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Discrete Vortex Modelling of Turbulent Flow in Two Dimensions

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A.A. Adegbola[1] and O.O. Ajide[2]
[1]Department of Mechanical Engineering, Ladoke Akintola University of Technology, Ogbomoso, Nigeria.
[2]Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria.
Correspondence email: saint080yinkus@yahoo.com

Abstract: There has been growing need to characterize the turbulent flow through a simplified model. This paper reports the characterization of turbulent flow in two dimensions using discrete vortex modelling. The free surface flow in channel is considered as a case study. Appropriate flow charts and FORTRAN source codes were developed to solve the main governing equations of fluid flow. The Reynolds number which is the control parameter from 100,000 (experimental transition point) at an interval of 10,000 to 1,000,000 is used and the result is displayed using Microsoft Excel Graph. After the four stages of transition process due to Helmholtz instability, the turbulent region is reached, which is characterized by irregularity, low momentum diffusion, high momentum convection, and rapid variation of velocity. The result has shown that turbulent flow can be characterized with ease by using discrete vortex modelling.

Key words: Turbulent flow, discrete vortex modeling, free surface flow in channel, Helmholtz instability, irregularity, high momentum convection.

INTRODUCTION

The two different types of real fluid flow are laminar flow and turbulent flow. The laminar flow is a flow over a smooth surface with vorticity that appear highly ordered. Turbulent flow is a flow regime characterized by chaotic, disordered and stochastic property changes. Fluid flow that is slow and has inertial effect tends to be laminar. As it speeds up, a transition takes place and its crinkles up into complicated random turbulent flow. Turbulence is the time dependent chaotic behaviour seen in many fluid flows. Turbulence is a different type of fluid flow to laminar flow, hence it is desirable to be able to quantity under what conditions it occurs. The (dimensionless) Reynolds number gives quantitative indication of laminar to turbulent transition and characterizes whether the flow conditions lead to laminar or turbulent flow. Transition to turbulence can occur over a range of Reynolds numbers, depending on many factors such as level surface roughness, heat transfer, vibration, noise and other disturbances. For flow in a pipe, the characteristic length is the pipe diameter while for flow over a flat plate, the characteristic length is usually the length of the plate and the characteristic velocity is the free stream velocity.

Diffusion methods such as core-spreading techniques, hair pin-removal, particle strength, exchange, vorticity redistribution method and random walk method have been developed to characterize fluid flow. The random walk method was introduced by Chorin (1978) to study slightly viscous flow. Morchore and Pulvient (1982), Goodman (1987) and Long (1988) have shown that for flow in free-space, the random walk solution converges to that of the Navier-Stokes equations as the number of vorticies is increased. Gagnon and Mercader (1996) used the random walk method to compute the starting flow behind a two-dimensional step. A two-dimensional random vortex method is used by Gagnon (1993) to simulate the flow over a single back-facing step and a double symmetrical backward-facing step. Cheer (1989) has implemented the random walk method for flows over a cylinder. Lewis (1990) has used the random walk method for flow over airfoil cascades. Abdolhosseini (1996) studies the turbulent statistics in a uniformly sheared flow with a two-dimensional using random walk method. Chui (1993) used the random walk method to study thermal boundary layers. Adegbola and Salau (2011) has implemented the random walk method to characterize the fluid flow into laminar, transition and turbulent region.

The random walk method has several advantages. It requires simple algorithms and it can easily handle flows around complicated boundaries. The method also conserves the total circulation. The free surface flow in channel is used as a case study in this paper. The objective of the work is to develop a simplified model for turbulent flow through the use of discrete vortex method. The study intends to discover the unique features of the turbulent flow. Turbulence has been described as “the most important unsolved problem of classical physics, hence there has been growing need to develop a simplified model which can be used to predict the fluid flow that moves in an erratic or random motion (turbulent flow). This paper reports a “random walk model” for predicting turbulent flow in two dimensions through the use of “discrete vortex modelling”.

MODEL FORMULATION

The boundary layer flow can be approximated by placing at appropriate locations some vortices in a parallel flow. This forms the basis of the vortex element method. The principle involved is to subject the entire free vortex elements to small random displacements, which produce a scatter equivalent to the diffusion of vorticity in the continuum, which we are seeking to represent.

Turbulent flow is always three-dimensional. However, when the equations are time aver-aged, it can be treated as two-dimensional.

Diffusion of a point vortex in two-dimensional flow

The motion of a diffusing vortex of initial strength (**Γ**) centered on the origin of the (r,) plane is described by the diffusion equations.

$$\frac{d\omega}{dt} = v\, \bar{\nabla}^2\, \omega \qquad (1)$$

from which we may obtain the wall solution in space and time.

$$\omega(r,t) = \frac{\Gamma}{4\pi vt}\, e^{\left(-r^2/4vt\right)} \qquad (2)$$

Vorticity strength is a function of radius r and time t, where π is the ratio of the circumference of a circle to its diameter and v is the kinematic viscosity.

