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Markov Chain Analysis of Manpower Data of a Nigerian University *(pp.107-123)*

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Abstract: The revolving door policy upon which the appointment of Vice-Chancellors of the Nigerian Universities is set appears to hasten the onset of dissonance and discontinuity in the general institutional arrangements and polity. Moreover, little effort appears to have been made to characterize manpower flow in relation to the manpower policies driving such system in order to ascertain if they are congruent with the raison d'être of the institutions. This study, which is an applied research, seeks to use the Markovian statistical tool to unravel the dynamics of staff stock and flow in a typical first generation Nigerian University with the ultimate intent of lucidly describing the existing manpower policy and pointing to its future direction. A forty-year data of staff transition within the six well defined states space, were transformed into frequency distributions which was ultimately used to estimate the transition probability matrix (TPM) that substantiated into a valued diagraph. Our research findings clearly suggest that about 47% of newly recruited staff exit the employment system through normal retirement while a disturbing 53% leave by either voluntary withdrawal or wastage. Possible attributions, or factors to blame, for this high attrition rate is principally staff seeking and discovering greener pastures, and disciplinary cases. The conclusion thereof is that, overall, in view of the favourable staff development programme, the existing manpower policy can be described as liberal and firm, and above all is tilted towards prime pumping capacity building. The research outcome, which is a strategic imperative for Nigerian universities, provides game-changing solutions and practical guides needful to align manpower policies to corporate goals.

Key words: staff stock, habituation decomposable, valued diagraph ergodic chain.

1 INTRODUCTION

The approach to manpower policy in most Nigerian Universities appears to be guided by the traditional method of putting the right number of people in the right place at the right time or arranging for suitable number of people to be allocated to various jobs usually in a hierarchical structure. The technique is outdated because it lags behind the state-of-the-art method that deals with manpower policy in the context of organizational strategy. The traditional method is deficit in the sense that it neither offers computational tools that will enable managers to determine possible line of action to be taken to steer manpower policy to desired ends nor provide tools to generate alternative policies and strategies. Government is concerned about these policies to the extent that the National Universities Commission (NUC) embarks on regular staff audit which focuses on fishing out of ghost workers, rightsizing to reduce overhead and aligning employment policy to strategic goals.

The method advocated is exploratory and through its computational tools, can generate outcomes that will enable normative models to be formulated. In this regard, prescriptive standard that can guide manpower policy to the desired direction can be easily established. The traditional methods are naïve and therefore lack this potency.

Manpower planning modelling has been accomplished by different approaches but the Markov Chain approach appears to offer more intuitive appeal than others like optimization method such as simulation, renewal theory, decision calculus, among others. Markov chain modelling is one of the most powerful tools for analyzing complex stochastic systems (Kim and Smith, 1989). The versatility of Markov chain is widely acclaimed. Slatyer (1977) is an excellent material that discussed a wide range of possible applications of Markov Chain. According to the source, it has been applied in the study of rainforest communities to characterize forest succession and hence predict its composition.The works: Williams et al. (1969), Stephens and Waggoner (1970), and Horn (1974, 1975) are typical of such applications. Other studies of the use of Markov models in forests include those of Pendern et al. (1973), Cassell and Moser (1974) and Lembersky and Johnson (1975). Later Binkley (1980),in addition to Buongiorno and Michie (1980) extended the forest application to incorporate such parameters as plot canopy gap level, individual tree level and stand level analysis of specific forest management problem.

Applications to other types of ecological system are even rarer than those of forests but Debussche et al (1977) describe an agricultural application to the southern end of the French Massif Central. Van deveer and Drummond (1978) have also used Markov process for estimating land use changes, particularly where a major impact such as a reservoir is imposed upon an existing system. Other biological applications of Markov processes include that by Rao and Kshirsagar (1978) who made a study of the population dynamics of predator/prey systems in which the attack cycle of a predator is assumed to consist of four different activities, namely search, pursuit, handle and eat, and digest.

Markov chain has also been applied successively to population genetics (Bartholomew and Forbes, 1979), geology and stratigraphy (Krumbein, 1967; Harbaugh and Bonham-Carter, 1970; Norris, 1971; Bhatta Chanya et al., 1970; Lloyd, 1977; Smeach and Jernigan, 1977). Moreover, Anthony and Taylor (1977) explored the use of Markov models in forecasting air pollution levels. The analysis of historical data concerning variations in air pollution indices suggests a pattern which might usefully be described by a transition probability matrix.

