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Optimisation of concrete mix cost using Scheffe's simplex lattice theory

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C.E. Okere[1], D.O. Onwuka[2], S.U. Onwuka[3], J.I. Arimanwa,[4]
[1] Civil Engineering, Federal University of Technology, P.M.B. 1526, Owerri, Nigeria
E-mail address: okereau@yahoo.com

[2] Civil Engineering, Federal University of Technology, Owerri, Nigeria.
E-mail address: Daviesonwuka@yahoo.com

[3] Project Management Technology, Federal University of Technology, Owerri, Nigeria.
E-mail address: dasyldavis@yahoo.com

[4] Civil Engineering, Federal University of Technology, Owerri, Nigeria
E-mail address: joanarimanwa@yahoo.com

Abstract: In order to obtain concrete which will meet a number of performance criteria simultaneously at minimum cost, it is necessary to carry out the optimization of concrete mixture design. In this work, the cost of concrete mixtures is optimised using Scheffe's optimisation theory. The cost optimisation is based on current market prices of concrete components. In case of price fluctuations over time, the current prices of concrete component can be obtained by multiplying the base prices with a price fluctuation factor (PFF). The optimization model can be used to determine the cost of producing one cubic meter of concrete. Conversely, it can be used to obtain the concrete mix proportions that can be afforded by a specified amount of money. The model values were approximately equal to estimated values. The use of the optimisation model eliminated the arbitrary choice of concrete design mixes and its associated disadvantages. In addition, it yielded optimum concrete mixtures, which minimises costs and satisfies specific performance requirements.

Key words: Optimisation; concrete mixtures; cost; Scheffe's optimisation theory; model.

INTRODUCTION

The earliest method of mix design of concrete was historical records. As at then, there was no means to achieve an efficient optimized mixture for a given criterion (FHWA, 2007). This was followed by conventional methods of designing concrete mixtures which were based on laid down rules, design standard and codes of practice. These methods took care of the shortcoming of the historical methods. Despite all of these advantages, the methods cannot be used to achieve an efficiently optimized mixture for a given criterion. Also, they require trial mixes (Simon, 2003).

Thus, in 1958 Scheffe developed the simplex design method, which uses the theory of statistics and experiments to obtain models that can be used to determine mix proportions for a specified criterion (Scheffe, 1958). The advantages of this method are that it eliminates the need for trial mixes, saves time and cost, and can be used with computers to determine concrete mixes for a specified criterion (i.e. property or cost).

Some researchers have applied Scheffe's simplex design method of optimisation and they came up with interesting results. Ezeh, et.al.(2010) optimized the compressive strength of laterite/sand hollow block using Scheffe's simplex method. In their work, a mathematical model was developed and used to optimize the mix proportion that will produce the maximum compressive strength of laterite/sand hollow blocks. The model predicts the compressive strength of the blocks when the mix ratios are known and vice versa. Mama and Osadebe (2011) formulated models for prediction of compressive strength of sandcrete blocks using Scheffe's and Osadebe's optimization theories. The results of the predictions were comparatively analysed and it was found that the two models are acceptable. Orie(2008) developed models for optimisation of compressive and flexural strength of mound soil concrete using Scheffe's method. Okere et. al.(2013) worked on concrete mixture design and generated a model for optimisation of concrete cube strength using Scheffe's optimisation theory. Statistical tools which were used to test the adequacy of the model agreed to the acceptance of same. Obam (1998) developed a model for optimisation of strength of palm kernel shell aggregate concrete using Scheffe's simplex theory.

