QUEUING MODEL IN CHUKWUEMEKA ODUMEGWU OJUKWU UNIVERSITY TEACHING HOSPITAL

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Abstract
This study examined the application of queuing model in Chukwuemeka Odumegwu Ojukwu University Teaching Hospital with the aim of assessing patient flow, service efficiency, and waiting time management within the hospital system. The study was motivated by the persistent challenge of long patient waiting time and congestion experienced in the hospital due to the increasing demand for healthcare services and limited-service facilities. The research adopted the M/M/1 single-server queueing model to analyze the service process using field data collected over three days. Findings revealed an average inter-arrival time of 6 minutes and an average service time of 4 minutes, resulting in an arrival rate of 0.17 patients per minute and a service rate of 0.25 patients per minute. The utilization factor was found to be 68%, indicating heavy use of available service resources, while the probability of system idleness was 32%. The average queue length was approximately 1.445 patients, the average waiting time in the queue was 8.5 minutes, and the average waiting time in the system was 12.5 minutes, showing moderate congestion and service delay. The study concluded that although the hospital demonstrates reasonable operational performance, improvements are required to reduce waiting time and enhance patient satisfaction. It recommended effective resource management, strategic capacity planning, improved patient communication, adoption of queue management technologies, continuous monitoring of performance indicators, and enhancement of customer care services to improve efficiency and ensure better healthcare service delivery.

Key words: *Queuing Model, Patient Flow, Service Efficiency, Waiting Time Management, healthcare service delivery*

Introduction

Queueing models are essential for studying and optimizing several operational facets in hospitals (Ala, Yazdani, Ahmadi, Poorianasab, and Attari, 2023). By examining the arrival process, service process, queue discipline, and queue capacity, With the aim of increasing patient flow, lowering waiting times, and promoting overall efficiency, queueing models provide vital insights into the dynamics of patient lines and resource allocation (Saini and Sharma, 2022). Hospitals are intricate systems comprising numerous departments, services, and resources that must be efficiently managed to provide high-quality patient care. By employing queueing theory, hospital administrators and healthcare workers can get a greater understanding of patient flow, resource usage, and waiting times, leading to informed decision-making and enhanced service delivery (Dong and Perry, 2020). A primary use of queueing models in hospitals is the examination of emergency departments (EDs).

Emergency departments frequently encounter issues including congestion, extended wait times, and resource constraints (Elalouf and Wachtel, 2021). Queueing models allow healthcare professionals to assess the impact of various factors, such as arrival patterns, triage processes, and resource availability on patient flow. Joseph 2020 Through the analysis of various queueing methodologies and the optimization of resource distribution, hospitals can diminish patient wait times, elevate patient happiness, and increase the overall quality of emergency services. We will examine the queuing model of Chukwuemeka Ojukwu University Teaching Hospital, with particular emphasis on the service process. Chukwuemeka Ojukwu University Teaching Hospital, is one of the best hospitals located in Anambra state with many customers coming in everyday for one particular ailment or the other but which faces challenges in timely services of their patients, patients tend to wait for a long time before being attended to due to the challenges of efficiently managing queues and ensuring a smooth servicing of their patients. The servicing procedure is essential in influencing client satisfaction and overall hospital efficiency. A well-designed queuing models can help hospitals make educated decisions, optimize resource allocation, and enhance overall operational efficiency.

Queuing models offer valuable insights into hospital operations by analyzing patient flow, resource utilization, and waiting times. Dong and Perry (2020). Utilizing queueing theory in emergency departments, outpatient clinics, and operating rooms enables hospitals to make educated decisions, optimize resource allocation, and improve overall operational efficiency. The application of queueing models in Chukwuemeka Ojukwu University Teaching Hospital operations will provide the necessary tools to analyze and optimize their functions. By comprehending patient arrival patterns, service durations, and resource allocation, hospitals can diminish waiting times, enhance patient satisfaction, and utilize their resources more efficiently.

