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Analysis of Elastic Anisotropy of Wood Material for Engineering Applications *(pp. 67-80)*

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Abstract: This paper presents a convenient method to describe the degree of the elastic anisotropy in a given type of wood and then discusses its practical values. Besides mechanical and elastic behaviour of wood are investigated in order to understand the optimum mechanical behaviour of it in selected directions. Bounds on the wood elastic constants have been constructed in terms of elasticity and compliance tensors for any type of woods by developing Hill (1952) approach. So for any type of wood with known elastic constants, it is possible to choose the best set of elastic constants (effective elastic constants) which determine the optimum mechanical and elastic properties of it. Bounds on the wood elastic constants as well as the degree of elastic anisotropy are significant and critical cases in design of any engineering and structural materials made up of wood.

Key words: elastic anisotropy, bounds, elastic constants, elasticity tensor, compliance tensor

1 INTRODUCTION

Wood is a cellulosic, semicrystalline, cellular material. The tissue making up the woody substance is oriented such that mechanical properties are generally higher along the bole of a tree than across the bole. The mechanical properties (elastic, strength and rheologic) exhibit strong orientation effects and are complicated by the addition of growth irregularities. Mechanically, clear wood obeys the laws of elastic orthotropic materials, and its failure characteristics are well described by strain energy of distortion-type theory. Wood also shows properties of high toughness and stiffness. These values vary greatly depending on the type of wood and the direction in which the wood is tested, as wood shows a high degree of anisotropy. Wood's properties are also strongly affected by the amount of water present in the wood. Generally, increasing the water content of wood lowers its strength. Wood shows viscoelasticity and has different properties when wet.

It is also a fibre-composite material (cellulose fibres in a lignin matrix) with complex overall structure and a cellular material. Cells form the basic unit of life and are immensely complicated. There are roughly 1012 cells of 4 main types in a tree. Cells display a great deal of self-organisation and assembly. Additionally, the constituents of a tree undergo continuous renewal, making a tree a dynamic system. Trees are divided into two classes: hardwoods and softwoods. The hardwoods such as Teak, birch, maple have broad leaves. The terms "hardwoods" and "softwoods" are not directly associated with the hardness or softness of the wood although in most cases hardwoods are actually harder and tougher than softwoods. In general softwoods originate from cone-bearing trees and hardwoods from trees that have their seeds contained in a seed-case.

The starting point in the paper is Voigt (1928) and Reuss (1929) schemes that are frequently used in averaging the single-crystal elastic constants for polycrystalline behaviour. In these averaging schemes, it is recalled that, Voigt assumed the uniform strain throughout a polycrystalline aggregate and Reuss assumed the uniform stress.
It is evident that Voigt and Reuss assumptions are true only when the aggregate concerned is made of isotropic crystals, but for an aggregate containing anisotropic crystals, their assumptions become immediately invalid. Hill (1952) has that for an aggregate of anisotropic crystals Voigt and Reuss assumptions result in theoretical maximum and minimum values of the isotropic elastic moduli of the polycrystalline aggregate, respectively, and suggested that a difference between these limiting values may be proportional to the degree of elastic anisotropy of the crystal.
In the present paper, anisotropic Hooke's law is summarized and Kelvin inspired formulation of anisotropic Hooke's law is presented in section 2. Bounds on the wood elastic constants have been constructed and the difference between Voigt and Reuss limits has been examined in detail for wood and used the result as the basis of the present method of describing the elastic anisotropy in sections 3 and 4 respectively. In addition, numerical examples are given in section 5. Finally, in the last section, the results of numerical implementations are discussed and conclusions pertinent to this work are stated.

