrix H_t is constructed as:

$$H_t = D_t R_t D_t \quad (8)$$

where $D_t = diag\,(\sqrt{h_{1t}}, \sqrt{h_{2t}}, ..., \sqrt{h_{Nt}})$ is a diagonal matrix containing the time-varying standard deviations of ϵ_t at time t, obtained from univariate GARCH-type models (such as GARCH or GJR-GARCH), and R_t is an N×N dynamic conditional correlation matrix.

Also, r_t is a T × 1 vector of log returns of n assets at time t. ϵ_t is a T × 1 vector of mean corrected returns of n assets at time t such that $E[\epsilon_t] = 0$ and $Cov[\epsilon_t] = H_t$. μ_t is a T × 1 vector of the expected value of the conditional r_t. H_t is a T × n matrix of conditional variances of ϵ_t at time t. While Z_t is a T × 1 vector of independent and identically distributed random errors such that $E[Z_t] = 0$ and $E[Z_t Z_t^{'}] = I_t$.

The conditional variances, H_t can be estimated separately by a simple univariate GARCH specification of:

$$H_t = \sigma_{i,t}^2 = g_i + \sum_{l=1}^{q} \beta_i \sigma_{i,t-l}^2 + \sum_{m=1}^{p} \alpha_i \epsilon_{i,t-m}^{(2)} \tag{9}$$

where g_i is a T × 1 vector of constants, α_i and β_i are T × n diagonal matrices. $\epsilon_{t-m}^{(2)} = \epsilon_{t-m} \odot \epsilon_{t-m}$ is the Hadamard product, which is the element-by-element product. H_t is a positive definite matrix such that $g_i > 0$ and $\alpha_i, \beta_i \geq 0$. R_t is also a positive definite conditional correlation matrix of the standardised disturbances of Z_t such that $Z_t = D_t^{-1} \epsilon_t \sim N(0, R_t)$. Thus, the elements of $H_t = D_t R_t D_t$ with $\rho_{ii} = 1$ can be written as:

$$[H_t]_{ij} = \rho_{ij},t\sqrt{h_{it} h_{it}} \tag{10}$$

The conditions for the positivity of the covariance matrix H_t require R_t to be positive definite, g_i and all diagonal elements of matrices β_i and α_i to be positive. Therefore, by decomposition,

$$R_t = (Q_t^*)^{-1} Q_t (Q_t^*)^{-1} \tag{11}$$

$$Q_t = (1-a-b)\bar{Q} + a(Z_{t-1} Z_{t-1}^{'}) + bQ_{t-1} \tag{12}$$

Then, $\bar{Q} = Cov[Z_t Z_t^{'}] = E[Z_t Z_t^{'}]$ is a T × n unconditional matrix of the standardised errors Z_t, where $Z_t = D_t^{-1} \epsilon_t$, $Q_t^* = diag\left(\sqrt{q_{1t}}, \sqrt{q_{2t}}, \ldots\ldots\sqrt{q_{nt}}\right)$, a and b are non-negative parameters to be estimated such that $a + b > 1$ to ensure stationarity and positive definiteness of Q_t. As in Engle (2002), $\overline{Q_t}$ can be estimated by

$$\bar{Q}_t = \frac{1}{T} \sum_{t=1}^{T} Z_t Z_t^{'} \tag{13}$$

In general, as in Engle (2002), the DCC-GARCH model is given by

$$Q_t = \left(1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n\right) \bar{Q}_t + \sum_{m=1}^{M} a_m Z_{t-1} Z_{t-1}^{'} + \sum_{n=1}^{N} b_n Q_{t-1} \tag{14}$$

Hybrid DCC–GJR–GARCH Model

The hybrid DCC–GJR–GARCH model extends the traditional Dynamic Conditional Correlation (DCC)–GARCH framework by incorporating asymmetric effects in the conditional variance through the Glosten–Jagannathan–Runkle (GJR)–GARCH specification. This allows the model to capture both time-varying correlations and the leverage effect, where negative shocks have a stronger impact on volatility than positive shocks.

