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Small and Large Deflections in Slabs and Cantilevers Using Isogeometric and Finite Element Analysis Approach: Implications for Buildings under Erosion Effects

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Umeononankume P. E[1*]. and Ezeagu C.A[2]
[1,2]Department of Civil Engineering, Nnamdi Azikiwe University Awka, Nigeria
[*]*Corresponding author E-mail*: paschalnkume@gmail.com

Abstract: *In this study, the deflection behaviour of structural elements was reviewed in detail in order to ascertain the possible failure behaviour of structures in erosion prone regions of Anambra State. The study was approached using isogeometric and finite element analysis using MATLAB and C++ programming language. The study focused more on deflection sensitive structural components such as plates and cantilevers. Using isogeometric analysis (IGA), the small deflection of a 6m x 6m clamped plate subjected to a serviceability pressure load was found to be 0.0627% more than the exact solution. IGA gave better result than finite element analysis carried out using StaadPro software (matrix size: 20 x 20) which was 1.1173% higher than the exact solution. Using Mindlin plate theory (incorporating shear deformation), IGA results were 0.1856% higher than finite element analysis result. On considering the large deformation of the square plate element at serviceability limit state at 100 iteration steps, the results show that the maximum deflection was 1.613mm, exactly the same with small deflection theory. At ultimate limit state (failure load), the result from large deflection analysis increased to 2.375mm (32.084% increase) at 100 iteration steps. However, a little consideration will show that the deflection at ultimate load was still small, since it was less than* $\frac{thicknes\ s\ of\ plate}{5}$ *(150/5 = 30mm). Also, considering the behaviour of cantilevers, the same behaviour was observed, the displacement from large deflection theory at ultimate load was observed to be within the small deflection limit (3.115mm). Therefore, we can conclude that the use of small deflection theory is sufficient for the analysis of buildings in erosion prone regions. From the study also, geometric non-linearity cannot likely be the major cause of failure of buildings in such areas but issues like loss of equilibrium, mass wasting, and loss of bearing capacity of foundations can be seen to be more prominent in causing failure of buildings at erosion sites.*

Keywords: Plates, Isogeometric analysis, Finite Element Analysis, Deflections, Erosion

1.0 INTRODUCTION

One of the basic necessities of life is shelter, and civil/structural engineers have always been determined to find many ways of solving complex problems they encounter in practice, in order to provide adequate and safe housing and infrastructure for mankind. Since the development of classical beam theories in the 19th century (Truesdell, 1960), the processes and ways at which complex structures are analysed have improved in a very encouraging way. Structural members such as beams and plates deform under the action of externally applied loads, and engineers often ensure that such deflections are controlled so that the aesthetics and functionality of such buildings are not compromised. This is generally identified as a serviceability requirement in the design of structures using limit state theory.

Soil erosion generally is caused by several factors working simultaneously or individually to detach, transport, and deposit soil particles in a different place other than where they were formed (Igwe, 2012). As a result of this, the foundations of buildings in erosion prone zones may be scoured away, and loss of bearing capacity will ensue. More often than not, not all parts of a foundation will be affected, and differential movement of building supports may become the case. Large magnitude of internal stresses is induced in statically indeterminate structures when subjected to differential support movement. As a result of this, large deformation may occur in some structural members, before final failure will occur. Large deformation of building structural members usually occurs at the non-linear stage. It is this phenomenon that this project explores, using isogeometric analysis (IGA) and finite element analysis to analyse the common structural members in a residential building (slabs, beams) subjected to small and large deformation due to external actions.

When beams and plates are deflected beyond a certain magnitude, the linear theory loses its validity and produces incorrect results. Linear theory can predict that the deflection of the member may exceed the length of the member, which is unrealistic. In order for an accurate large deflection solution, one needs to include the coupling between axial and transverse motion, which is geometric nonlinearity (Nishawala, 2011). If the edges are allowed to move freely within the plane of the undeformed member, this boundary condition is called ‘stress-free’. If the edges are restricted from moving, the edges require an equivalent axial load to prevent motion, which is called ‘immovable’ boundary conditions. There are several sources of nonlinear behaviour. One source is geometric nonlinearity. This characteristic is important to systems with large deformations, or systems that may fail due to buckling. In beams and plates, the nonlinearity is from the nonlinear strain equations, where the transverse displacement is coupled to the axial strains (Nishawala, 2011). As a result, mid-plane stretching of the beam or plate may occur. The von Karman, or large deformation, theory of plates uses geometric nonlinearity in its derivation.