For a vortex of unit strength split in N elements, let us assume that n vortex elements are scattered into the small area r$\Delta\theta\Delta$r after time t. The total amount of vorticity in the element of area is:

$$P_v = \frac{n}{N} = \left[\frac{1}{4\pi vt} e^{\left(-r^2/4vt\right)}\right] r\Delta\theta\Delta r \qquad (3)$$

It is obvious from symmetry that scattering in the θ direction ought to be done with equal probability. Hence θ_i values may be defined independently of r_i values by the equations:

$$\theta_i = 2\pi Q_i \qquad (4)$$

Where θ_i is a random number within the range $0 < \theta_i < 1$. The probability P that on element will lie within a circle of radius r is given by the equation:

$$P = 1 - e^{\left(-r^2/4vt\right)} \qquad (5)$$

Hence for the vortex element (5) becomes

$$P_i = 1 - e^{\left(-r_1^2/4vt\right)} \qquad (6)$$

From which we may obtain it radial shift

$$r_i = \left\{4vt\, In\left(\frac{1}{1 - P_i}\right)\right\}^{1/2} \qquad (7)$$

Random Number Generation

Algorithms were developed to produce long sequences of apparently random results which are fact completely determined by a shorted initial value known as a seed.

Diffusion over a series of Time steps

The displacements of element i during time Δt for diffusion over a succession of small time increments is given by:

$$\Delta\theta_i = 2\pi Q_i \qquad (8)$$

$$\Delta r_i = \left\{4v\Delta t ln\left(\frac{1}{1 - P_i}\right)\right\}^{1/2} \qquad (9)$$

After the increment Δt, the new co-ordinate location (x_i', y_i') of the ith element will become:

$$x_1' = x_1 + \Delta r_i \cos\Delta\theta_i \quad (10)$$

$$y_1' = y_1 + \Delta r_i \sin\Delta\theta_i \quad (11)$$

where x_i and y_i are the old x –coordinate the ith element and old y - coordinate of the ith element respectively. The displacement of the ith element from the origin is given by the equation.

$$D_i = \sqrt{\left(x_1' - x_0\right)^2 + \left(y_1' - y_0\right)^2} \quad (12)$$

Where x_o and y_o are the origin.

Boundary layers by discrete vortex modelling

Convective motions were completely ignored for the diffusing point which we have just considered, an assumption which is permissible in view of symmetry in these special cases and justified for very low Reynolds number. Boundary layer flows on the other hand are more complex involving two additional features:

i. Externally imposed convention due to the main stream U, the significance of which is determined by the body scale Reynolds number (UL/v).
ii. Continuous creation of vorticity at the contact surface between fluid and wall, replacing the vorticity removed by diffusion and convection.

Vorticity Creation

The treatment of viscous boundaries in a vortex method is done by defining at the boundary, special sheets of vorticity which stay at the boundary and diffuse their vorticity to free elements.

Random walk method

Application of the random walk will result in the loss of half of the newly created vorticity due to diffusion across the wall and therefore out of the active flow domains. Since vorticity creation, diffusion and convection are being considered independently and in sequence in the computational scheme, vorticity must be considered. A single strength sheet is used by making sure that vortices which attempt to cross the wall are bounced back by assigning the value y_i = abs (y_i).

Selection of element size and time step

A reasonable approach to the selection of an appropriate time step Δt is to focus attention on the average displacements of the discrete vortices due to convection and diffusion.

The average convective displacement may be approximated by:

$$\delta_c = \frac{1}{2} U\Delta t \quad (13)$$

The average diffusive displacement is

$$\delta_D = \sqrt{(4v\Delta t\ In2)} \quad (14)$$

In order to maintain equal discretisation of the fluid motion due to convection and diffusion, we equate δc and δ_D resulting in the expression:

$$\Delta t = \frac{16 L In 2}{U Re} \quad (15)$$

Where Re = UL/v is the plate Reynolds number.

It is more reasonable to select the surface element size at twice δc leading to:

$$\Delta s = U\Delta t = \frac{16 L In 2}{Re} \quad (16)$$

The required number of surface elements for satisfactory discretisation of the plate is given by:

$$M = \frac{L}{\Delta s} = \frac{Re}{16\ In\ 2} \quad (17)$$

It is obvious that enforcing equal discretisation scales δ_o and δ_D for convection and diffusion will lead to computational difficulties at large Reynolds number. The large Reynolds number will impose severe pressure upon computational requirements. Hence practical computational limitation will rule out vortex modelling for typical engineering system Reynolds numbers if we attempt to impose the constraint $\delta_D = \delta_c$

Some considerations for high Reynolds numbers

The difficulties for high Reynolds numbers can be eased by selecting different time steps for diffusion (Δt_D) and convection (Δt_c). Since convection now dominates the flow, it is better to select the scale of convection displacements through:

$$K = \frac{\delta_c}{\Delta_s} \quad (18)$$

where previously K has been set to 0.5. The convective time step then follows from (13)

$$\Delta t_c = \frac{2K\Delta_s}{\mu} = \frac{2K}{M}\left(\frac{\iota}{\mu}\right) \quad (19)$$

The average random walk diffusive displacement over the same interval follows from (14), namely

$$\frac{\delta_D}{\Delta s} = \sqrt{\left(\frac{8\ MK\ In 2}{Re}\right)} \quad (20)$$

Although, it would be perfectly to perform both the convection and random walk processes over the same time step Δt_c, a saving in computational effort could be achieved by undertaking only one random walk for every N_t convection steps with:

$$\Delta t_D = N_t \Delta t_c \quad (21)$$

Calculation of Velocity Profile

The relationship between average displacements of the ith element from the origin to the number of time steps is given by:

$D_{av} \propto t_H$

$$\therefore \; D_{av} = Kt^{H} \qquad (22)$$

K and H are diffusion constant and Index respectively. From (23), taking the log of both sides

$$\log D_{av} = Hlogt + Log\ K \qquad (23)$$

Equation on a straight line is given as:

$$y = mx + c \qquad (24)$$

Comparing (23) and (24)

y = log D_{av}, c = log K, x = log t and m = N

Having developed the governing equations for the fluid flow using the Reynolds number as the tuning parameter, the algorithms is formulated for the model which is illustrated by the flow chart. The Reynolds number of 100,000 (Experimental transition points) at an interval of 10,000 to 1,000,000 is used. The flow chart is used in writing the FORTRAN program, the program is then run to generate the desired output. The result obtained was used to plot the graphs through the use of Microsoft Excel.

RESULTS AND DISCUSSION

Table 1 shows the Reynolds number, Time increment, number of time steps, number of elements or trials, log of average distance against log of time steps and index (velocity). The index is the slope obtained from the graph of log of average distance against log of time steps. There is fluctuation of index (velocity) from the transition region to turbulent region.

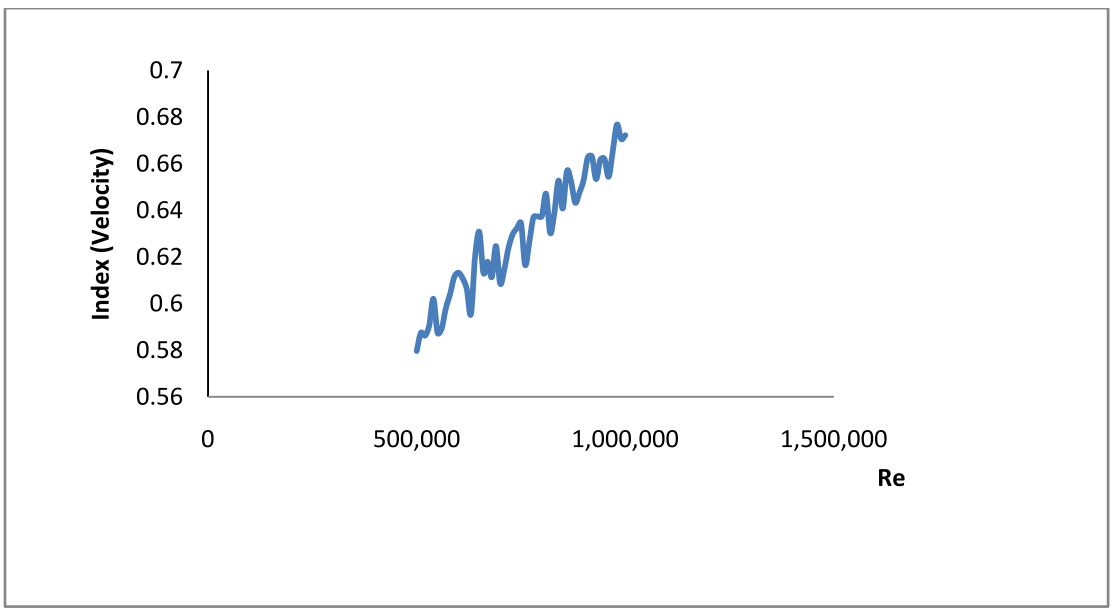


Figure 3: Depiction of Index (Velocity) against Reynolds Number for Turbulent Region

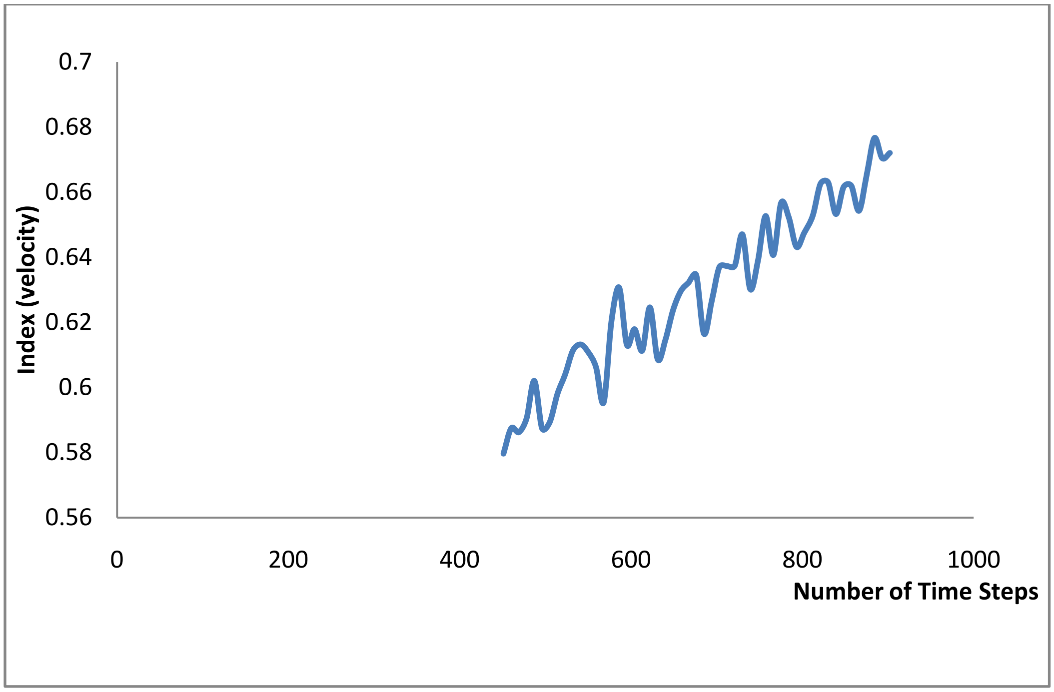


Figure 4: Depiction of Index (Velocity) against number of time steps for Turbulent Region