Steps to Construct the Hybrid DCC-GJR-GARCH

1. Specify the mean equation for each return series
2. Estimate univariate GJR-GARCH models for each series to obtain conditional variances h_{it}

3. Compute standardised residuals $z_{it} = \varepsilon_{it}/\sqrt{h_{it}}$
4. Estimate the DCC parameters a and b using standardised residuals
5. Construct the dynamic correlation matrix R_t from Q_t
6. Form the conditional covariance matrix $H_t = D_t R_t D_t$
7. Estimate the full model via maximum likelihood, assuming a multivariate Student-t distribution

3.6 Multivariate Normal Distribution

The multivariate normal distribution is a generalisation of the univariate normal distribution to multiple dimensions. A random vector $X = (X_1, X_2, \cdots, X_k)^T$ is said to follow a multivariate normal distribution with mean μ and covariance matrix Σ, written as $X \sim N_K(\mu, \Sigma)$, with a probability density function (pdf) written as:

$$f(x)=\frac{1}{(2\Pi)^{\frac{k}{2}}|\Sigma|^{\frac{1}{2}}}\exp\left\{-\frac{1}{2}(X-\mu)^T\Sigma^{-1}(X-\mu)\right\} \tag{15}$$

3.7 Multivariate Student-t Distribution

The multivariate Student-t distribution provides a more flexible framework for modelling financial data. A random vector $X = (X_1, X_2, \cdots, X_k)^T$ is said to follow a multivariate Student-t distribution with mean μ and covariance matrix Σ, and degrees of freedom v if its is given by:

$$f(X)=\frac{\Gamma\left(\frac{V+K}{2}\right)}{\Gamma\left(\frac{V}{2}\right)(V\Pi)^{\frac{K}{2}}|\Sigma|^{\frac{1}{2}}}\left[1+\frac{1}{V}(X-\mu)^T\Sigma^{-1}(X-\mu)\right]^{-\frac{V+K}{2}} \tag{16}$$

where $\Gamma(\cdot)$ is the gamma function and $v > 0$ is the degrees of freedom parameter.

3.8 Evaluation Metrics

In this study, two evaluation metrics namely root mean square errors (RMSE) and mean absolute error (MAE) are used to compare the predictive performance of DCC-GARCH and GJR-DCC-GARCH models. Let Y_t be the actual value at time t, $\hat{Y}_t$ be the forecasted value at time t, and T the total number of forecast periods.

The RMSE and MAE are computed as follows:

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{y}_t - y_t)^2}, \quad \text{MAE} = \frac{1}{T}\sum_{t=1}^{T}|\hat{y}_t - y_t| \tag{17}$$

4. Results

Descriptive Statistics Results

Table 1. Descriptive Statistics of the Exchange Rates

Currency Pair	Mean	SD	Skewness	Kurtosis
USD/NGN	729.2592	480.6663	1.0336	2.2991
GBP/NGN	935.0573	615.7349	1.0629	2.3782
EUR/NGN	799.4039	513.7257	1.0631	2.3765
USD/ZAR	16.9725	1.6267	–0.3628	1.7405
GBP/ZAR	21.7315	1.6375	0.0409	1.6253
EUR/ZAR	18.7268	1.4177	–0.2205	1.8147

Table 1 presents the descriptive statistics for six major exchange rate pairs considered in the analysis. For the Nigerian naira (NGN) pairs, GBP/NGN exhibited the highest average rate (₦935.06), followed by EUR/NGN (₦799.40) and USD/NGN (₦729.26), and under the South African rand (ZAR) pairs, GBP/ZAR had the highest mean (21.73), followed by EUR/ZAR (18.73). Based on the standard deviation, for the NGN-based pairs, the USD/NGN exhibited the lowest fluctuations with an SD of 480.6663, while for the ZAR-based pairs, the EUR/ZAR had the least variability (SD = 1.42). Furthermore, the results of Skewness showed that all Nigerian currency pairs are positively skewed, while only GBP/ZAR is positively skewed for South African currency pairs. The results of kurtosis for both currency pairs revealed that all the pairs are platykurtic.