1.1 Isogeometric Analysis

Isogeometric Analysis (IGA) is a computational approach that integrates Finite Element Analysis (FEA) and Computer Aided Design (CAD). It was introduced by Hughes and co-workers to bridge the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA) (Hughes et al 2005). Isogeometric Analysis is developed in the purpose of utilizing the same data set in both design and analysis (Raknes, 2011). In today’s CAD and FEA packages one have to convert the data generated in design to a data set suitable for FEA. Converting the data is not trivial, as the computational geometric approach is different in CAD and FEA. IGA makes it possible to utilize the NURBS geometry, which is the most used basis in CAD packages, in FEA directly. Isogeometric analysis is thus a great tool for optimizing models, as one easily can make refinements and perform testing and analysis during design and development. The core idea of the method is to use the same basis functions for the representation of geometry in CAD and the approximation of solutions fields in FEA. This strategy bypasses the mesh generation process required for standard FEA and supports a tightly connected interaction between CAD and FEA tools which could potentially reduce the time required for the analysis of complex engineering designs by up to 80%. In addition, it has been shown that the use of a smooth, higher-order geometric basis is superior to standard discretizations (Evans et al, 2009). This has been demonstrated for a variety of application areas such as structural vibrations, incompressibility, shells, fluid-structure interaction, turbulence, phase fields, contact, fracture, and optimization.

According to Raknes (2011), isoparametric analysis involves using the same basis functions to represent design as well as to perform analysis, whereas isogeometric analysis also implies letting the geometry be the deciding factor on exactly what kind of basis functions to use. The main idea in isogeometric analysis is to use Non Uniform Rational B-Splines (NURBS) as basis functions in both design and in the finite element method (Raknes, 2011). NURBS are, as the name indicate, built from B-splines.

According to Kiendl (2011), B-Spline curves are defined by a linear combination of control points and basis functions over a parametric space. The basis functions are called B-Splines (short for Basis-Splines). The parametric space is divided into intervals and the B-Splines are defined piecewise on these intervals, with certain continuity requirements between the intervals. Since the number of intervals is arbitrary, the polynomial degree can be chosen independently of the number of control points.

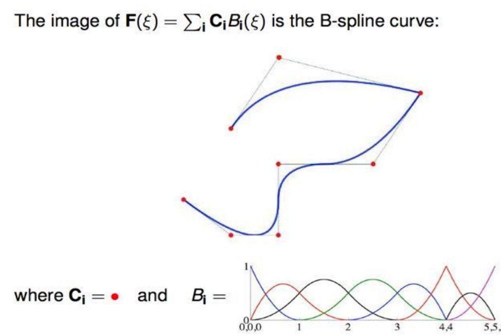


Figure 1: Example and nature of B-spline curves (Lovadina et al, 2014)

The parametric space is defined by the so called knot vector;

$$\Xi = [\xi_1, \xi_2, \ldots, \xi_{n+p+1}] \quad (1)$$

It is a set of parametric coordinates ξ_i in non-descending order which divide the parametric space into sections. If all knots are equally spaced, the knot vector is called uniform. A B-Spline basis function is C^∞ continuous inside a knot span, i.e. between two distinct knots, and C^{p-1} continuous at a single knot. A knot value can appear more than once and is then called a multiple knot. At a knot of multiplicity k the continuity is C^{p-k}, i.e. by increasing the multiplicity of a knot the continuity can be decreased. If the first and the last knot have the multiplicity p + 1, the knot vector is called open, clamped, or non-periodic.