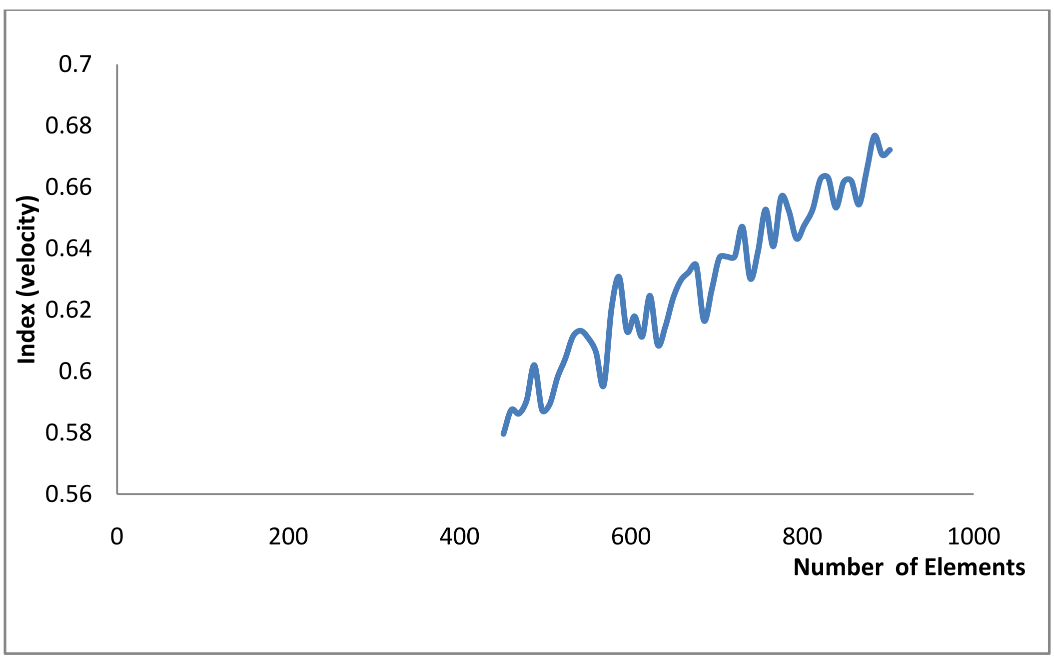


Figure 5: Depiction of Index (velocity) against Number of elements for Turbulent Region

The initial stage of the natural transition process (receptive phase) consist of the transformation of external disturbances in the outer free stream flow over the boundary layer into internal instability oscillations within the boundary layer from the Reynolds number of 100,000. The second stage of the process is the exponential growth of the few unstable disturbances from Reynolds number of 180,000. In the third stage from Reynolds number of 290,000, the amplitude of the disturbances now large enough to introduce non-linearlity effects. Due to the distortion of the boundary layer, inflexional mean profiles develop and a fourth stage is reached from Reynolds number of 420,000 where the boundary layer becomes unstable to high frequency disturbances. Finally, an explosive growth of these high frequency disturbances initiate the fifth and final phase from Reynolds number of 500,000 (the breakdown into turbulence). Hence it can be perceived how transition of a boundary layer from laminar to turbulent motion take place at very high Reynolds number, bearing in mind the Kelvin-Helmholtz instability. The graphs (fig 3, fig 4 and fig 5) show distinct region of turbulent flow which have the following features.

- ***Irregularity:*** Turbulent flow is highly irregular and chaotic.
- ***Diffusion and convection:*** Turbulent flow has low momentum diffusion and high momentum convection
- ***Unsteady and Non uniform:*** There is rapid variation of velocity in space and time.
- ***Energy Cascade:*** It is made up of super position of a spectrum of velocity fluctuation and eddies an over mean flow. Their hierarchy can be described by energy spectrum that measures the energy in velocity fluctuations of wave number.

CONCLUSION

The study has explored discrete vortex model to characterize the turbulent flow in two dimensions. The transition to turbulent motion takes place at high Reynolds number at the fifth stage due to Helmholtz instability. The turbulent region is characterized by irregularity, low momentum diffusion, high momentum convection and rapid variation of velocity.

REFERENCES

Abdolhoseini R. and Milane, R. (1996). Prediction of growth of turbulence in a uniformly sheared flow using the vortex-in-cell method, ESAIM Proc 1. 381-400.

Adegbola, A,A. and Salau, T.A.O. (2011). Characterization of fluid flow using Random Walk Dynamics, Accepted paper for publication in journal of Applied Science and Technology, University of Ibadan, Nigeria.

Cheer, A.Y. (1989). Unsteady Separated wake behind an impulsively started cylinder in slightly viscous fluid, J. Fluid Mech. 201: 485 – 505.