Zanakis and Marret (1980) noted that personnel supply in an organization can be forecasted using Markov Chains to model the flow of people through various states (usually skill or position levels). Such applications are treated in Rowland and Sovereign (1969), Nielson and Young (1973) and Merch (1970). While manpower flow through minority status is reported in Churchill and Shank (1975), flow based on years of service is studied by Merch (1970), Leeson (1979) and Uyar (1972).

Parthasarathy et al (2010) adopted stochastic models to manpower planning in an organization and showed that for a two grade manpower system, exponentiated exponential distribution can be used to determine when the cumulative loss of manpower crosses a random threshold level that is detrimental to organizational performance. Related previous work in this direction include Rao (1990), Esary (1973), Sathiyamoorthi and Parthasarathy (2002), Parthasarathy and Vinoth (2009), Gupta and Kundu (2001), Gumpta and Kundu (1999).

Markov chain models have become popular in manpower system planning. Several researchers have adopted Markov chain models to clarify manpower policy issues: Kim and Smith (1989), Heyman and Sobel (1982), Zanakis and Maret (1980), Calantone and Darmon (1984), Trivedi et al (1987) and Skulj et al (2008).

In summary, one can say that there have been several attempts at applying Markov chain to a wide range of problem situations. A myriad of these situations exist and so do the associated data may appear, completely eclectic and therefore require skill in the estimation of transition probability. The current paper collected data spread over large state space and we saw the need for aggregation to achieve parsimony. Accordingly, the state space was reduced to six from seventeen.

In this paper we developed the theory of canonical forms of transition probability matrix of a finite ergodic recurrent set decomposable Markov chain of order n with steady state probability row vector. We fitted a 40-year data of personnel transitions over 6 states obtained from one of the foremost Nigerian Universities. The computational results were used to interprete the manpower policies of the organization studied.

2 METHODS

This is both basic and applied research in the sense that the paper offers theoretical knowledge upon which application to solve fundamental human resources planning problems in the organization studied is established. The structure and strategy of our investigation comprise description of the manpower structure of the university being studied. For mathematical tractability, the states space investigated was condensed from sixteen states to mere six states which are enumerated as follows: (i) Recruitment (Rc) (ii) Staff Stock (Ss) (iii) Training leave (T) (iv) Suspension (S) (v) Wastage (W) (vi) Retirement (R).

By states, it is specifically implied the condition, the status, the position or situation a Markov Chain (object or staff) undergoes in the transition process towards the final position of retirement, if the staff can ever get there. The staff movement, in steps, in this system is considered to be a non-cyclic ergodic chain that appears regular. A forty year data of staff movements, as specified by state space described above, was obtained from the organization studied. The population of staff is 4,000. Bayesian method of determining probability, complemented with heuristic arguments, was used to determine step transition probabilities. The probabilities were organized in canonical form in order to conform to the theory of Markov Chain transitions that will be presented in the paper. The data obtained were checked for stochastic regularities. Basic assumptions were explicitly stated and applicable theorems and formulae governing the computation were presented. The data obtained appear prestine although we cannot vouch for perfection in the way the records were kept.

A purposive and quota sampling methods were used. Markov Chain model was used to analyse the data collected. A valued diagraph (Markov Transition Diagram) was developed to enable one discern the structure of transition probability matrix. Finally, the data obtained were used to compute the transition probability matrix (TPM) which we depict in Table 1. The estimates of the transition probabilities were based on frequency distributions or tabulations of the number of transitions from one state to the other in the system considered. The frequency were converted to TPM by dividing each row by its total .

Table 1: Pristine States space

States	1970 - 2010	States	1970 - 2010
Contract	282	Special Leave	5
Dismissal	2	Study Leave	2
Late	109	Suspension	2
Leave of Absence	65	Temporary	617
Left	108	Termination Appointment	3
Permanent	3051	Training Leave	63
Resigned	12	Visiting Appointment	12
Retired	446	Withdrawal of Service	7
Sabatical	49		

Table 2: Lumped States space

States	Manpower
Retirement	458
Wastage	227
Disciplinary case	463
Leave	191
Staff stock	3109
Recruitment	911
Total	5359

2.1 Analytical Formulation

The theory governing movement of personnel among well defined states (i.e. status, conditions and positions) in the organisation studied is hereby briefly presented.