The B-spline basis functions are defined recursively by (Hughes, 2005);

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \quad (2)$$

For *p = 1,2,3,...*

For p = 0, we have that;

$$N_{i,0}(\xi) = \begin{cases} 1 & if\ \xi_i \le \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

If the denominators in the factor we multiply the basis functions by are zero we define the factor to be zero. That is;

$$\frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} \equiv 0 \text{ if } \xi_{i+p} - \xi_i = 0, \quad (4)$$

$$\frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} \equiv 0 \text{ if } \xi_{i+p+1} - \xi_{i+1} = 0 \quad (5)$$

This formula is also known as the *Cox-de Boor recursion formula* (Hughes, 2005). Dynamic programming is recommended to improve the running time of this recursively formula. Else wise the same values will be calculated several times. We have *n* basis functions, with Ni,p being the i^{th} basis function of order *p*, $i \in [1, n]$. The number of basis functions is determined by the order and the number of knots; we have $n + p + 1$ knots resulting in *n* basis functions. Increasing the number of knots will consequently also increase the number of basis function. It is worth noticing that the basis functions are non-negative and that they form a partition of unity, i.e.

$$N_{i,p}(\xi) \ge 0\ \forall \xi,$$

$$\sum_{i=1}^{n} Ni,p(\xi) = 1.0 \quad (6)$$

Two other properties of the basis functions are that they have local support and local knots. The properties are stated in Lemma 2.6 in (Lyche and Morken, 2008) and involve the following; Assume that we have the knot vector $\Xi = [\xi_1, \xi_2, \ldots, \xi_{n+p+1}]$. Then $N_i^p(\xi) = 0$ if ξ is outside the interval $[\xi_i, \xi_{n+p+1}]$. Thus, the i^{th} B-spline $N_i^p(\xi)$ depends only on the knots $[\xi_i, \xi_{n+p+1}]$.

Table 1: Comparison between IGA and FEA (Raknes, 2011)

Isogeometric Analysis	Finite Element Analysis
Control Points	Nodal Points
Control Variables	Nodal Variables
Knots	Mesh
Exact geometry	Approximated geometry
NURBS basis functions	Lagrange basis functions
Basis not interpolating control points	Basis interpolating nodes
Patches	Subdomains

The comparison between the classical Finite Element Analysis (FEA) and Isogeometric Analysis (IGA) is shown in Table 1. This is what this project explores; employing the classical method of FEA, and comparing the results obtained using IGA. This can give us a good idea on the efficiency of the two methods based on accuracy, and also comparison of small and large deflection results.

1.2 Aims

The aim of the paper is to access the impact of erosion on building in the vicinity of erosion prone area with the centre pointed on its large deformation using Isogeomentric analysis tools

1.3 Objective

1. To examine the effect of erosion on buildings in a erosion prone area (Ekwulobia and Oko, Community)

2. To investigate the effects of structural deformation due to erosion with study area.
3. to reveal Isogeomentric analysis in its simplest form an a r4cent approach compared with the already existing finite element and classical method through the analysis of the common structure members (i.e) slab, beam and cantilever using Isogeomentric analysis with computer program written in MATLAB which will be developed for each element.

1.4 Scope of Work

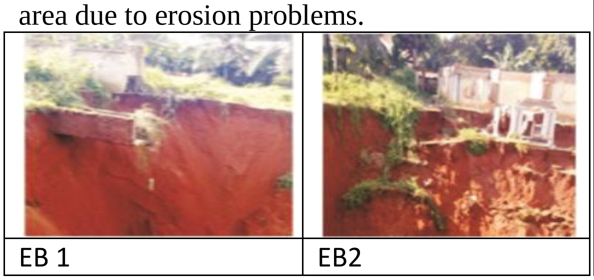
This work was approved through Isogeomentric analysis with erosion in Ekwulobia/Oko community at Aguta L.G.A Anambra State on the case of study.

2.0 RESEARCH METHODOLOGY

In this paper, a residential building is modelled and subjected to serviceability and ultimate limit state load for the investigation of small and large deflection analysis. Large deflection analysis has been carried on the basis on geometric non-linearity after the field inspection carried out in the buildings at Oko and Ekwulobia erosion regions. They are label EB and OB respectively.

2.1 Structural Defects due to erosion

The pictures below shows the nature of some buildings at Ekwulobia/Oko study area due to erosion problems.