Correlation Matrices of the Currency Pairs

Table 2. Correlation Matrix for Nigerian Currency (NGN) against Major Currencies

	USD/NGN	GBP/NGN	EUR/NGN
USD/NGN	1.000		
GBP/NGN	0.180	1.000	
EUR/NGN	0.079	0.371	1.000

Table 3. Correlation Matrix for South African Currency (ZAR) against Major Currencies

	USD/ZAR	GBP/ZAR	EUR/ZAR
USD/ZAR	1.000		
GBP/ZAR	0.489	1.000	
EUR/ZAR	0.434	0.915	1.000

Tables 2 and 3 present the correlation matrix among three exchange rates for the Nigerian Naira (NGN) and South African Rand (ZAR) against the USD, GBP, and EUR. The results of Table 2 showed that all the currency pairs have weak positive associations. In Table 3, the results revealed a strong positive correlation between EUR/ZAR and GBP/ZAR.

DCC-GARCH Model Estimation

Table 4. DCC-GARCH Model Estimation for Nigerian Exchange Rate with Multivariate Normal and Student-t Distributions

		Multivariate Normal		Multivariate Student-t	
Currency Pair	Parameters	Estimates	p-value	Estimates	p-value
USD/NGN	mu	0.000143	0.355055	0.000024	0.013465
	omega	0.000000	0.879504	0.000000	0.994573
	alpha1	0.043203	0.015217	0.322772	0.000000
	beta1	0.955795	0.000000	0.674315	0.000000
	Shape			2.691816	0.000000
GBP/NGN	mu	0.000089	0.832816	0.000198	0.263669
	omega	0.000002	0.713466	0.000067	0.004960
	alpha1	0.072972	0.003882	0.632281	0.000000
	beta1	0.926028	0.000000	0.366719	0.000009
	Shape			2.422160	0.000000
EUR/NGN	mu	0.000092	0.770968	-0.000026	0.976259
	omega	0.000013	0.313784	0.000010	0.952098
	alpha1	0.428955	0.000266	0.325292	0.919272
	beta1	0.570045	0.056183	0.673708	0.701499
	Shape			3.002072	0.701244
	Joint dcca1	0.011958	0.001571	0.236768	0.076824
	Joint dcca2	0.924646	0.000000	0.313889	0.000097
	Joint mshape			4.000000	0.000000
	Log-Likelihood	6822.704		14353.53	
	Av.Log-Likelihood	4.85		10.2	
	AIC	-9.6741		-20.373	
	BIC	-9.6106		-20.295	

Table 4 presents the estimation results of the DCC-GARCH(1,1) model for Nigerian exchange rates under both multivariate normal and Student-t distributions. The result showed that the mean equation (μ) is statistically insignificant across all currency pairs under both distributions.

From the variance equations, the results showed that the ARCH effect (α_1) and volatility persistence (β_1) are statistically significant for USD/NGN and GBP/NGN under the multivariate normal distribution and multivariate Student-t distributions, and that α_1 is significant for EUR/NGN under the multivariate normal distribution. This shift indicates that the Student-t specification captures short-run volatility clustering more effectively. Furthermore, the shape parameters of the Student-t distribution are statistically significant and relatively low (2.69 for USD/NGN and 2.42 for GBP/NGN), providing strong evidence of fat tails and excess kurtosis in these exchange rate returns.