Record of staff movement over a period of 40 years (n=40) was obtained from a typical Nigerian University and the Markov Chain model was fitted into the record.

Consider a staff recruited and who begins transition from initial distribution d_o among the six states. The long run distribution is expressed as:

$$\bar{T} = d_0 T^n \quad (1)$$

Where $\bar{T}$ = stabilized transition, in this case n = 40 years (T^n). It is assumed that after this number of movements, the matrix T would have attained stochastic regularity (stationarity).

$$T = \left[\begin{array}{c|c} I & O \\ \hline R & Q \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0.04632 & 0.09447 & 0.03897 & 0.63436 & 0.18588 \\ 0.08924 & 0 & 0.09022 & 0.03722 & 0.60581 & 0.17751 \\ 0.09355 & 0.04636 & 0 & 0.03901 & 0.63501 & 0.18607 \end{array}\right]$$

Thus

$$d_o = R = \begin{bmatrix} 0 & 0.04632 & 0.09447 \\ 0.08924 & 0 & 0.09022 \\ 0.09355 & 0.04636 & 0 \end{bmatrix} \tag{2}$$

$$Q = \begin{bmatrix} 0.03897 & 0.63436 & 0.18588 \\ 0.03722 & 0.60581 & 0.17751 \\ 0.03901 & 0.63501 & 0.18607 \end{bmatrix} \tag{3}$$

From this initial distribution d_0, if it transits to other states as follows:

$$T^2 = \left[\begin{array}{c|c} I & O \\ \hline R & Q \end{array}\right]^2$$

$$= \left[\begin{array}{c|c} I & O \\ \hline (I+Q)R & Q^2 \end{array}\right] \tag{4}$$

$$T^3 = \left[\begin{array}{c|c} I & O \\ \hline (I+Q+Q^2)R & Q^3 \end{array}\right]$$

Also $T^4 = \left[\begin{array}{c|c} I & O \\ \hline (I+Q+Q^2+Q^3)R & Q^4 \end{array}\right]$ (5)

Thus in general,

$$T^n = \left[\begin{array}{c|c} I & O \\ \hline (I+Q+Q^2+\ldots+Q^{n-1})R & Q^n \end{array}\right] \tag{6}$$

Clearly,

$$(I-Q)(I+Q+Q^2+\ldots+Q^{n-1})R = I - Q^n$$

$$\therefore T^n = \begin{array}{c} \\ abs \\ nonabs \end{array} \begin{array}{c} \begin{array}{cc} abs & nonabs \end{array} \\ \left[\begin{array}{c:c} I & O \\ \hdashline (I-Q)R & Q^n \end{array}\right] \end{array} \quad (7)$$

It is well known that every object in a nonabsorbing state eventually enters any of the absorbing state after a long run transition. Hence from (7), by this notion, long run transition from the 4th quadrant to the first is represented as

$Q^n \to 0$ as depicted in (7).

Since

$$(I-Q)(I+Q+Q^2+...+Q^{n-1}) = I-Q^n$$

And since $Q^n = 0$

$$(I+Q+Q^2+...+Q^{n-1}) = \frac{I}{(I-Q)} = (I-Q)^{-1}$$

$$\therefore T^n = \left[\begin{array}{c:c} I & O \\ \hdashline (I-Q)^{-1}R & Q^n \end{array}\right]$$

$$= \left[\begin{array}{c:c} I & O \\ \hdashline (I-Q)^{-1}R & O \end{array}\right]$$

Set $N = (I-Q)^{-1}$, and

$B = NR = (I-Q)^{-1}R$

Where $R = d_o$ = long-run distribution to absorbing states

3 RESULTS

Figure 1 depicts the valued diagraph of a Markov Chain transiting among states space, $S = \{S_1, S_2, . . S_6\}$.