In this work, cost of concrete mixture is optimized using Scheffe's method of optimisation. Scheffe considered experiment with mixtures in which the desired property depends on the proportions of the constituent materials present as atoms of the mixture. A simplex lattice which can be described as a structural representation of lines joining the atoms of a mixture can be used as a mathematical space in model experiments involving mixtures by considering the atoms as the constituent components of the mixture. For instance in normal concrete mixture, the constituent elements are water, cement, fine and coarse aggregates and so normal concrete mixture gives a simplex of four components. Hence the simplex lattice of this four- component mixture is a three-dimensional solid equilateral tetrahedron. Mixture components are subject to the constraint that the sum of all the components must be equal to one (Scheffe 1958). In order words: $\sum X_i = 1$, where q is the number of components of a mixture and i ranges from 1 to q. X_i is the proportion of the ith component in the mixture. Simplex lattice designs are characterised by a well chosen polynomial equation to represent the response surface over the entire simplex region. The response is the property of mixture sought and in this case, the cost of concrete mixture. The processed equation is given as 'equation (1)'.

METHODOLOGY

The cement used for the analysis was eagle cement brand of Ordinary Portland Cement conforming to BS 12 (1978). The fine aggregate used was river sand free from deleterious matters such as dirts, clay and organic matters. The fine aggregate falls into zone 3 of the grading curve. The coarse aggregate was normal weight, crushed irregular shaped coarse aggregate with a maximum size of 20mm. Both the fine and coarse aggregate were hard and durable, and conform to the specifications of BS 882 (1992).

Portable drinking water was used for the analysis. Here, optimization method is used in formulating a mathematical model for predicting the cost per cubic metre of concrete. The model is based on Henry Scheffe's Simplex Lattice Theory, using a quadratic polynomial and a (4,2) simplex lattice.

Formulation of cost model

- Scheffe's theory

The polynomial equation is given by Scheffe (1958)

$$Y = \alpha_1 \mathcal{X}_1 + \alpha_2 \mathcal{X}_2 + \alpha_3 \mathcal{X}_3 + \alpha_4 \mathcal{X}_4 + \alpha_{12} \mathcal{X}_1 \mathcal{X}_2 + \alpha_{13} \mathcal{X}_1 \mathcal{X}_3 + \alpha_{14} \mathcal{X}_1 \mathcal{X}_4 + \alpha_{23} \mathcal{X}_2 \mathcal{X}_3 + \alpha_{24} \mathcal{X}_2 \mathcal{X}_4 + \alpha_{34} \mathcal{X}_3 \mathcal{X}_4 \quad (1)$$

Where α_i and α_{ij} are coefficients and $\mathcal{X}_i$ are the pseudo components of mixture.

The pseudo and actual components were determined and presented in (Table 1)(Okere, 2006). The actual components Z determined were converted to actual mix ratios in kg for $1m^3$ of concrete as shown in Table 2 together with their corresponding concrete cube strength as investigated by Okere et.al., (2013) and Onwuka et.al. (2011) .

Cost analysis

The current market prices of concrete constituent materials were obtained and presented in Table 3. They were used to determine the total cost of producing one cubic meter ($1m^3$) of concrete in naira for the different mix ratios and presented in Table 4. In case of price fluctuations over time, the current prices of concrete components can be obtained by the base prices with a price fluctuation factor (PFF) given in Table 5.

RESULTS AND ANALYSIS

The result of the cost analysis for one cubic meter of concrete is presented in Table 4.

Determination of the coefficients of optimization model

Using Table 3 and equations for coefficients (equation 2), the coefficients of the second degree polynomial was determined as follows:

$$\alpha_i = y_i \ \ and$$

$$\alpha_{ij} = 4y_{ij} - 2y_i - 2y_j \qquad (2)$$

$$\alpha_1 = 20{,}532.39, \qquad \alpha_2 = 18{,}560,$$

$$\alpha_3 = 17{,}957.34 \ and \ \alpha_4 = 22{,}261.55$$

$$\alpha_{12} = 4(19{,}408.25) - 2(20{,}532.39) - 2(18{,}560) = -551.78$$

$$\alpha_{13} = 4(19{,}068.18) - 2(20{,}532.39) - 2(18{,}560) = -1912.06$$

$$\alpha_{14} = 4(21{,}339.80) - 2(20{,}532.39) - 2(18{,}560) = 7174.42$$

$$\alpha_{23} = 4(18{,}259.52) - 2(18{,}560) - 2(17{,}957.34) = 3.4$$

$$\alpha_{24} = 4(20{,}032.77) - 2(18{,}560) - 2(22{,}261.55) = -1512.02$$

$$\alpha_{34} = 4(19{,}674.97) - 2(17{,}957.34) - 2(22{,}261.55) = -1737.9$$

Substituting the values of these coefficients into 'equation (1)' yields:

$$Y = 20{,}532.39\mathcal{X}_1 + 18{,}560\mathcal{X}_2 + 17{,}957.34\mathcal{X}_3 + 22{,}261.55\mathcal{X}_4 - 551.78\mathcal{X}_1\mathcal{X}_2 - 1912.06\mathcal{X}_1\mathcal{X}_3 + 7174.42\mathcal{X}_1\mathcal{X}_4 - 3.4\mathcal{X}_2\mathcal{X}_3 - 1512.02\mathcal{X}_2\mathcal{X}_4 - 1737.9\mathcal{X}_3\mathcal{X}_4 \qquad (3)$$

'Equation (3)' is the Scheffe's mathematical model for cost per cubic meter of concrete.

Test of the adequacy of the model

The model equation was tested to see if the models agree with the actual experimental results. Let the statistical Null Hypothesis be denoted by H_0: and the alternative by H_1

H_0: There is no significant difference between the analytical and the predicted results.

H_1: There is a significant difference between the analytical and predicted results.

The fisher test statistic was used to test the adequacy of the model (Ogoamaka, 2004). The predicted values ($Y_{predicted}$) for the test control points were obtained by substituting the values of X_1 into the model equation ie 'equation (3)'. These values were compared with the analytical result ($Y_{observed}$) given in (Table 4).

- **Fisher Test**

The mean (y) of the response (Y) is given by:

$$y = \frac{\sum Y}{n} \qquad (4)$$

Where n is the number of responses.

Using variance,

$$S^2 = \left[\frac{1}{n-1}\right][\sum(Y-y)^2] \quad and$$

$$y = \frac{\sum Y}{n} \quad for\ 1 \leq i \leq n \qquad (5)$$

Therefore from (Table 6),

$$S^2_{(obs)} = \frac{4617967.37}{9} = 513{,}107.485 \quad and$$

$$S^2_{(pre)} = \frac{4984957.513}{9} = 553{,}884.168$$

But the fisher test statistics is given by

$$F = \frac{S_1^2}{S_2^2} \qquad (6)$$

Where S_1^2 is the larger variance

Hence

$S_1^2 = 553{,}884.164\ and$

$S_2^2 = 513{,}107.485$

Therefore, $F = \frac{553{,}884.168}{513{,}107.485} = 1.08$

From standard Fisher Table, $F_{0.95}\,(9{,}9) = 3.18$. This is higher than the calculated value. We accept the Null hypothesis. Hence the model equation is adequate.

Comparison of results

The results obtained from the model were compared with those obtained from the analysis as presented in (Table 7).

A comparison of the predicted results with the analytical results shows that the percentage difference ranges from a minimum of 0.1% to a maximum of 1.95%, which is insignificant.

CONCLUSION

(1) Scheffe's optimisation theory was used to formulate mathematical model for optimisation of cost per cubic meter of concrete.

(2) The cost of concrete mix is a function of the proportions of the ingredients (cement, water, sand and coarse aggregate) of the concrete.

(3) The fisher test used in the statistical hypothesis showed that the model developed is adequate.

(4) The cheapest cost obtainable with the model is 17,957.34 Naira with a corresponding mix ratio of 1:3:5.5 at 0.45 water/cement ratio.