Chorin, A.J. (1978). Vortex sheet approximation of boundary layers, J. Comput. Phys. 27:428 – 442.

Chui W.K. (1993). A numerical model of two dimensional incompressible flow and heat transfer in a boundary layer, Ph.D. Thesis Tulare University.

Gagnon, Y. Giovanni, A Hebred, P. (1993) Numerical simulation and physical analysis of high Reynolds number recirculation flows behind sudden expansions. Phys. Fluid A 5(10) 2377-2389.

Gagnon, Y and Mercandier, Y. (1996). Numerical simulation of the starting flow down a step. ESAIM Proc 1. 377 – 386.

Goodman, J. (1987). Convergence of the random vortex method in two dimensions, J. Amer Math Soc. 804- 1779.

Long, D.G. (1988). Convergence of the random vortex method in dimensions, J. Amer, Math, Soc, 804-1,779.

Marchiora, C. and Pulvienti, M (1982). Hydrodynamics in two dimensions and vortex theory, comm... Math. Phys. 84:483-503.

Table 1: Parameters Estimation for Transition and Turbulent Regions

S/N	Reynolds number (Re)	Time Increment (Δt)	Number of time steps (N)	Number of elements or trials (M)	Log of average distance against log of time steps (y = mx + c)	Index (m)
1	100,000	0.01109	90	90	y = 0.4681x + 0.01918	0.4681
2	110,000	0.01008	99	99	y = 0.4542 x + 0.2166	0.4542
3	120,000	0.00924	108	108	y = 0.4906 x +0.1595	0.4906
4	130,000	0.00853	117	117	y =0.4423 x + 0.0.2401	0.4423

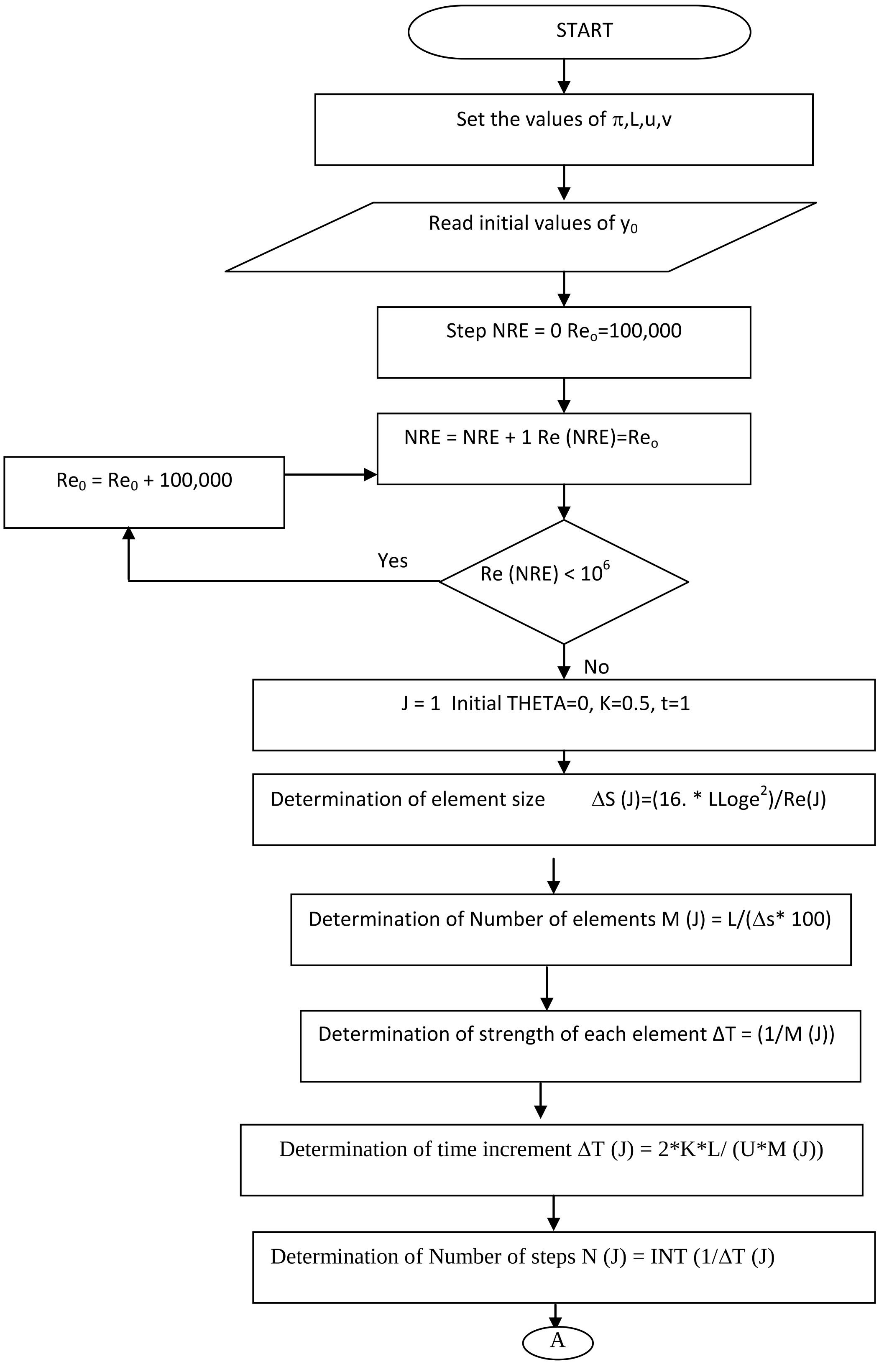
5.	140,000	0.00792	126	126	y = 0.4773 x +0.177	0.4773
6	150,000	0.00739	135	135	y = 0.4655 x + 0.182	0.4655
7	160,000	0.00693	144	144	y = 0.4821 x +0.1588	0.4821
8	170,000	0.00652	153	153	y =0.4755 x + 0.1779	0.4755
9	180,000	0.00616	162	162	y = 0.5054 x + 0.1389	0.5054
10	190,000	0.00584	171	171	y = 0.4808 x + 0.1572	0.4808
11	200,000	0.00555	180	180	y = 0.4925 x + 0.1122	0.4925
12	210,000	0.00528	189	189	y = 0.4936 x + 0.1423	0.4936
13	220,000	0.00504	198	198	y = 0.4993 x + 0.155	0.4993
14	230,000	0.00482	207	207	y = 0.5036 x + 0.1315	05036
15	240,000	0.00462	216	216	y = 0.5332 x + 1896	0.5332
16	250,000	0.00444	225	225	y = 0.5146 x + 0.125	0.5146
17	260,000	0.00427	234	234	y = 0.5306 x + 0.097	0.5306
18	270,000	0.00411	243	243	y = 0.5295 x + 0.1165	0.5295
19	280,000	0.00396	252	252	y = 0.551 x + 0.655	0.551
20	290,000	0.00382	261	261	y = 0.5348 x + 0.1018	0.5348
21	300,000	0.00370	271	271	y = 0.5275 x + 0.0882	0.5275
22	310,000	0.00358	280	280	y = 0.565 x + 0.051	0.565
23	320,000	0.00347	289	289	y = 0.5459 x + 0.0544	0.5459
24	330,000	0.00336	298	298	y = 0.5262 x + 0.189	0.5262
25	340,000	0.00326	307	307	y = 0.5642 x + 0.0405	0.5642
26	350,000	0.00317	316	316	y = 0.5384 x + 0.0661	0.5484