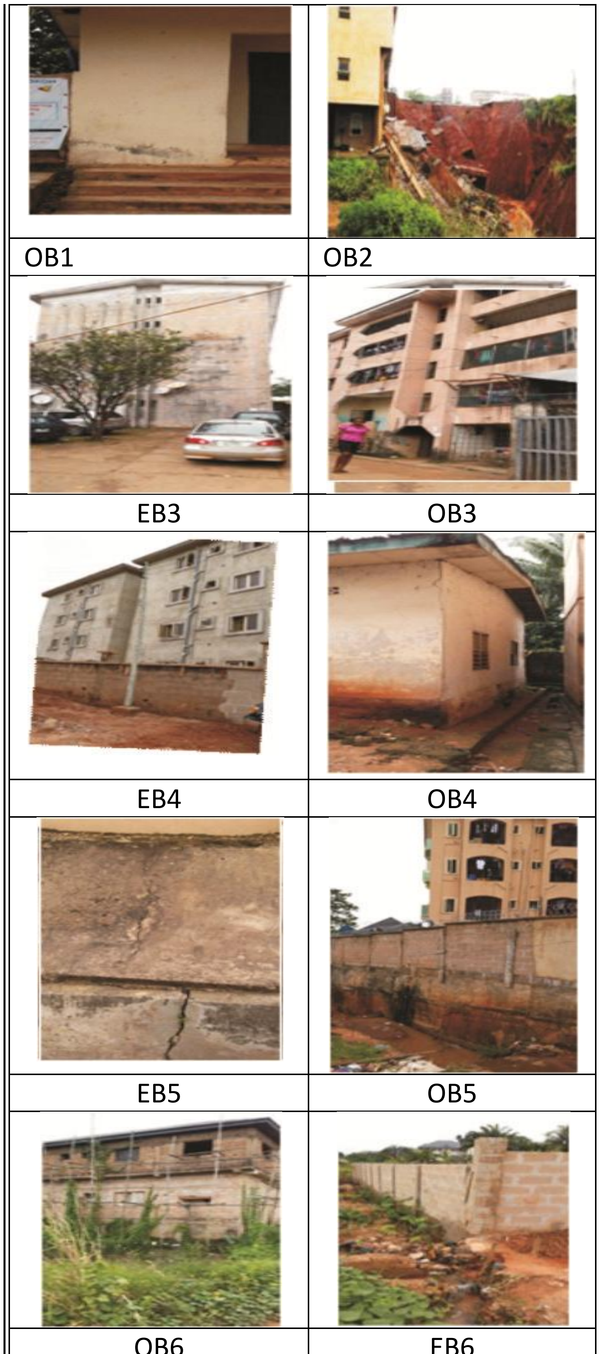


Figure 2: the nature of some buildings at Ekwulobia/Oko study area due to erosion problems

Table 4 shows summary of the field inspection carried out, the buildings under consideration have been surveyed by visual inspection. In order to make the analysis more simplified, the buildings were modified and assembled into a single building that will represent the major structural elements that are often encountered in structural engineering practice in Nigeria. This structural element is shown in Figure 3;

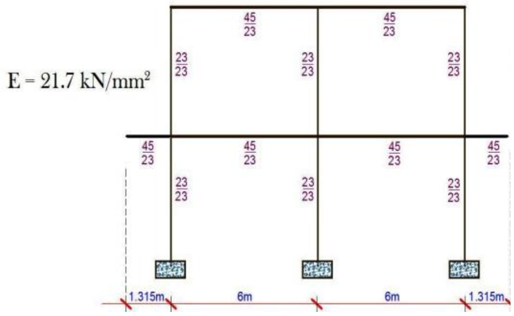


Figure 3: 2D view of the frame of the model showing section sizes (cm)

In this the section, the methods employed in formulation of the MATLAB and C++ programs for the application of the isogeometric analysis for the linear elastic deflection of structural members are presented. Here, we consider the domain Ω with boundary Γ in the physical spa

$$\sum_{i=1}^{ngp} \mathbf{R}^{eT} \left(\xi\left(\hat{\xi}_i\right), \eta(\hat{\eta} =$$

$bt\,\mathbf{J}\; x, \xi e\xi i,\eta = b\rho i.$ (6)

for the part of the boundary we η is constant.