2 THEORETICAL BACKGROUND

2. 1 Anisotropic Hooke's Law

The anisotropic form of Hooke's law in linear elasticity is often written in indicial notation as

$$T_{ij} = c_{ijkm} E_{km} \tag{1}$$

where T_{ij} are components of stress tensor, E_{km} are components of infinitesimal strain tensor and c_{ijkm} are the components of elasticity tensor (Mehrabadi, 1995). In other words, c_{ijkm} are the components of a fourth-rank tensor called the elastic constant tensor (stiffness tensor) and $i, j, k, m = 1,2,3.$

$$\begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{13} \\ T_{12} \end{bmatrix} = \begin{bmatrix} c_{1111} & c_{1122} & c_{1133} & c_{1123} & c_{1113} & c_{1112} \\ c_{2211} & c_{2222} & c_{2233} & c_{2223} & c_{2213} & c_{2212} \\ c_{3311} & c_{3322} & c_{3333} & c_{3323} & c_{3313} & c_{3312} \\ c_{2311} & c_{2322} & c_{2333} & c_{2323} & c_{2313} & c_{2312} \\ c_{1311} & c_{1322} & c_{1333} & c_{1323} & c_{1313} & c_{1312} \\ c_{1211} & c_{1222} & c_{1233} & c_{1223} & c_{1213} & c_{1212} \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 2E_{23} \\ 2E_{13} \\ 2E_{11} \end{bmatrix} \qquad (2)$$

The indices are abbreviated according to the replacement rule given in the following Table:

Table 1: Abbreviation of Indices for Four Index and Double Index Notations (Nye, 1957)

Four index notation	11	22	33	23, 32	13, 31	12, 12
Double index notation	1	2	3	4	5	6

Alternatively, Hooke's law is also written in indicial notation as

$$E_{ij=} s_{ijkm} T_{km} \qquad (3)$$

where s_{ijkm} are the compliance constants of the anisotropic material. There are three important symmetry restrictions on the elastic constant tensor. These are

$$c_{ijkm=} c_{jikm} \quad c_{ijkm=} c_{ijmk} \quad c_{ijkm=} c_{kmij} \qquad (4)$$

which follow from the symmetry of the stress tensor, the symmetry of the strain tensor and the elastic strain energy. These restrictions reduce the number of independent elastic constants c_{ijkm} from 81 to 21 (Nye, 1957).

2.2 Tensorial Presentation of the Kelvin Formulation

It is written as a linear transformation in six dimensions, Hooke's law has the representation $\mathbf{T = cE}$ and in Voigt notation equation (2) is represented as follows

$$\begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{13} \\ T_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 2E_{23} \\ 2E_{13} \\ 2E_{12} \end{bmatrix} \tag{5}$$

The relationships of the components of c_{ijkm} to the components of the symmetric matrix c are given Table 2. By introducing new notation, equation (5) can be rewritten in the form $T = \hat{c}E$, where the shearing components of these new six dimensional stress and strain vectors which are denoted by T and E, respectively. They are multiplied by $\sqrt{2}$, $\hat{c}$ a new six-by-six matrix is obtained (Mehrabadi, 1995). The matrix form of $T = \hat{c}E$ is given by

$$\begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ \sqrt{2}T_{23} \\ \sqrt{2}T_{13} \\ \sqrt{2}T_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \sqrt{2}c_{14} & \sqrt{2}c_{15} & \sqrt{2}c_{16} \\ c_{12} & c_{22} & c_{23} & \sqrt{2}c_{24} & \sqrt{2}c_{25} & \sqrt{2}c_{26} \\ c_{13} & c_{23} & c_{33} & \sqrt{2}c_{34} & \sqrt{2}c_{35} & \sqrt{2}c_{36} \\ \sqrt{2}c_{14} & \sqrt{2}c_{15} & \sqrt{2}c_{16} & 2c_{44} & 2c_{45} & 2c_{46} \\ \sqrt{2}c_{24} & \sqrt{2}c_{25} & \sqrt{2}c_{26} & 2c_{45} & 2c_{55} & 2c_{56} \\ \sqrt{2}c_{34} & \sqrt{2}c_{35} & \sqrt{2}c_{36} & 2c_{46} & 2c_{56} & 2c_{66} \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ \sqrt{2}E_{23} \\ \sqrt{2}E_{23} \\ \sqrt{2}E_{23} \end{bmatrix} \tag{6}$$