Literature Review

Concept of Queuing Model

Queuing theory is a discipline within operations research that examines and analyzes waiting lines, or queues, together with the mathematical models that characterize their behavior (Ala, Yazdani, Ahmadi, Poorianasab, & Attari, 2023). It offers a framework for comprehending and enhancing the performance of systems characterized by congestion, including transportation networks, call centers, service centers, and manufacturing processes (Ala, Yazdani, Ahmadi, Poorianasab, & Attari, 2023).

Components of a Queuing Model as opined by (Aliyu, & Diockou, 2023)

Arrival Process

The arrival process in a queuing model refers to the pattern or rate at which customers enter the system. It is essential to understand the characteristics of customer arrivals to effectively manage queues. The arrival process can be modeled using various probability distributions, such as Poisson distribution, which assumes arrivals occur randomly and independently over time. Understanding the arrival process helps organizations predict when and how many customers will arrive, allowing them to allocate resources and manage service capacity accordingly.

Service Process

The service process in a queuing model represents the time it takes to serve each customer or complete a task. It is crucial to accurately estimate the service times to optimize resource allocation and minimize waiting times. The service process can be modeled using probability distributions such as exponential distribution, where service times are assumed to follow an exponential pattern. Organizations can use historical data or conduct time studies to determine the service distribution and estimate the average service time. Understanding the service process helps in determining the number of servers required and their efficiency.

Queue Discipline

Queue discipline determines how customers are served from the queue. It defines the order in which customers are selected for service and has a significant impact on customer satisfaction and system performance. The most common discipline is first-in-first-out (FIFO), where the customer who arrived first is served first. Other queue disciplines include priority queues, where customers with higher priority receive preferential treatment, and last-in-first-out (LIFO), where the customer who arrived last is served first. The choice of queue discipline depends on the nature of the system and the organization's objectives.

Queue Capacity

Queue capacity refers to the maximum number of customers that can be accommodated in the queue or system at a given time. Managing queue capacity is crucial to avoid congestion and ensure efficient service. Queue capacity can be determined by physical constraints, such as the number of available seats, or operational constraints, such as the number of servers or resources available. Understanding queue capacity helps organizations plan resource allocation, adjust staffing levels, and implement strategies to handle fluctuations in customer demand.

Types of Queuing Models by (Vilaplana, Solsona, Teixidó, Mateo, Abella, & Rius, 2014)

Single-Server Queues

Single-server queues involve a single server serving customers from a single queue. Examples include a single checkout counter at a grocery store or a help desk with one support agent. Single-server queues are relatively straightforward to analyze mathematically and provide insights into average waiting times, queue lengths, and server utilization. The most common model for single-server queues is the M/M/1 (Markovian arrivals, Markovian service, single server) model, where arrivals and service times follow exponential distributions.

Multi-Server Queues

Multi-server queues have multiple servers serving customers from a single queue. They are commonly used in scenarios where parallel processing or simultaneous service is possible. Examples include multiple ATMs in a bank or multiple ticketing agents at an airport. Multi-server queues can provide faster service and reduce waiting times compared to single-server queues. The analysis of multi-server queues is more complex than single-server queues and often requires simulation or approximate methods. The M/M/c model, where arrivals and service times follow exponential distributions and there are c servers, is a common model for multi-server queues.

Networks of Queues

In complex systems, multiple queues may be interconnected, forming a network. This occurs in situations like traffic intersections, where vehicles arrive from different directions and join separate queues. Networks of queues pose additional challenges for analysis and optimization. The behavior of one queue can affect the behavior of others, and the overall system performance depends on the interactions between queues. Network queuing models involve studying the flow of customers between queues, analyzing routing decisions, and optimizing resource allocation to maximize system efficiency.

Applications of Queuing Models

Capacity Planning

Queuing models are used in capacity planning to determine the appropriate number of servers or resources needed to meet service level targets and minimize waiting times. By analyzing queuing models, organizations can estimate the required capacity to handle customer demand effectively, ensuring that resources are neither underutilized nor overwhelmed.