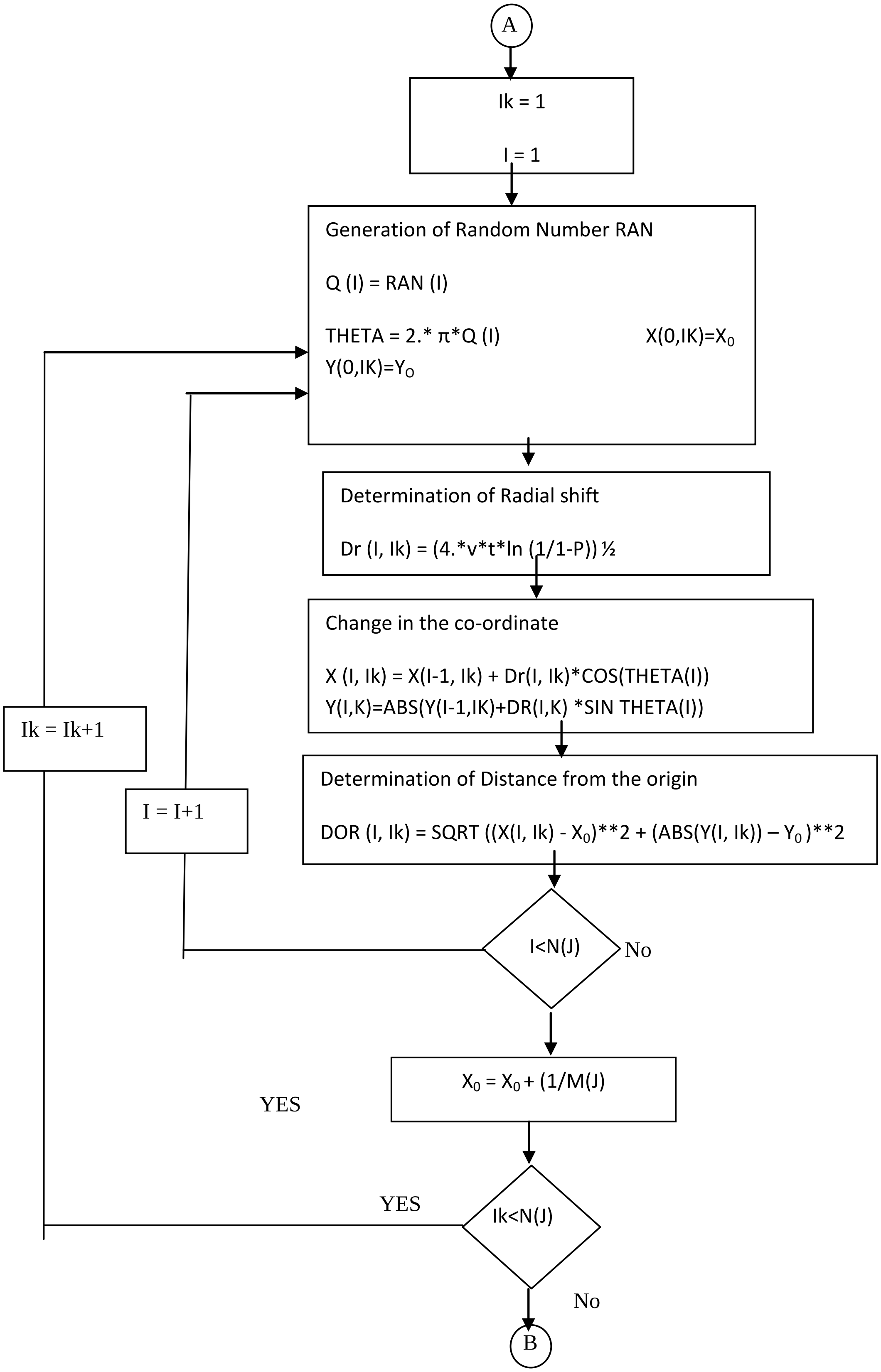
27	360,000	0.00308	325	325	y = 0.548 x + 0. 0661	0.548
28	370,000	0.00300	334	334	y = 0.5633 x +0.0507	0.5633
29	380,000	0.00292	343	343	y = 0.562 x + 0.0609	0.562
30	390,000	0.00284	352	352	y = 0.584 x + 0.0132	0.584
31	400,000	0.00277	361	361	y = 0.5674 x + 0.0685	0.5674
32	410,000	0.00270	370	370	y = 0.5549 x + 0.0514	0.5549
33	420,000	0.00264	379	379	y = 0.5632 x + 0.0438	0.5632
34	430,000	0.00258	388	388	y = 0.5628 x + 0.0454	0.5628
35	440,000	0.00252	397	397	y =0.5522 x + 0.0641	0.5522
36	450,000	0.00246	406	406	y = 0.5847 x + 0.0133	0.58
37	460,000	0.00241	415	415	y = 5696 x + 0.0332	0.5696
38	470,000	0.00236	424	424	y = 0.5663 x + 0.0354	0.5663
39	480,000	0.00231	433	433	y = 0.5797 x – 0.0231	0.5797
40	490,000	0.00226	441	441	y = 0.5927 x – 0.017	0.5927
41	500,000	0.00222	451	451	y = 0.5796 x + 0.0211	0.5796
42	510,000	0.00217	460	460	y = 0.5874 x + 0.0086	0.5874
43	520,000	0.00213	469	469	y = 0.5862 x – 0.0027	0.5862
44	530,000	0.00209	478	478	y = 0.5905 x + 0.0202	0.5905
45	540,000	0.00205	487	487	y = 0.602 x – 0.0287	0.602
46	550,000	0.00202	496	496	y = 0.5874 x + 0.0153	0.5874
47	560,000	0.00198	505	505	y = 0.5893 x + 0.0085	0.5893