The matrix $\hat{c}$ is called the *matrix of elastic constants* and its inverse $\hat{s}$, $E = \hat{s}T$, $\hat{s} = \hat{c}^{-1}$ is called the *compliance* matrix. A table relating these various notations for the

specific elastic constants is given in Table 2. The symmetric matrices $\hat{c}$ and $\hat{s}$ can be shown to represent the components of a second-rank tensor in a six dimensional space. In this thesis, these matrices are used in the following chapters. Since the components of the matrix **c** appearing in equation (5) do not form a tensor (Mehrabadi, 1995).

Table 2: The Elasticity and Compliance in Different Notations

1	2	3	1	2	3
c_{1111}	c_{11}	$\hat{c}_{11}$	s_{1111}	s_{11}	$\hat{s}_{11}$

c_{2222}	c_{22}	$\hat{c}_{22}$	s_{2222}	s_{22}	$\hat{s}_{22}$
c_{3333}	c_{33}	$\hat{c}_{33}$	s_{3333}	s_{33}	$\hat{s}_{33}$
c_{1122}	c_{12}	$\hat{c}_{12}$	s_{1122}	s_{12}	$\hat{s}_{12}$
c_{1133}	c_{13}	$\hat{c}_{13}$	s_{1133}	s_{13}	$\hat{s}_{13}$
c_{2233}	c_{23}	$\hat{c}_{23}$	s_{2233}	s_{23}	$\hat{s}_{23}$
c_{2323}	c_{44}	$\frac{1}{2}\hat{c}_{44}$	s_{2323}	$\frac{1}{4}s_{44}$	$\frac{1}{2}\hat{s}_{44}$
c_{1313}	c_{55}	$\frac{1}{2}\hat{c}_{55}$	s_{1313}	$\frac{1}{4}s_{55}$	$\frac{1}{2}\hat{s}_{55}$
c_{1212}	c_{66}	$\frac{1}{2}\hat{c}_{66}$	s_{1212}	$\frac{1}{4}s_{66}$	$\frac{1}{2}\hat{s}_{66}$
c_{1323}	c_{54}	$\frac{1}{2}\hat{c}_{54}$	s_{1323}	$\frac{1}{4}s_{54}$	$\frac{1}{2}\hat{s}_{54}$
c_{1312}	c_{56}	$\frac{1}{2}\hat{c}_{56}$	s_{1312}	$\frac{1}{4}s_{56}$	$\frac{1}{2}\hat{s}_{56}$
c_{1223}	c_{64}	$\frac{1}{2}\hat{c}_{64}$	s_{1223}	$\frac{1}{4}s_{64}$	$\frac{1}{2}\hat{s}_{64}$
c_{2311}	c_{41}	$\frac{1}{\sqrt{2}}\hat{c}_{41}$	s_{2311}	$\frac{1}{2}s_{41}$	$\frac{1}{\sqrt{2}}\hat{s}_{41}$
c_{1312}	c_{56}	$\frac{1}{2}\hat{c}_{56}$	s_{1312}	$\frac{1}{4}s_{56}$	$\frac{1}{2}\hat{s}_{56}$
c_{1311}	c_{51}	$\frac{1}{\sqrt{2}}\hat{c}_{51}$	s_{1311}	$\frac{1}{2}s_{51}$	$\frac{1}{\sqrt{2}}\hat{s}_{51}$

c_{1211}	c_{61}	$\frac{1}{\sqrt{2}}\hat{c}_{61}$	s_{1211}	$\frac{1}{2}s_{61}$	$\frac{1}{\sqrt{2}}\hat{s}_{61}$