Performance Evaluation

Queuing models enable the assessment of key performance metrics, such as average waiting time, queue length, and server utilization. These metrics provide insights into the efficiency and effectiveness of the system. By analyzing queuing models, organizations can identify bottlenecks, inefficiencies, and areas for improvement. This information can guide decision-making to optimize resource allocation, streamline processes, and enhance overall system performance.

Service Level Management

Queuing models help organizations define and manage service level agreements (SLAs) by estimating the probability of service delays, customer waiting times, and system performance under different scenarios. By understanding the behavior of queues, organizations can set appropriate service level targets, allocate resources effectively, and make informed decisions to meet customer expectations.

Cost Optimization

Queuing models assist organizations in cost optimization by analyzing resource allocation, staffing levels, and operational processes. By understanding queuing behavior, organizations can make data-driven decisions to minimize costs while maintaining service quality. Queuing models help identify areas of inefficiency, such as overstaffing or underutilization of resources, allowing organizations to allocate resources optimally and reduce operational expenses.

Application of Queueing Theory

Resource Allocation: Organizations must optimally deploy resources, including personnel and equipment, to prevent excessive queuing. This entails assessing staffing levels according to projected patient demand, ensuring sufficient availability of essential equipment, and instituting systems for dynamic resource allocation adjustments based on real-time demand.

Process Enhancement: Organizations can evaluate their processes to pinpoint inefficiencies or bottlenecks that lead to excessive queuing. By enhancing procedures, maximizing patient throughput, and eliminating superfluous delays or redundancies, hospitals can augment operational efficiency and reduce waiting times.

Capacity Planning: Organizations can perform comprehensive capacity planning to ensure that the quantity of service channels, such as physicians, nurses, or examination rooms, aligns with anticipated patient demand. By precisely estimating the arrival and service rates, hospitals can deploy resources efficiently and prevent scenarios where demand surpasses capacity.

Ongoing Monitoring and Evaluation: Organizations must consistently assess important performance indicators, including waiting times, line lengths, and patient satisfaction metrics. Through the analysis of these data, companies can pinpoint areas for enhancement, execute modifications, and assess the efficacy of actions to reduce over-queueing.

Table 1 Data presentation and analyses

Days	Arrival Rate	Service Rate	Balking	Emergency
Day 1	4mins	5mins	2	3
Day 2	8mins	4mins	3	-
Day 3	6mins	3mins	2	-
Average Rate	6mins	4mins	2mins	1mins

Source: Field Work 2025

* Average inter-arrival time: 6 minutes

* Average service time: 4 minutes

* Single-server queuing model (M/M/1)

Based on these data, we can calculate the arrival rate (λ) and the service rate (μ) :

λ = 1 / average inter-arrival time

= 1 / 6 minutes

= 0.17 arrivals per minute

μ = 1 / average service time

= 1 / 4 minutes

= 0.25 services per minute

Now, we can use these rates to calculate other performance measures. For a single-server queuing model (M/M/1), we can calculate the following:

1. Utilization factor (ρ): This represents the utilization of the server it means that the facility was not idle and is calculated as the ratio of arrival rate (λ) to service rate (μ).