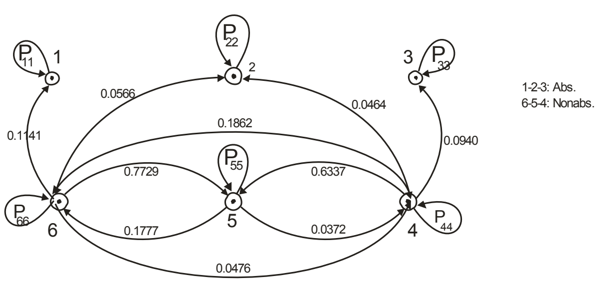

Figure 1: Markov Transition Diagram

It is obvious from the diagraph that it is a non-cyclic ergodic chain (regular). Moreover, the open circuit at states 1, 2, 3 is a sure evidence that they are absorbing states and therefore, Markov Chain properties apply. States 4, 5, 6 are looped showing that objects habituate in step-wise fashion among these states before being trapped in states 1, 2, 3.
In the method section, we showed that the long run TPM in canonical form resulted to matrix shown in equation (7).

$$\left[\begin{array}{c|c} I & 0 \\ \hline (I-Q)^{-1}R & Q^n \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0.6037 & 0.1421 & 0.5667 & 0 & 0 & 0 \\ 0.6656 & 0.0914 & 0.5409 & 0 & 0 & 0 \\ 0.8504 & 0.1733 & 0.5758 & 0 & 0 & 0 \end{array}\right], Q^n = 0$$

Three sets of results were obtained that were considered very significant. The first deals with the mean and variance of the level of habituations of Markov Chain among transient states.

Accordingly:

Mean of habituation

(i) Mean, $N = (I-Q)^{-1}$

$$N = (I-Q)^{-1} = \begin{array}{c} \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccc} 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1.3027 & 4.9113 & 1.4432 \\ 0.2890 & 5.6871 & 1.3772 \\ 0.3692 & 5.6895 & 2.7599 \end{bmatrix} \end{array}$$

The interpretation of this fundamental matrix is significant. For instance, it is evident from this fundamental matrix (N) that a newly recruited staff works in 5.9895 i.e.6 different job positions while being on probation before confirmation. Movement from 6 to 4 (i.e newly recruited staff going on leave) is unusual as shown by the value (0.3692) which is lower than one.

$N(2N_{dg}-I)$:

$$N = \begin{bmatrix} 1.2304 & 3.7503 & 1.0989 \\ 0.2200 & 4.5815 & 1.0494 \\ 0.2306 & 3.7541 & 2.100 \end{bmatrix}$$

$$N_{dg} = \begin{bmatrix} 1.2304 & 0 & 0 \\ 0 & 4.5815 & 0 \\ 0 & 0 & 2.100 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2N_{dg} = \begin{bmatrix} 2.4608 & 0 & 0 \\ 0 & 9.1630 & 0 \\ 0 & 0 & 4.200 \end{bmatrix}$$

$$2N_{dg}\text{-}I = \begin{bmatrix} 1.4608 & 0 & 0 \\ 0 & 8.163 & 0 \\ 0 & 0 & 3.200 \end{bmatrix}$$

$$N(2N_{dg}\text{-}I) = = \begin{bmatrix} 1.2304 & 3.7503 & 1.0989 \\ 0.2200 & 4.5815 & 1.0494 \\ 0.2306 & 3.7541 & 2.100 \end{bmatrix} \begin{bmatrix} 1.4608 & 0 & 0 \\ 0 & 8.163 & 0 \\ 0 & 0 & 3.200 \end{bmatrix} = \begin{bmatrix} 1.7974 & 30.6137 & 3.5165 \\ 0.3214 & 37.3988 & 3.3581 \\ 0.3369 & 30.6447 & 6.7200 \end{bmatrix}$$

$$N_2 = N(2N_{dg}\text{-}I) - N_{sq}$$

$$N_{sq} = \begin{bmatrix} 1.2304^2 & 3.7503^2 & 1.0989^2 \\ 0.2200^2 & 4.5815^2 & 1.0494^2 \\ 0.2306^2 & 3.7541^2 & 2.100^2 \end{bmatrix} = \begin{bmatrix} 1.5139 & 14.0648 & 1.2076 \\ 0.0484 & 20.9901 & 1.1012 \\ 0.0532 & 14.0933 & 4.4100 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} 1.7974 & 30.6137 & 3.5165 \\ 0.3214 & 37.3988 & 3.3581 \\ 0.3369 & 30.6447 & 6.7200 \end{bmatrix} - \begin{bmatrix} 1.5139 & 14.0648 & 1.2076 \\ 0.0484 & 20.9901 & 1.1012 \\ 0.0532 & 14.0933 & 4.4100 \end{bmatrix} = \begin{bmatrix} 0.2835 & 16.5489 & 2.3089 \\ 0.2730 & 16.4087 & 2.2569 \\ 0.2837 & 16.5514 & 2.3100 \end{bmatrix}$$