(5) Since the maximum percentage difference between the analytical cost and the predicted cost is insignificant (i.e. 1.95), the optimisation model will yield accurate values of concrete mix cost if given the mix proportions and vice versa.

Table 1: Pseudo Components with their corresponding Actual Component Values

n	x_1	x_2	x_3	x_4	*Response*	z_1	z_2	z_3	z_4
1	1	0	0	0	Y_1	0.549	1	2	4
2	0	1	0	0	Y_2	0.501	1	2.5	6
3	0	0	1	0	Y_3	0.45	1	3	5.5
4	0	0	0	1	Y_4	0.6	1	1.5	3.5
5	0.5	0.5	0	0	Y_{12}	0.525	1	2.25	5
6	0.5	0	0.5	0	Y_{13}	0.449	1	2.5	4.75
7	0.5	0	0	0.5	Y_{14}	0.575	1	1.75	3.75
8	0	0.5	0.5	0	Y_{23}	0.475	1	2.75	5.75
9	0	0.5	0	0.5	Y_{24}	0.551	1	2	4.75
10	0	0	0.5	0.5	Y_{34}	0.525	1	2.25	4.5
Control points within the factor space									
11	0.5	0.25	0.25	0	C_1	0.5125	1	2.375	4.875
12	0.25	0.25	0.25	0.25	C_2	0.525	1	2.25	4.75
13	0	0.25	0.25	0.5	C_3	0.5375	1	2.125	4.625
14	0	0.25	0	0.75	C_4	0.575	1	1.75	4.125
15	0.75	0	0.25	0	C_5	0.525	1	2.25	4.375
16	0	0.5	0.25	0.25	C_6	0.5125	1	2.375	5.25
17	0.25	0	0.5	0.25	C_7	0.5125	1	2.375	4.625
18	0.75	0.25	0	0	C_8	0.5375	1	2.125	4.5
19	0	0.75	0.25	0	C_9	0.4875	1	2.625	5.875
20	0	0.4	0.4	0.2	C_{10}	0.5	1	2.5	5.3

Table 2: Mix ratios in kg for 1m^3 of concrete

S/N	Water (kg)	Cement (kg)	Fine Aggregates (kg)	Coarse Aggregates (Kg)	Concrete cube Strength (N/mm^2)
1	174.83	317.88	635.76	1271.52	26.22
2	120	240	600	1440	30.22
3	108.54	241.2	723.6	1326.6	24
4	218.2	363.6	545.4	1272.6	27.55
5	143.59	273.5	615.37	1367.5	28.89
6	137.14	274.28	685.7	1302.83	24.44
7	195.05	339.22	593.64	1272.08	21.77
8	114.28	240.6	661.65	1383.45	31.11
9	159.04	289.16	578.32	1373.51	22.44
10	152.27	290.03	625.57	1305.14	26.00
Control Points					
11	140.37	273.89	650.49	1335.21	25.81
12	147.8	281.52	633.42	1337.22	26.39
13	155.65	289.59	615.38	1339.35	26.98
14	185.24	322.15	563.76	1328.87	23.32
15	154.6	294.48	622.58	1288.35	28.24
16	134.61	262.65	623.79	1375.9	27.75
17	144.49	281.94	669.61	1303.97	23.69
18	158.04	294.03	624.81	1323.14	24.02

19	117.15	240.3	630.79	1411.76	25.96
20	129.03	258.06	645.15	1367.72	26.57

Table 3: Current market prices of concrete components

Components of concrete	Unit cost (Naira per kg)
Cement	36
Fine Aggregate	0.833
Coarse Aggregate	6.25
Water	3.50

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Table 4: Cost estimate (in Naira) for 1m^3 of concrete of the different mix ratios