After assembling all contributions to **K** and **f** we can solve with respect to **d** and further calculate strain and stresses. Recall that $\in^e = \mathbf{B}^e\mathbf{d}^e$ and $\sigma^e = \mathbf{D}^e\in^e = \mathbf{D}^e\mathbf{B}^e\mathbf{d}^e$. This can be comfortably programmed in MATLAB.

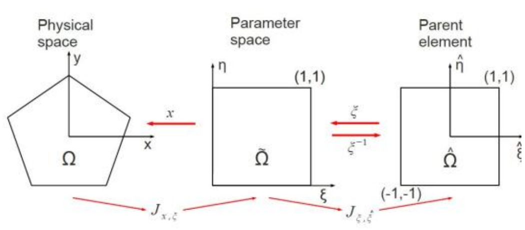


Figure 4: Computational domain

3.0 RESULTS

Consider the general arrangement of a floor plan as shown below in Figure 5. The panels have been idealised as thin plates clamped at all edges.

3.1 Analysis of the plates at serviceability limit state

Load Analysis

Self weight of concrete = 25 × 0.15 = 3.6 KN/m^2

Finishes (say) = 1.2 KN/m^2

Total dead load (gk) = 3.6 + 1.2 = 4.8 KN/m^2

Imposed load (residential apartment) (qk) = 1.5 KN/m^2

At serviceability limit state = 1.0gk + 1.0qk = 1.0(4.8) + 1.0(1.5) = 6.2 KN/m^2

We will be treating the slab as a square plate that is clamped at all edges, subjected to a uniform pressure of 6.2 KN/m^2.

Material Data;

Modulus of Elasticity **E** = 21.7 KN/mm^2 = 2.17 × 10^7 KN/m^2

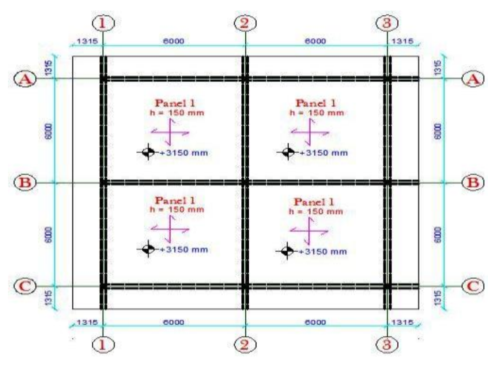


Figure 5: Plan view of the floor plan general arrangement

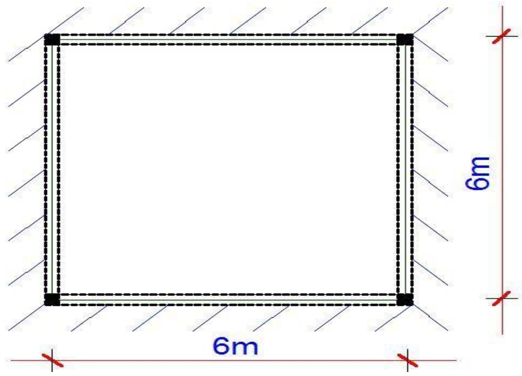


Figure 6: 2-way square plate clamped at all edges for deflection analysis

Panel 1: All edges clamped

Poisson ratio $v = 0.2$

Plate dimensions = a = b = 6.0m

Solution

(a) By Kirchoff's small deflection thin plate theory;
Maximum deflection at mid span = -0.001593681267073 m

$W_{max} = 1.59368$ mm

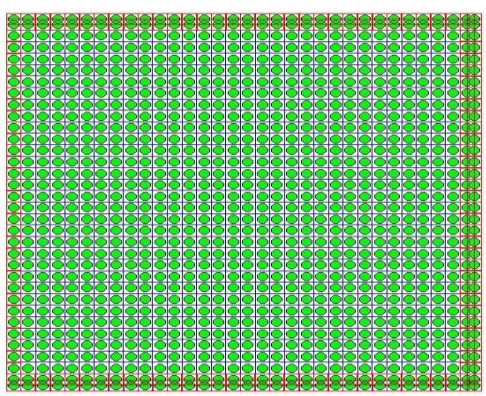


Figure 7: Meshing of the plate (IGA)