$$\tau_2 = (2N - I)\tau - \tau_{sq}, \ \tau = N\xi, \ \xi = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\tau = N\xi = \begin{bmatrix} 1.2304 & 3.7503 & 1.0989 \\ 0.2200 & 4.5815 & 1.0494 \\ 0.2306 & 3.7541 & 2.100 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6.0796 \\ 5.8509 \\ 6.0847 \end{pmatrix}$$

$$\tau_{sq} = \begin{pmatrix} 36.9615 \\ 34.2330 \\ 37.0236 \end{pmatrix}$$

(2N-I):

$$2N = \begin{pmatrix} 2.4608 & 7.5006 & 2.1978 \\ 0.4400 & 9.1630 & 2.0988 \\ 0.4612 & 7.5082 & 4.2000 \end{pmatrix}$$

$$\mathrm{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(2\mathrm{N}\text{-}\mathrm{I}) = \begin{pmatrix} 2.4608 & 7.5006 & 2.1978 \\ 0.4400 & 9.1630 & 2.0988 \\ 0.4612 & 7.5082 & 4.2000 \end{pmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.4608 & 7.5006 & 2.1978 \\ 0.4400 & 8.1630 & 2.0988 \\ 0.4612 & 7.5082 & 3.2000 \end{bmatrix}$$

And $(2N-I)\tau = \begin{bmatrix} 1.4608 & 7.5006 & 2.1978 \\ 0.4400 & 8.1630 & 2.0988 \\ 0.4612 & 7.5082 & 3.2000 \end{bmatrix} \begin{pmatrix} 6.0796 \\ 5.8509 \\ 6.0847 \end{pmatrix} = \begin{bmatrix} 66.1393 \\ 63.2065 \\ 66.2047 \end{bmatrix}$

$$\tau_2 = (2N-I)\tau - \tau_{sq} = \begin{bmatrix} 66.1393 \\ 63.2065 \\ 66.2047 \end{bmatrix} - \begin{pmatrix} 36.9615 \\ 34.2330 \\ 37.0236 \end{pmatrix} = \begin{bmatrix} 30.1157 \\ 28.9735 \\ 29.1811 \end{bmatrix}$$

τ_2 represents the variances associated with transition of objects among the three non-absorbing states.

The long run distribution of TPM (T^n) has been shown earlier to be

$$\bar{T} = T^n = \left[\begin{array}{c|c} I & O \\ \hline (I-Q)^{-1}R & Q^n = 0 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & O \\ 0 & 0 & 1 & \\ \hline 1.2304 & 3.5703 & 1.0989 & \\ 0.2200 & 4.5815 & 1.0494 & O \\ 0.2306 & 3.7541 & 2.100 & \end{array} \right]$$

$$\mathrm{R} = \begin{bmatrix} 0 & 0.04632 & 0.09447 \\ 0.08924 & 0 & 0.09022 \\ 0.09355 & 0.04636 & 0 \end{bmatrix} = \mathrm{d_o}$$

The fundamental matrix, N, is :

$$N = (I-Q)^{-1} = \begin{array}{c} \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{matrix} 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1.2304 & 3.7503 & 1.0989 \\ 0.2200 & 4.5815 & 1.0494 \\ 0.2306 & 3.7541 & 2.1000 \end{bmatrix} \end{array}$$

The matrix N_2 provides interesting information about the variance of the estimate of $N = (I-Q)^{-1}$. We use the N_2 derivatives, namely standard deviation (σ) to report the possible variability of the estimates of N.

$$B = NR = Nd_o = \begin{bmatrix} 1.2304 & 3.7503 & 1.0989 \\ 0.2200 & 4.5815 & 1.0494 \\ 0.2306 & 3.7541 & 2.100 \end{bmatrix}\begin{bmatrix} 0 & 0.04632 & 0.09447 \\ 0.08924 & 0 & 0.09022 \\ 0.09355 & 0.04636 & 0 \end{bmatrix}$$

$$B = \begin{array}{c} \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 0.4375 & 0.1079 & 0.4546 \\ 0.5070 & 0.0588 & 0.4341 \\ 0.5315 & 0.1087 & 0.3605 \end{bmatrix} \end{array}$$

The matrix B provides the long run distribution of recruited staff.
Finally, $B = NR = Nd_o$

Flowing from this background information we undertake the following interpretations

(i) Number of habituations staff make in a particular non-absorbing state as well as the standard deviation of such estimates.