c_{2322}	c_{42}	$\frac{1}{\sqrt{2}}\hat{c}_{42}$	s_{2322}	$\frac{1}{2}s_{42}$	$\frac{1}{\sqrt{2}}\hat{s}_{42}$
c_{1322}	c_{52}	$\frac{1}{\sqrt{2}}\hat{c}_{52}$	s_{1322}	$\frac{1}{2}s_{52}$	$\frac{1}{\sqrt{2}}\hat{s}_{52}$
c_{1222}	c_{62}	$\frac{1}{\sqrt{2}}\hat{c}_{62}$	s_{1222}	$\frac{1}{2}s_{62}$	$\frac{1}{\sqrt{2}}\hat{s}_{62}$
c_{2333}	c_{43}	$\frac{1}{\sqrt{2}}\hat{c}_{43}$	s_{2333}	$\frac{1}{2}s_{43}$	$\frac{1}{\sqrt{2}}\hat{s}_{43}$
c_{1333}	c_{53}	$\frac{1}{\sqrt{2}}\hat{c}_{53}$	s_{1333}	$\frac{1}{2}s_{53}$	$\frac{1}{\sqrt{2}}\hat{s}_{53}$
c_{1233}	c_{63}	$\frac{1}{\sqrt{2}}\hat{c}_{63}$	s_{1233}	$\frac{1}{2}s_{63}$	$\frac{1}{\sqrt{2}}\hat{s}_{63}$
c_{1222}	c_{62}	$\frac{1}{\sqrt{2}}\hat{c}_{62}$	s_{1222}	$\frac{1}{2}s_{62}$	$\frac{1}{\sqrt{2}}\hat{s}_{62}$
c_{2333}	c_{43}	$\frac{1}{\sqrt{2}}\hat{c}_{43}$	s_{2333}	$\frac{1}{2}s_{43}$	$\frac{1}{\sqrt{2}}\hat{s}_{43}$
c_{1333}	c_{53}	$\frac{1}{\sqrt{2}}\hat{c}_{53}$	s_{1333}	$\frac{1}{2}s_{53}$	$\frac{1}{\sqrt{2}}\hat{s}_{53}$
c_{1233}	c_{63}	$\frac{1}{\sqrt{2}}\hat{c}_{63}$	s_{1233}	$\frac{1}{2}s_{63}$	$\frac{1}{\sqrt{2}}\hat{s}_{63}$

In Table 2, column 1 illustrates the Voigt notation of these quantities as fourth rank tensor components in a three dimensional Cartesian space. Column 2 represents the same Voigt matrix in double index notation. Column 3 illustrates the Kelvin-inspired notation for these quantties as second rank tensor components in a six dimensional cartesian space.

2.3 Bounds on the Wood Elastic Constants

For wood, there are nine independent components of elastic constant tensor. These components are used to construct bounds for wood elastic constants. The upper bounds are denoted by K_V (bulk modulus of Voigt), G_V (shear modulus of Voigt) , ν_R (Poisson's ratio of Reuss) , E_V (Young's modulus of Voigt). The lower bounds are denoted by K_R (bulk modulus of Reuss) , G_R (shear modulus of Reuss) , ν_V (Poisson's ratio of Voigt) , E_R (Young's modulus of Reuss). Bulk modulus, shear modulus, Poisson's ratio and Young's modulus are also called engineering constants (Nye, 1957). The effective anisotropic elastic constants, K_{eff} (effective bulk modulus), G_{eff} (effective shear modulus) , ν_{eff} (effective Poisson's ratio) and E_{eff} (effective Young's modulus)of wood must satisfy the following bounds:

$$K_R \le K_{eff} \le K_V, \qquad G_R \le G_{eff} \le G_V, \tag{7}$$

$$E_R \le E_{eff} \le E_V, \qquad \nu_V \le \nu_{eff} \le \nu_R. \tag{8}$$

K_{eff} , G_{eff} , ν_{eff} and E_{eff} are the best sets of elastic constants of wood which should lie between the above bounds.