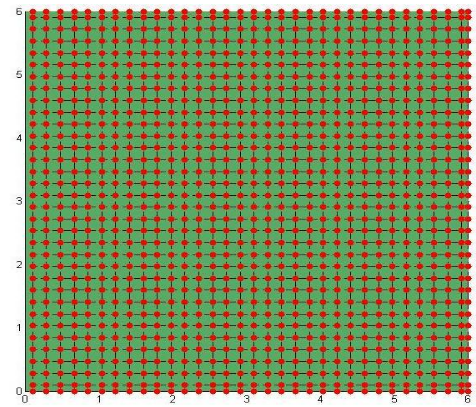


Figure 8: Enforcement of boundary conditions

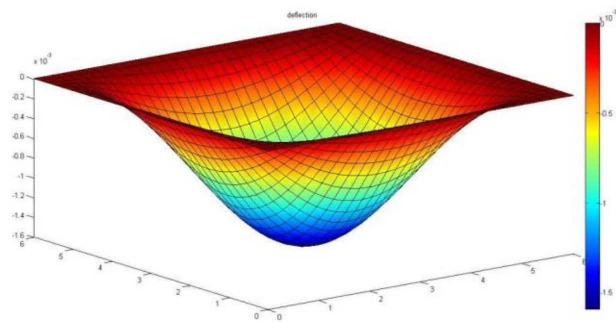


Figure 9: Deflection profile of the plate under consideration using isogeometric analysis

The result obtained using isogeometric analysis has been compared with classical solution and result from finite element analysis using Staad Pro Software.

Table 2: Comparison of results from present study with other known methods

From present study (isogeometric analysis)	Classical Method (Timoshenko, 1959)	Staad Pro (Finite Element Analysis) 20 x 20
W = 1.594 mm	W = 1.593 mm	W = 1.611 mm

(b) By Midlin plate theory (incorporating shear deformation)

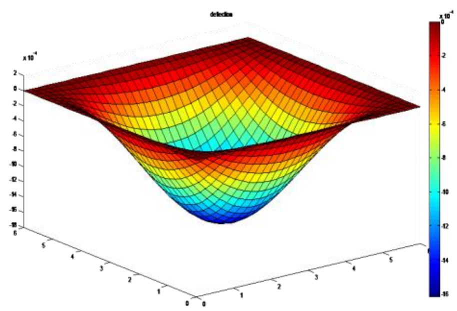


Figure 10: Deflection profile of the thin plate using Mindlin plate theory (IGA)

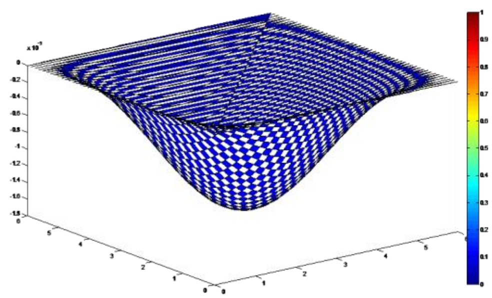


Figure 11: Deflection profile of the thin plate using Mindlin plate theory (FEM)

The results from the analysis are compared in Table 3.

Table 3: Comparison of results from present study with other known methods

From present study (isogeometric analysis)	(Finite Element Analysis) 20 x 20
W = 1.616 mm	W = 1.613 mm

3.2 Large deformation analysis using finite element analysis

No of iteration steps = 100; Maximum load = 6.2 KN/m^2 (Service load)

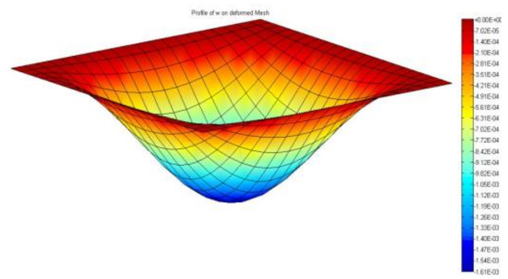


Figure 12: SLS Large deflection profile of the plate (FEM)