(ii) The total number of habituations staff undertook among all the three non-absorbing states before being trapped in any of the absorbing state. The associated standard variances, represented by τ_2 , are also important.

(iii) Using $B = Nd_o$, we estimate the probability that staff starting in any of the non-absorbing states can end up in any of the absorbing states.
Accordingly:

(i) Movement of staff within the non-absorbing states

a. **6→4 with entry 0.3692**: little number of newly recruited staff, perhaps less than four in every ten instances, benefit from categories of leave prior to confirmation. This estimate has low standard deviation and it therefore hardly varies.

b. **6→5 with entry 5.9895**: for staff who are still on probation following recruitment, they undergo, on the average, six movements to different job positions as confirmed staff. This is particularly more so with non-academic staff than academic who change job position by promotions only. This average estimate of six is subject to variance of 26.2622 (σ=5). In other words, staff may experience at least one change in job position or up to eleven (6+5=11) positions.

c. **6→6 with entry 2.7599 (i.e. persistence):** in this category, staff on temporary appointment or visiting appointment average about three visits to different positions while still remaining a temporary or visiting staff.

d. **5→4 with entry 0.2890:** This represents the number of times on the average staff may go on study leave, training leave, special leave, sabbatical leave or leave of absence. This happens perhaps about three in every ten occasions.

e. **5→5 with entry 5.6871 (persistence):** This variety applies to staff who remain in the system and change job positions. On the average, such staff change positions six times but this estimate may vary from 1 to 11 (σ= $\sqrt{26.648} \approx 5$), i.e. 6±5

f. **5→6 with entry 1.3772**: A confirmed staff may go back to temporary appointment, this time around, as contract staff. On the average, this may happen once before the contract may be renewed. The estimate is subject to standard deviation of 2.

g. **4→4 with entry 1.3027:** This result suggest that staff do not enjoy more than two categories of leave at the same time and this estimate is associated with a standard deviation of $\sigma = \sqrt{0.5}$.

h. **4→5 with entry (4.9113):** This result suggests that staff who are on any type of the various categories of leave, on the average, return about five times to the system to undertake some assignments. For example, there are records of academic staff on leave of absence or sabbatical that come to the system to pursue their promotion documents. This estimate is associated with a standard deviation of 5. Impliedly, staff may have nil or ten of such transactions from leave.

i. **4→6 with entry of 1.4432**: staff who are newly recruited may benefit from government or institution's scholarship in which such beneficiary proceeds on training leave and returns to the system at the expiration of the leave. Upon such return, the staff remains unconfirmed staff until he or she is due. This appears to happen only once for any staff.

(ii) The total number of movement of staff within the non-absorbing states.

The matrix: $\tau = \begin{pmatrix} 6.0796 \\ 5.8509 \\ 6.0847 \end{pmatrix}$ reveal that generally all staff, irrespective of the starting state, undergo on the average 6 transition among the three non-absorbing states before being trapped in any of the absorbing states. However, these estimates are subject to a standard deviation of $\sqrt{\tau_2} \approx 5$.

(iii) Transition from non-absorbing to absorbing states.

Finally, 53% of newly employed staff, 51% of staff stock and 44% of those in various categories, leave the services of the institution through normal retirement. Moreover, 11% each of newly recruited staff and those on various categories of leave, exit the organization through wastage. Furthermore, about 36% of newly recruited, 43% of confirmed staff and 45% of those on various categories of leave do leave the services of the organization through suspension. It should be pointed out that although suspension cases take protracted period to resolve, few eventually come back after long administrative and legal struggle.

4 DISCUSSION

It was claimed at the outset that the method proposed in this study is normative and therefore prescribes standards required to steer the existing manpower policy towards the desired direction. An attempt to justify this claim is pursued in this section.