S/N	Water	Cement	Fine Aggregate	Coarse Aggregate	Response Symbol	Total cost (response) Naira
1	611.91	11,443.68	529.80	7,947	Y_1	20,532.39
2	420	8,640	500	9,000	Y_2	18,560
3	379.89	8,683.2	603	8,291.25	Y_3	17,957.34
4	763.7	13,089.6	454.5	7,953.75	Y_4	22,261.55
5	502.56	9,846	512.81	8,546.88	Y_{12}	19,408.25
6	479.99	9,874.08	571.42	8,142.69	Y_{13}	19,068.18
7	682.68	12,211.92	494.7	7,950.5	Y_{14}	21,339.80
8	399.98	8,661.6	551.38	8,646.56	Y_{23}	18,259.52
9	556.64	10,409.76	481.93	8,584.44	Y_{24}	20,032.77

10	532.95	10,441.08	543.81	8,157.13	Y_{34}	19,674.97
Control Points						
11	491.30	9,860.04	542.08	8,345.06	C_1	19,238.48
12	517.3	10,134.72	527.85	8,357.63	C_2	19,537.5
13	544.78	10,425.29	512.82	8,370.94	C_3	19,853.78
14	648.34	11,597.4	469.8	8,305.44	C_4	21,020.98
15	541.1	10,601.28	552.15	8,052.19	C_5	19,746.72
16	471.13	9,455.4	519.83	8,599.38	C_6	19,045.74
17	505.72	10,149.84	558.01	8,149.81	C_7	19,363.38
18	553.14	10,585.08	520.68	8,269.63	C_8	19,928.53
19	410.03	8650.8	525.66	8,823.50	C_9	18,409.99
20	451.61	9,290.16	537.63	8,548.25	C_{10}	18,827.65

Table 5: Optimal values of PFF for years 1-10

Number of years	PFF
Year 1	1.2326
Year 2	1.4638
Year 4	1.9300
Year 6	2.3939
Year 8	3.3260
Year 10	3.3260

Source: Nworu, G.E. and Unaeze, G.O.. (1977)

Table 6: F-Statistics for the controlled points

Response Symbol	$Y_{(observed)}$	$Y_{(predicted)}$	$Y_{(obs)}-y_{(obs)}$	$Y_{(pre)}-y_{(pre)}$	$Y_{(obs)}-y_{(obs)}^2$	$(Y_{(pre)}-y_{(pre)})^2$
C_1	19,238.48	19,087.76	-258.795	-441.878	66974.852	195256.1669
C_2	19,537.50	19,919.32	40.225	389.682	1618.0506	151852.0611
C_3	19,853.78	19,854.08	356.505	324.442	127095.815	105262.6114
C_4	21,020.98	21,052.66	1523.705	1523.022	2321676.93	2319596.012
C_5	19,746.72	19,530.12	249.445	0.482	62222.808	0.2323
C_6	19,045.74	19,037.53	-451.535	-492.108	203883.856	242170.284
C_7	19,363.38	19,669.31	-133.895	139.672	17927.8710	19508.268
C_8	19,928.53	19,935.83	431.255	406.192	185980.875	164991.941
C_9	18,409.99	18,409.97	-1087.285	1119.668	1182188.67	1253656.43
C_{10}	18,827.65	18,799.80	-669.625	-729.838	448397.641	532663.506
Sum	194972.75	195296.38			4617967.37	4984957.513
Mean	$y_{(obs)}=$ 19,497.27	$y_{(pre)}=$ 19,529.64				

Table 7: Comparison of Predicted Costs with Analytical Costs

S/N	Analytical Cost (in Naira)	Predicted Cost (in Naira)	Percentage Difference
1	19,238.48	19,087.76	0.78
2	19537.50	19,919.32	1.95
3	19853.78	19,854.08	0.00

4	21,020.98	21,052.66	0.15
5	19,746.72	19,530.12	1.1
6	19,045.74	19,037.53	0.043
7	19,363.38	19,669.31	1.58
8	19,928.53	19,935.83	0.04
9	18.409.99	18,409.97	0.00
10	18,827.65	18,799.80	0.15

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