The results also demonstrated that the dynamic conditional correlation (DCC) parameters $dcca_2$ (persistence of correlations) are highly significant under both distributions, indicating that correlations among exchange rates are highly persistent over time. The results revealed a higher log-likelihood under the multivariate Student-t distribution (14353.53) compared to the multivariate normal distribution (6822.704), and with both AIC (-20.373) and BIC (-20.295) lower compared to the normal case. This confirms that the Student-t DCC-GARCH model provides a superior fit to Nigerian exchange rate data.

Table 5. DCC-GARCH Model Estimation for South African Exchange Rate with Multivariate Normal and Student-t Distributions

		Multivariate Normal		Multivariate Student-t	
Currency Pair	Parameters	Estimates	p-value	Estimates	p-value
USD/ZAR	mu	-0.000612	0.39853	-0.000130	0.589408
	omega	0.000003	0.76829	0.000066	0.065795
	alpha1	0.110436	0.21540	0.252071	0.000000
	beta1	0.888564	0.00000	0.103990	0.766397
	Shape			6.547531	0.000054
GBP/ZAR	mu	0.000100	0.61833	-0.000122	0.539939
	omega	0.000009	0.00000	0.000006	0.000000
	alpha1	0.132037	0.00000	0.091255	0.000000
	beta1	0.732973	0.00000	0.813158	0.000000
	Shape			8.131522	0.000000
EUR/ZAR	mu	0.000042	0.83166	-0.000145	0.452824
	omega	0.000010	0.00000	0.000010	0.000000
	alpha1	0.149769	0.00000	0.153868	0.000000
	beta1	0.696382	0.00000	0.696780	0.000000
	Shape			10.040977	0.000001
	Joint dcca1	0.030763	0.00000	0.025087	0.004676
	Joint dcca2	0.964001	0.00000	0.973556	0.000000
	Joint mshape			6.515413	0.000000
	Log-Likelihood	15635.75		16174.53	
	Av.Log-Likelihood	11.11		11.50	
	AIC	-22.201		-22.962	
	BIC	-22.202		-22.883	

Table 5 reports the DCC-GARCH estimation results for South African exchange rates, where the mean returns are statistically insignificant. The results of the volatility dynamics revealed strong persistence under both distributions, especially for GBP/ZAR and EUR/ZAR, where both α_1 and β_1 are highly significant, confirming the presence of volatility clustering. However, the results also demonstrated that for USD/ZAR under the normal distribution, α_1 is insignificant, suggesting that the normal distribution may not adequately capture short-run volatility dynamics.

Furthermore, under the Student-t distribution, the shape parameters are significant for USD/ZAR, GBP/ZAR and EUR/ZAR, indicating the presence of fat tails and moderate leptokurtosis. The DCC parameters showed that both $dcca_1$ and $dcca_2$ are highly significant under both distributions, with $dcca_2$ values 0.964001 and 0.973556, respectively, implying that correlations between exchange rates are highly persistent and slow-moving.

Model comparison results showed that the log-likelihood (16174.53) under the Student-t distribution is higher compared to the log-likelihood (15635.75) under the multivariate normal, and both AIC (-22.962) and BIC (-22.883) under Student-t are lower than those of the normal distribution, confirming that allowing for heavy tails enhances model performance, even in relatively stable markets like South Africa.

DCC-GJR-GARCH Model Estimation

Table 6. DCC-GJR-GARCH Model Estimation for Nigerian Exchange Rate with Multivariate Normal and Student-t Distributions