For various reasons, ranging from discovery of greener pastures, disciplinary cases and attrition, 47% of newly recruited staff left the organization prior to retirement. It is also observed that 49% of those who had been confirmed and 56% of those on various categories of leave do not reach retirement stage before leaving the system. These figures are significant and go to show that there is high rate of job mobility in the employment system. Several attributions can be made. These include: payment system not being attractive, staff not meeting opportunities for self actualization, low economic fortune, and the likes. It is also obvious form our computation that wastages appear reasonably low: 11% each of those who are newly recruited and 6% of confirmed staff and 11% of those on different leave categories leave by severance. Evidently this attrition rate is low by all standards. Our deduction therefore is that the organization has a liberal policy as far as wastage is concerned. On the other hand, there appears to be high rate of attrition through suspension cases in the sense that 36%, 43% and 45% of recruits, confirmed staff and those on leave respectively leave by suspension. We had earlier pointed out that few of these people are recalled, sometimes on compassionate grounds.

The overall picture:

(i) Less than four (4) in every ten(10) applications for study leave succeeded in respect of staff who are recruited and are yet to be confirmed;

(ii) Non academic staff undergo job rotation

(iii) Grant for sabbatical leave appears to be relatively marginal. There is also evidence that the organization is liberal with training leave.

Overall, the manpower policy appears liberal but firm.

5 CONCLUSION

Arising from the foregoing analysis and discussion, it is evident that the Markov Chain model applied has been able to provide useful insight into the behavior pattern for a long run manpower policy for the organization studied. The transition probability matrix, after several sets of transitions, shows the properties of stochastic regularities and ergodicity. The stabilized matrix (i.e. $(I-Q)^{-1}$) when multiplied with the original distribution, d_o, gave us a dependable probability matrix that provides reliable manpower forecast outcomes in line with the theory of Markov Chain. The results of this study appear useful for discerning the long run manpower policy for a university system.

6 REFERENCES

Anthony, T. F., Taylor, B. W. (1977). Analyzing the predictive capabilities of Markovian analysis for air pollution level variations. *J. Environ. Manage,* 2, 139-149.

Bartholomew, D. J., and Forbes, A. F. (1979). *Statistical techniques for manpower planning.* New York, Chichester: John Wiley.

Bhattacharya, R. N., *et al.* (1976). A Markovian stochastic basis for the transport of water through unsaturated soil. *J. Amer. Soc. Soil Science,* 40, 465-467.

Binkley, C. S. (1980). Is succession in hardwood forests a stationary Markov process? *Forest Science,* 26, 566-570.

Buongiorno, J. and Michie, B. R., (1980). A matrix model of uneven-aged forest management. *Forest Science,* 26, 609-625.

Calantone ,R. and Darmon, R. (1984). Salesforce Decisions: A Markovian Approach Journal of Academy of Marketing Science Vol. 12, No. 4, pp. 124- 144.

Cassell, R. F. and Moser, J. W., (1974). A programmed Markov model for predicting diameter distribution and species composition in uneven-aged forests. *Purdue Univ. Agric. Exp. Stn. Res. Bull.* 915.

Churchill, N. C. and Shank, J.K. (1975). Accounting for Affirmative Action Programmes: A stochastic flow approach. Actg Rev. Vol.50, pp. 643-656.

Cooke, D. (In press). The description of plant succession data using a Markov chain model of plant-by-plant replacement.

Debussche, M., Godron, M., Lepart, J. and Romane, F. (1977). An account of the use of a transition matrix. *Agro-Ecosystems,* 3, 81-92.

Essay, J.D., Marshall, A.W. and Proschan F. (1973). Shock models and wear processes. Ann. Probability, 1(14), 627-649.

Glaz, J. (1979). Probabilities and moments for absorption in finite homogeneous birth-death processes. *Biometrics,* 35, 813-816.

Gupta , R. D. and Kundu D. (1999). Generalized exponential distributions, *Australian and New Zealand* Journal of Statistics, 41, 173-188.

Gupta, R.D. and kundu D. (2001). Exponentiated exponential family; an alternative to Gamma and Weibull'', Biometrical Journal, 43, 117-130.

Harbaugh, J. W. and Bonham-Carter, G. (1970). *Computer simulation in geology.* London, Chichester. John Wiley & Sons.

Heyman. D.P. and Sobel, M.J. (1982). Stochastic Models in Operations Research, Vol. 1 McGraw- Hill.

Horn, H. S. (1974). The ecology of secondary succession. *Ann, Rev. Ecol. Syst.,* 5, 25-37.

Horn, H. S. (1975). Markovian properties of forest succession. In: *Ecology and evolution of communities* (edited by M. L. Cody and J. M. Diamond), pp. 126-211. Cambridge, Mass.: Harvard University Press.