$$K_V = \frac{c_{11}+c_{22}+c_{33}}{9} + \frac{2(c_{12}+c_{23}+c_{13})}{9}, \tag{9}$$

$$G_V = \frac{c_{11}+c_{22}+c_{33}}{15} - \frac{c_{12}+c_{23}+c_{13}}{15} + \frac{3(c_{44}+c_{55}+c_{66})}{15}. \tag{10}$$

The bulk modulus of Voigt for wood can be obtained by substituting t, u, v into the equation (9), so it takes the form

$$K_V = \frac{t}{9} + \frac{u}{9}, \tag{11}$$

where $t = c_{11} + c_{22} + c_{33}$, $u = c_{12} + c_{23} + c_{13}$ and $v = c_{44} + c_{55} + c_{66}$.

The shear modulus of Voigt for wood can be found by substituting t, u, v into equation (10) then this equation becomes

$$G_V = \frac{t - u + 3v}{15}. \tag{12}$$

Poisson ratio of Voigt for wood is obtained by substituting the values of K_V and G_V in

$$\nu_V = \frac{1}{2}\left[1 - \frac{3G_V}{3K_V + G_V}\right], \tag{13}$$

where ν_V represents the Poisson's ratio of Voigt for wood and it is also expressed by substituting the appropriate elastic constants

$$\nu_V = \frac{1}{2}\left[\frac{5t - 8u - 2v}{6t - 9u + v}\right]. \tag{14}$$

Young's modulus of Voigt for wood is found by substituting the values of K_V and G_V in

$$E_V = \frac{27 G_V K_V}{3G_V + 9K_V}, \tag{15}$$

where E_V represents the Young's modulus of Voigt for wood and it is also expressed by putting the appropriate elastic constants

$$E_V = \frac{(t + 2u)(t - u + 3v)}{6t + 9u + 3v}. \tag{16}$$

K_V, G_V, ν_R, E_V are upper bounds. In similar way, for wood, Reuss bounds in terms of compliance tensors are

$$K_R = \frac{1}{s_{11} + s_{22} + s_{33} + 2(s_{12} + s_{23} + s_{13})}, \tag{17}$$

$$G_R = \frac{15}{4(s_{11} + s_{22} + s_{33}) - 4(s_{12} + s_{23} + s_{13}) + 3(s_{44} + s_{55} + s_{66})}. \tag{18}$$

The bulk modulus of Reuss for wood can be obtained by substituting x, q, w into the equation (17), then the equation takes the form

$$K_R = \frac{1}{x+2q}. \tag{19}$$

Where $x = s_{11} + s_{22} + s_{33}$, $q = s_{12} + s_{23} + s_{13}$ and

$w = s_{44} + s_{55} + s_{66}$.

For wood, the shear modulus of Reuss can be found by substituting x, q, w into equation (18), then it becomes

$$G_R = \frac{15}{4x - 4q + 3w}. \tag{20}$$

Poisson ratio of Reuss for wood is obtained by substituting the values of K_R and G_R in

$$v_R = \frac{1}{2}\left[1 - \frac{3G_R}{3K_R + G_R}\right], \tag{21}$$

Where v_R represents the Poisson's ratio of Reuss for wood and it is also expressed in terms of the appropriate elastic constants

$$v_R = \frac{1}{2}\left[\frac{w - 2x - 8q}{3x + 2q + w}\right]. \tag{22}$$

For wood, Young's modulus of Reuss is found by substituting the values of K_R and G_R in

$$E_R = \frac{27 G_R K_R}{3G_R + 9K_R}, \tag{23}$$

where E_R represents the Young's modulus of Reuss for wood and it is also expressed in terms of compliance tensors as

$$E_R = \frac{15}{3x + 2q + w}. \tag{24}$$

3 ELASTIC ANISOTROPY OF WOOD

Wood is probably the most commonly recognized anisotropic composite material on earth. As wood possesses a complex fiber-composite structure, it varies in its most properties with the directions, called anisotropy. It is the best described mechanically as an orthotropic material and given the orthogonal symmetry of wood, the orthorhombic (a kind of elastic anisotropy) elasticity concepts developed to describe crystal characteristics.