48	570,000	0.00195	514	514	y = 0.598 x – 0.0155	0.598
49	580,000	0.00191	523	523	y = 0.604 x – 0.0226	0.604
50	590,000	0.00188	532	532	y = 0.6113 x – 0.0326	0.6113
51	600,000	0.00185	541	541	y = 0.6132 x – 0.0434	0.6132
52	610,000	0.00182	550	550	y = 0.6108 x – 0.0307	0.6108
53	620,000	0.00179	559	559	y = 0.6062 x – 0.0253	0.6062
54	630,000	0.00176	568	568	y = 0.5954 x – 0.007	0.5954
55	640,000	0.00173	577	577	0.6203 x – 0.073	0.6203
56	650,000	0.00171	586	586	y = 0.6307 x – 0.0713	0.6307
57	660,000	0.00168	595	595	y = 0.6131 x – 0.0494	0.6131
58	670,000	0.00166	604	604	y = 0.6179 x – 0.042	0.6179
59	680,000	0.00163	613	613	y = 0.6113 x – 0.0362	0.6113
60	690,000	0.00161	622	622	y = 0.6246 x – 0.0687	0.6246
61	700,000	0.00158	631	631	y = 0.6086 x – 0.0407	0.6086
62	710,000	0.00156	640	640	y = 0.6146x – 0.0582	0.6146
63	720,000	0.00154	649	649	y = 0.6238x – 0.0637	0.6238
64	730,000	0.00152	658	658	y = 0.6296 x – 0.0665	0.6296
65	740,000	0.00150	667	667	y = 0.6322 x – 0.0563	0.6322
66	750,000	0.00148	676	676	y = 0.6345 x – 0.0827	0.6345
67	760,000	0.00146	685	685	y = 0.6165 x – 0.0471	0.6165
68	770,000	0.00144	694	694	y = 0.6264 x – 0.0756	0.6264
69	780,000	0.00142	703	703	y = 0.637 x – 0.0898	0.637