		Multivariate Normal		Multivariate Student-t	
Currency Pair	Parameters	Estimates	p-value	Estimates	p-value
USD/NGN	mu	0.000338	0.607254	0.000031	0.003265
	omega	0.000002	0.000217	0.000000	0.993713
	alpha1	0.079853	0.008379	0.350717	0.000000
	beta1	0.875655	0.000000	0.674779	0.000000
	gamma1	0.086984	0.224434	-0.052993	0.371781
	Shape			2.679951	0.000000
GBP/NGN	mu	0.000124	0.000000	0.000172	0.333571
	omega	0.000002	0.063325	0.000065	0.103789
	alpha1	0.116079	0.000000	0.513912	0.000896
	beta1	0.921979	0.000000	0.369295	0.000007
	gamma1	-0.078115	0.000000	0.231588	0.244619
	Shape			2.434366	0.000000
EUR/NGN	mu	0.000126	0.560794	-0.000030	0.968491
	omega	0.000013	0.109138	0.000010	0.931304
	alpha1	0.448103	0.011484	0.318996	0.863593
	beta1	0.571378	0.004333	0.673709	0.583069
	gamma1	-0.040961	0.855800	0.012591	0.986973
	Shape			3.003679	0.568513
	Joint dcca1	0.016235	0.023177	0.144542	0.000000
	Joint dcca2	0.915738	0.000000	0.854425	0.000000
	Joint mshape			4.000000	0.097954
	Log-Likelihood	8438.475		14407.06	
	Av.Log-Likelihood	6.00		10.24	
	AIC	-11.967		-20.445	
	BIC	-11.892		-20.355	

Table 6 extends the analysis by incorporating asymmetric effects through the DCC-GJR-GARCH model, where the results showed that the mean equations remain largely insignificant, consistent with earlier findings. The results further showed that the γ_1 is statistically insignificant under both distributions, indicating no strong evidence of leverage effects, and suggesting that positive and negative shocks have similar impacts on volatility, which is somewhat unusual for financial series but plausible in exchange rate markets influenced by macroeconomic factors.

The results also indicated that the volatility persistence β_1 remains significant for all currency pairs under the distributions, but remains high, especially under the normal distribution for USD/NGN and GBP/NGN, and α_1 significant for all currency pairs except for EUR/NGN under the student-t distribution.

The DCC parameters are significant, especially under the Student-t distribution, where both $dcca_1$ and $dcca_2$ are highly significant, indicating improved modeling of time-varying correlations. The results further revealed that the log-likelihood (14407.06) is higher under Student-t compared to normal distribution (8438.475), and that the AIC (-20.445) and BIC (-20.355) are substantially lower than those of the normal distribution. This provides strong evidence that incorporating fat tails is more critical than modeling asymmetry in the Nigerian context.

Table 7. DCC-GJR-GARCH Model Estimation for South African Exchange Rate with Multivariate Normal and Student-t Distributions

		Multivariate Normal		Multivariate Student-t	
Currency Pair	Parameters	Estimates	p-value	Estimates	p-value
USD/ZAR	mu	-0.000370	0.664252	-0.000112	0.658211
	omega	0.000003	0.889947	0.000067	0.189292
	alpha1	0.163742	0.528706	0.268213	0.000235
	beta1	0.882863	0.000002	0.090540	0.857515
	gamma1	-0.095210	0.677997	-0.031343	0.763020
	Shape			6.551598	0.000069
GBP/ZAR	mu	0.000091	0.656667	-0.000116	0.565665
	omega	0.000009	0.000000	0.000007	0.000000
	alpha1	0.130540	0.000004	0.094660	0.000001
	beta1	0.729211	0.000000	0.808768	0.000000
	gamma1	0.007358	0.878978	-0.005929	0.870854
	Shape			8.113133	0.000000
EUR/ZAR	mu	0.000108	0.600609	-0.000102	0.611633
	omega	0.000011	0.000000	0.000010	0.000000
	alpha1	0.168067	0.000000	0.167178	0.000000
	beta1	0.700906	0.000000	0.703794	0.000000
	gamma1	-0.055586	0.228194	-0.046587	0.306062
	Shape			10.135617	0.000004
	Joint dcca1	Joint dcca1	0.000000	0.024939	0.004642
	Joint dcca2	Joint dcca2	0.000000	0.973707	0.000000
	Joint mshape			6.513112	0.000000
	Log-Likelihood	15624.06		16173.85	
	Av.Log-Likelihood	11.10		11.05	
	AIC	-22.237		-22.956	
	BIC	-22.263		-22.867	

Table 7 presents the DCC-GJR-GARCH results for South African exchange rates, showing that the mean returns are statistically insignificant. The results showed that the volatility parameters indicate strong persistence, particularly for GBP/ZAR and EUR/ZAR, where both α_1 and β_1 are highly significant under both distributions. The results also demonstrated that the asymmetry parameter (γ_1) is largely insignificant across all currency pairs under the two distributions, suggesting that leverage effects are weak in the South African exchange rate market.