In previous section, G_V and G_R are needed to construct bounds in order to find the effective shear modulus. Furthermore G_V and G_R represent the averaged polycrystalline shear moduli according to the Voigt and Reuss schemes, respectively. For isotropic materials, G_V is exactly the same as G_R. On the other hand, for anisotropic materials, it is expected from Voigt and Reuss assumptions that G_V exceeds G_R and they represent the theoretical maximum and minimum limits of the true shear modulus for the isotropic polycrystalline aggregate, respectively. Here, for lower symmetry materials such as orthorhombic materials, these two limiting shear moduli G_V and G_R are taken to examine the difference between them as a measure of the elastic anisotropy. For highly anisotropic crystals, however, the difference ($G_V - G_R$) would be a sizable quantity proportional to the magnitude of the elastic anisotropy possessed by the crystals.

Now a dimensionless quantity A can be defined, such that

$$A = \frac{G_V - G_R}{G_V + G_R} \tag{25}$$

Equation (25) is an important result of the present analysis and it represents the degree of elastic anisotropy for wood. However, in an attempt to find a better parameter, A can be described, such that

$$A(\text{in percent}) = (100)\left[\frac{G_V - G_R}{G_V + G_R}\right] \tag{26}$$

The elastic anisotropy specified by the equation (25) has the following properties of a practical importance:

a) $A = 0$ for materials which are elastically isotropic. (If the magnitude of A is closer to 0 , the material property is said to be more isotropic.)

b) $A > 0$ for materials which exhibit anisotropy. (If A is greater than 0 in magnitude, the material exhibits more anisotropic property.)

c) A gives the relative magnitude of the elastic anisotropy.

4 NUMERICAL ANALYSIS

To construct bounds numerically and illustrate the properties of A, several examples are immediately obvious. In the following tables, a few of these are considered. The elastic constants (both elastic stiffnesses and compliances) of hardwoods' and softwoods' species (Hearmon, 1948) can be displayed in Tables 3, 4, 5 and 6 respectively. The units are GPa.

Table 3: The Elastic Constants (Elastic Stiffnesses) of Hardwoods Species

Hard woods	c_{11}	c_{22}	c_{33}	c_{12}	c_{13}	c_{23}	c_{44}	c_{55}	c_{66}
Maple	1.451	2.256	11.492	1.197	1.267	1.818	2.460	2.194	0.584
Oak	0.350	2.983	16.958	1.007	1.005	1.463	2.380	1.532	0.784
Ash	1.439	2.439	17.000	1.037	1.485	1.968	1.720	1.218	0.500
Beech	1.659	3.301	15.437	1.279	1.433	2.142	3.216	2.112	0.912

Table 4: The Elastic Constants (Elastic Compliances) of Hardwoods Species

Hard woods	s_{11}	s_{22}	s_{33}	s_{12}	s_{13}	s_{23}	s_{44}	s_{55}	s_{66}
Maple	1.242	0.762	0.078	0.551	0.030	0.027	2.1-6	1.489	2.342
Oak	1.093	0.614	0.064	0.552	0.024	0.020	0.415	0.551	1.969
Ash	0.933	0.469	0.050	0.553	0.018	0.013	0.334	0.432	1.587
Beech	1.000	0.529	0.055	0.552	0.020	0.016	0.368	0.481	1.745