ρ = λ / μ

= 0.17 / 0.25

= 0.68 or 68%

2. Probability that the system is idle

1- λ / μ

1-0.68

0.32 or 32%

3. The average/waiting time in the que (wq)

$$wq = \frac{\lambda}{m(m-\lambda)}$$

$$wq = \frac{0.17}{0.25\ (0.25-0.17)}$$

$$wq = \frac{0.17}{0.25\ (0.08)}$$

$$wq = \frac{0.17}{0.02}$$

$$wq = 8.5\, mniutes$$

$$\simeq 9\, minutes$$

4. The length/average number of customers in the system (LQ)

$$Lq = \frac{\partial^2}{m(m - \partial)}$$

$$Lq = \frac{0.17^2}{0.25\,(0.25 - 0.17)}$$

$$Lq = \frac{0.0289}{0.25\,(0.08)}$$

$$Lq = \frac{0.0289}{0.02}$$

$$Lq = 1.445\, customers$$

5. Length of customers in service system (LS)

$$LS = \frac{\partial}{m - \partial}$$

$$LS = \frac{0.17}{0.25 - 0.17}$$

$$LS = \frac{0.17}{0.08}$$

$$LS = 2.125\, customers$$

6. Mean waiting time in the system (WS)

$$ws = \frac{1}{m(m - \partial)}$$

$$ws = \frac{1}{0.25 - 0.17}$$

$$ws = \frac{1}{0.08}$$

$$ws = 12.5\ mniutes/per\ unit\ of\ time$$

Findings

1. Resource Utilization (ρ): The Resource Utilization is approximately 0.68 or 68 %, indicating that the server is utilized quite heavily.: The utilization factor indicates that the hospital's resources, such as staff, equipment, and facilities, are being utilized at a relatively high level. This suggests that a significant portion of the available resources is actively engaged in providing service to patients.
2. If the probability that the system is idle is approximately 0.32 or 32%, it means that there is a relatively low chance of finding the system idle at any given time.
3. Average Number of Customers in the System (Lq): Based on the calculated length of the queue (Lq) of approximately 1.445 customers, the findings in a hospital is that there is an average of 1.445 patients waiting in the queue at any given time.
4. Average Waiting Time in the System (Ws): The average waiting time in the system is approximately 12.5 per units of time and this indicates that there is a significant number of customers waiting in the queue at any given time.
5. Average Waiting Time in the Queue (Wq): The average waiting time in the queue is approximately 8.5 minutes. This indicates that customers are experiencing a significant delay before they can begin receiving service or treatment
6. Average number of customers in the system (Ls) The average number of customers in the service system of 2.125 indicates that there is a moderate level of customer demand for the services provided.

Conclusion

The utilization factor indicates the efficiency of resource usage inside the institution. A high utilization factor signifies that the hospital's space is fully used, perhaps leading to extended wait times and congested circumstances. The hospital must assess its capacity and consider strategies for more efficiently managing demand. Prolonged wait times may indicate an excessive number of patients being treated at the hospital or insufficient staff to accommodate the demand. This issue must be addressed to enhance patient satisfaction and provide timely access to healthcare services. A longer mean wait time within the system may indicate a disruption in the hospital's operations or a deficiency in staffing levels. To enhance patient outcomes, the system's waiting time must be minimized.

Recommendation;

1. **Resource Management:** Due to elevated utilization, it is imperative to implement excellent resource management and scheduling to efficiently satisfy demand. They must deploy resources, including personnel and equipment, efficiently to prevent excessive queuing.

2. **Capacity Planning:** Evaluate the hospital system's capacity and determine whether supplementary resources or infrastructure are necessary to adequately meet the demand. This may entail extending facilities, augmenting the number of beds or treatment rooms, or exploring collaborations with other healthcare providers to manage the patient volume.

3. **Client Communication:** Employ efficient communication tactics to regulate client expectations concerning wait times. Delivering explicit and honest information regarding waiting times can enhance customer satisfaction and mitigate frustration.

4. **Technology Implementation:** Investigate the utilization of technological solutions, including queue management systems or online appointment scheduling, to enhance system efficiency and reduce waiting times. This may also encompass the establishment of a transparent queuing system design, the provision of regular information regarding wait times, and the investigation of alternatives such as virtual queuing or appointment-based systems.

5. **Ongoing Surveillance:** Persistently observe and evaluate critical performance indicators, including the average number of clients within the system, wait times, and service rates. This will facilitate the identification of developing trends or areas for enhancement and enable proactive modifications to guarantee optimal system performance.

6. **Customer care Enhancements;** Emphasize the improvement of the overall customer experience through attentive and individualized care. This can reduce consumer unhappiness and enhance customer loyalty, especially during times of

moderate demand.

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