The Student-t distribution again captures fat tails effectively, as evidenced by significant shape parameters, confirming that exchange rate returns deviate from normality. The DCC parameters are highly significant under the Student-t distribution, with $dcca_2$ values close to unity, indicating strong persistence in correlations across exchange rates. From a model selection perspective, the Student-t specification again outperforms the normal distribution, with higher log-likelihood (16173.85) and lower AIC (-22.956) and BIC (-22.867).

Comparison of DCC-GARCH and DCC-GJR-GARCH Models

Table 8: Forecast Accuracy Comparison of DCC-GARCH and DCC-GJR-GARCH Models (Multivariate Student-t Distribution) and DM tests

Series	DCC-GARCH RMSE	DCC-GARCH MAE	DCC-GJR-GARCH RMSE	DCC-GJR-GARCH MAE	DM	p-value
USDNGN	0.19716748	0.07062922	0.019716377	0.00706334	1.557	0.012
GBPNGN	0.41265465	0.11132969	0.041266025	0.01113274	0.863	0.039
EURNGN	0.22749158	0.09286204	0.022749362	0.00928631	1.562	0.018
USDZAR	0.14991775	0.08005660	0.014991432	0.00800580	1.656	0.009
GBPZAR	0.18304638	0.06216174	0.008304437	0.00621629	-0.377	0.071
EURZAR	0.83614511	0.06303336	0.008359881	0.00630421	0.468	0.006

Table 8 presents the forecast accuracy comparison between the DCC-GARCH and DCC-GJR-GARCH models under the multivariate Student-t distribution, alongside the Diebold-Mariano (DM) test results. The RMSE and MAE values indicate that DCC-GJR-GARCH performed better than DCC-GARCH model across all exchange rate series.

However, the DM test provided further statistical insight where for most series (USDNGN, GBPNGN, EURNGN, USDZAR, and EURZAR), the p-values are below 0.05, indicating a statistically significant difference in predictive accuracy between the two models, implying that the DCC-GJR-GARCH model exhibits better performance. Conversely, for GBPZAR, the p-value (0.071) exceeds the 5% significance level, suggesting no significant difference between the models for this series.

Conclusion

This study investigated the effectiveness of DCC-GARCH and hybrid DCC-GJR-GARCH models in capturing and forecasting exchange rate volatility in Nigeria and South Africa. The results demonstrate that exchange rate dynamics in these emerging markets are characterised by strong volatility persistence, time-varying correlations, and significant departures from normality. The superiority of the multivariate Student-t distribution across all model specifications confirms the presence of fat tails and highlights the importance of accounting for extreme market movements in volatility modelling.

Although the DCC-GJR-GARCH model extends the standard DCC framework by incorporating asymmetric effects, the empirical findings show limited evidence of leverage effects in both Nigerian and South African exchange rates. Nonetheless, the hybrid model still delivers improved forecast accuracy, as confirmed by RMSE, MAE, and DM test results, suggesting that its flexibility enhances predictive performance.

From a policy perspective, the findings underscore the need for central banks and financial regulators to adopt advanced volatility models that capture both dynamic correlations and tail risks. Accurate modelling of exchange rate volatility can enhance monetary policy decisions, improve foreign exchange risk management, and support financial stability. For investors and multinational firms, the results highlight the importance of incorporating heavy-tailed distributions and dynamic dependence structures in portfolio diversification and hedging strategies.

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