Table 5: The Elastic Constants (Elastic Stiffnesses) of Softwoods Species

Soft woods	c_{11}	c_{22}	c_{33}	c_{12}	c_{13}	c_{23}	c_{44}	c_{55}	c_{66}
Balsa	0.127	0.360	6.380	0.086	0.091	0.154	0.624	0.406	0.066
DouglasFir	1.226	1.775	17.004	0.753	0.747	0.941	2.348	1.816	0.160
Spruce	0.443	0.775	16.286	0.192	0.321	0.442	1.234	1.52	0.072
Pine	0.721	1.405	16.929	0.454	0.535	0.857	3.484	1.344	0.132

Table 6: The Elastic Constants (Elastic Compliances) of Softwoods Species

Softwoods	s_{11}	s_{22}	s_{33}	s_{12}	s_{13}	s_{23}	s_{44}	s_{55}	s_{66}
Balsa	7.634	4.120	0.159	2.70	-0.123	0.086	1.692	2.179	17.540
DouglasFir	1.202	0.737	0.0532	0.354	0.022	0.018	0.429	0.500	0.547
Spruce	1.984	1.175	0.072	0.614	0.035	0.030	0.622	0.746	10.310
Pine	1.399	0.848	0.058	0.418	0.025	0.021	0.480	0.567	9.010

Table 7: **Bounds on Hardwood Elastic Constants and Elastic Anisotropy for Hardwoods Species**

Hardwoods	K_R	K_V	G_R	G_V	E_R	E_V	v_R	v_V	A	A (%)
Maple	1.155	2.675	0.525	1.796	1.368	4.403	0.303	0.226	0.548	54.8
Oak	1.727	3.138	0.821	2.127	2.126	5.205	0.295	0.224	0.443	44.3
Ash	3.318	3.520	0.987	1.780	2.707	4.530	0.372	0.272	0.287	28.7
Beech	2.451	3.345	0.911	2.284	2.431	5.582	0.335	0.222	0.423	42.3

Table 8: **Bounds on Softwood Elastic Constants and Elastic Anisotropy for Softwoods Species**

Softwoods	K_R	K_V	G_R	G_V	E_R	E_V	v_R	v_V	A	A (%)
Balsa	0.164	0.837	0.121	0.655	0.292	1.558	0.203	0.190	0.687	68.7
Douglas Fir	0.830	2.765	0.395	2.036	1.023	4.904	0.297	0.204	0.675	67.5
Spruce	0.534	2.157	0.296	1.669	0.750	3.979	0.266	0.193	0.699	69.9
Pine	0.726	2.527	0.364	2.139	0.935	5.006	0.285	0.170	0.709	70.9

Effective elastic constants for hardwood and softwood can be selected between the corresponding values found in Tables 7 and 8 respectively.

5 DISCUSSION AND CONCLUDING REMARKS

Results of the Tables illustrated in section 4 can be interpreted as follows: According to Table 7; Maple exhibits the most anisotropy among the other hardwoods' species since it has the greatest degree of elastic anisotropy with 54.8%. Whereas Ash is close to isotropy more than both hardwoods' and softwoods' species because of its degree of elastic anisotropy with 28.7%.

From Table 8; it is seen that Pine shows the most anisotropy among all wood species with the degree of elastic anisotropy 70.9% . So it's mechanical and elastic behavior are expected to be more anisotropic. These results prove that different wood types selected from same anisotropic elastic symmetry (orthorhombic symmetry), depending upon the degree of elastic anisotropy, can exhibit whether close to isotropy or anisotropy.

In this paper, it has been also shown that it is possible to construct bounds on the wood elastic constants in terms of elasticity and compliance tensors. It has been mainly focussed on engineering elastic properties of selected materials and represented anisotropy in terms of engineering properties: K , G , and ν , E in order to construct bounds. Constructing bounds on the anisotropic elastic constants provides a deeper understanding about mechanical behavior of anisotropic materials. Besides, the degree of elastic anisotropy also has significant effects on many applications in different fields such as:
1) design of wood-based composite materials,
2) determination of wood types which are highly anisotropic or close to isotropic,
3) understanding the mechanical and elastic behaviour of natural composites such as wood types.

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