Kim , D.S. and Smith, R.L. (1989). An Exact Aggregation Algorithm for a Special Class of Markov Chains. Technical Report 89-2, Department of Industrial and Operations Engineering University of Michigan.

Krumbein, W. C. (1967). FORTRAN IV computer programs for Markov chain experiments in geology. *Computer Contribution,* 13, Kansas Geological Survey.

Leeson G.W. (1979).Wastage in a hierarchical manpower system. *Journal of Operational Research Society, 30. 341-348.*

Lembersky, M. R. and Johnson, K. N. (1975). Optimal policies for managed stands: an infinite horizon Markov decision process approach. *Forest Science,* 21, 109-122.

Lloyd, E. H. (1977). Reservoirs with seasonally varying Markovian inflows and their first passage times. *International Institute of Applied Systems Analysis,* RR-77-4.

Merck J.W. (1970). A Markovian model for projecting movements of personnel through a system. In personnel management: A management Science

Approach (P. GREENLAW and R. SMITH, Eds). International Textbook, Scranton. P A. U.S.A.

Neilson G.L. and Young A.R. (1973). Manpower planning: A Manpower Chain Application. *Publ. pers. Mgmt, 133-143*.

Norris, J. M. (1971). Functional relationships in the interpretation of principal component analysis. *Area,* 3, 217-220.

Parthasarathy, S.and Vinoth R. (2009). Determination of expected time to recruitment in manpower planning, International conference on Operations Research applications in Engineering and management (ICOREM). Anna University Tiruchirappalli, Tamilnadu, India. May 27-29.

Peden, L. M., Williams, J. S. and Frayer, W. E. (1973). A Markov model for stand projection. *Forest Science,* 19, 303-314

Poornachandra Rao P. (1990). A Dynamic programming approach to determine optimal manpower recruitment policies, Journal of Operations Research Society, 41, 983-988.

Rao, C. R. and Kshirsagar, A. M. (1978). A semi-Markovian model for predator-prey interactions. *Biometrics,* 34, 611-620.

Rowland K. M. and Sovereign (1969). Markov-chain analysis of internal manpower supply. *Ind. Relat. 9, 88-99.*

Sathiyamoorthi, R. and Parthasarathy S. (2002). On the Expected time to recruitment in a two grade marketing organization-shock model approach, IAPQR Transactions, 27(1), 77-81

Slatyer, R. (1977), Dynamic Changes in Terrestial ecosystems: patterns of change, techniques for study and application to management Edited by Raph Slatyer and published by UNESCO in Collaboration with SCOPE in 1977 as MAB Technical Note 4. (http://www.icsu-Scope.org/download pubs/scope34(3/3/2011).

Smeach, S. C. and Jernigan, R. W. (1977). Further aspects of a Markovian sampling policy for water quality monitoring. *Biometrics,* 33, 41-46.

Stephens, G. R., and Waggoner, P. E. (1970). The forests anticipated from 40 years on natural transitions in mixed hardwoods. *Bull. Conn. Agric. Exp. Stn,* 77, 1.58.

Tavare, S. (1979). A note on finite homogeneous continuous-time Markov chains. *Biometrics,* 35, 831-834.

Usher, M. B. (1979). Markovian approaches to ecological succession. *Journal of Animal ecology,* 48, 413-426.

Usher, M. B. and Parr, T. W. (1977). Are there successional changes in arthropod decomposer communities? *Journal of Environmental Management,* 5, 151-160.

Uyar K.M. (1972). Markov Chain forecasts of employee replacement needs. *Ind. Relat. 11, 96-106.*

Vandeveer, L. R. and Drummond, H. E. (1978). The use of Markov processes in estimating land use change. *Tech. Bull. No.148.* Oklahoma: Agricultural Experimental Station.

Waggoner, P. E. and Stevens, G. R. (1970). Transitional probabilities for a forest. *Nature,* 225, 1160-1161.\

Williams, W. T., Lance, G. N., Tracey, J. G. and Connell, J. H. (1969). Studies in the numerical analysis of complex rain-forest communities. IV. A method for the elucidation of small scale forest pattern. *J. Ecol.* , 57, 635-654.

Zanakis S.H. and Maret M.W. (1980). A Markov Chain Application to Manpower Supply Planning. *The Journal of the Operational Research Society, vol. 31, No. 12, pp. 1095-1102.*