70	790,000	0.00140	712	712	y = 0.6373 x – 0.0768	0.6373
71	800,000	0.00139	721	721	y = 0.6374 x – 0.0848	0.6374
72	810,000	0.00137	730	730	y = 0.647 x – 0.1019	0.647
73	820,000	0.00135	739	739	y = 0.6302 x – 0.0814	0.6302
74	830,000	0.00134	748	748	y = 0.6389 x – 0.1007	0.6389
75	840,000	0.00132	757	757	y = 0.6527 x – 0.1166	0.6527
76	850,000	0.00130	766	766	y = 0.6407 x – 0.1038	0.6407
77	860,000	0.00129	775	775	y = 0.6567 x – 0.1206	0.6567
78	870,000	0.00127	784	784	y = 0.6523 x – 0.1211	0.6523
79	880,000	0.00126	793	793	y = 0.6432 x – 0.1078	0.6432
80	890,000	0.00125	802	802	y = 0.6475 x – 0.1076	0.6475
81	900,000	0.00123	812	812	y = 0.6528 x – 0.1325	0.6528
82	910,000	0.00122	821	821	y = 0.6626 x – 0.466	0.6626
83	920,000	0.00121	830	830	y = 0.663 x – 0.1417	0.663
84	930,000	0.00119	839	839	y = 0.6533 x – 0.1233	0.6533
85	940,000	0.00118	848	848	y = 0.6616 x – 0.1317	0.6616
86	950,000	0.00117	857	857	y = 0.662 x – 0.384	0.662
87	960,000	0.00116	866	866	y = 0.6543x – 0.114	0.6543
88	970,000	0.00114	875	875	y = 0.6654 x – 0.1337	0.6654
89	980,000	0.00113	884	884	y = 0.6767 x – 0.0417	0.6767
90	990,000	0.00112	893	893	y = 0.6705 x – 0.1579	0.6705
91	1,000,000	0.00111	902	902	y = 0.6721 x – 0.1612	0.6721

APPENDIX A


START
Set the values of π,L,u,v
Read initial values of y0
Step NRE = 0 Reo=100,000
NRE = NRE + 1 Re (NRE)=Reo
Re0 = Re0 + 100,000
Yes
Re (NRE) < 10^6
No
J = 1 Initial THETA=0, K=0.5, t=1
Determination of element size ΔS (J)=(16. * LLoge^2)/Re(J)
Determination of Number of elements M (J) = L/(Δs* 100)
Determination of strength of each element ΔT = (1/M (J))
Determination of time increment ΔT (J) = 2*K*L/ (U*M (J))
Determination of Number of steps N (J) = INT (1/ΔT (J)
A


A
Ik = 1
I = 1
Generation of Random Number RAN
Q (I) = RAN (I)
THETA = 2.* π*Q (I)
X(0,IK)=X0
Y(0,IK)=YO
Determination of Radial shift
Dr (I, Ik) = (4.*v*t*ln (1/1-P)) ½
Change in the co-ordinate
X (I, Ik) = X(I-1, Ik) + Dr(I, Ik)*COS(THETA(I))
Y(I,K)=ABS(Y(I-1,IK)+DR(I,K) *SIN THETA(I))
Ik = Ik+1
I = I+1
Determination of Distance from the origin
DOR (I, Ik) = SQRT ((X(I, Ik) - X0)**2 + (ABS(Y(I, Ik)) – Y0)**2
I<N(J)
No
X0 = X0 + (1/M(J)
YES
YES
Ik<N(J)
No
B

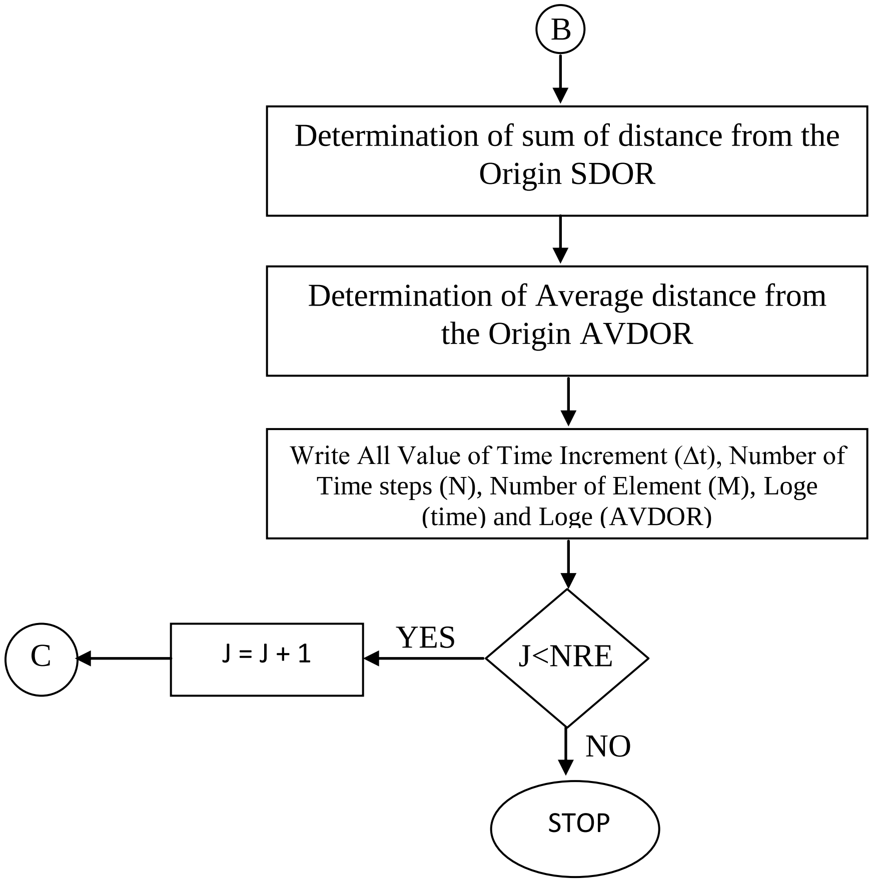


Figure1: Discrete Vortex Model Flow Chart