Maximum deflection at serviceability limit state = **1.6137 mm**

At ultimate limit state

n = 1.4gk + 1.6qk = 1.4(4.8) + 1.6(1.5) = 9.12 KN/m^2

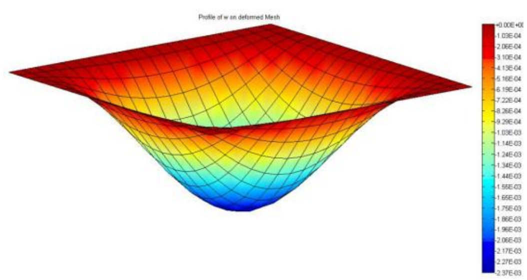


Figure 13: ULS Large deflection profile of the plate (FEM)

The maximum deflection at the centre of the plate is **2.375mm.**

3.3 Cantilever Plate Element (isogeometric analysis) subjected to concentrated load

Unit weight of block work = 3.47 KN/m^2

Height of wall = 3m

Concentrated load on cantilever = 3.47 × 3 = 10.41 KN/m

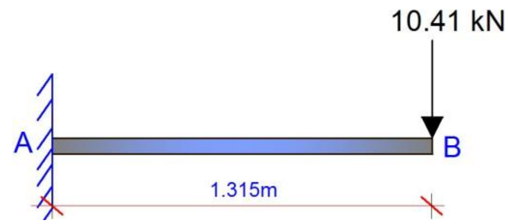


Figure 14: Cantilever plate subjected to block work load at the free end

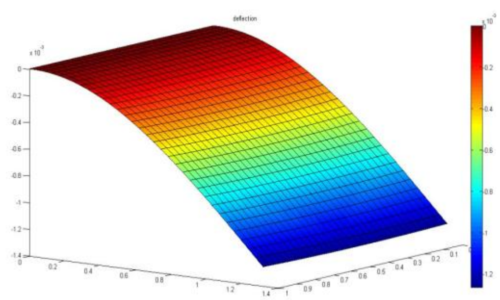


Figure 15: Elastic deflection profile of cantilever plate subjected to point load (IGA)

Maximum deflection at the free end (isogeometric analysis) = 1.2667 mm

By finite element analysis using Staad Pro

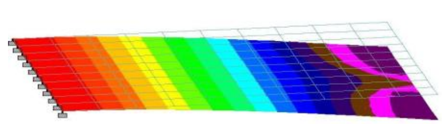


Figure 16: Elastic deflection profile of cantilever plate subjected to point load (FEM)

Maximum deflection at the free end (finite element analysis) = 1.262 mm

Exact solution = $\frac{PL^3}{3EI}$ = 1.2928 mm

Large deflection results using finite element analysis

Number of iterations = 10

P = 10.41 KN (SLS)

Maximum deflection at the free end = 2.243mm

P at ultimate limit state = 1.4 × 10.41 = 14.547 KN (ULS)

Maximum deflection at the free end = 3.115mm

4.0 DISCUSSION OF RESULTS

Using isogeometric analysis (IGA), the small deflection of a 6m x 6m clamped plate subjected to a serviceability pressure load of 6.2 KN/m^2 was found to be 1.59368mm. This result was 0.0627% more than the exact solution at 1.593mm. IGA gave better result than finite element analysis carried out using StaadPro software. Employing a matrix size of 20 x 20, the deflection was observed to be 1.611mm, which was 1.1173% higher than the exact solution. Using Mindlin plate theory (incorporating shear deformation), the maximum deflection at the centre of the plate was observed to be 1.616mm. This was 1.423% higher than the result from Kirchoff's plate theory. The result obtained using finite element analysis was 1.613 mm, this shows that the result for IGA was 0.1856% higher than finite element analysis result.

On considering the large deformation of the square plate element at serviceability limit state at 100 iteration steps, the results show that the maximum deflection was 1.613mm, exactly the same with small deflection theory. At ultimate limit state (failure load), the result from large deflection analysis increased to 2.375mm (32.084%

increase) at 100 iteration steps. However, a little consideration will show that the deflection at ultimate load was still small, since it was less than $\frac{thickness\ of\ plate}{5}$ (150/5 = 30mm). Therefore, it is not a surprise that the results from linear and nonlinear theory are very similar. These deflections are so small that the membrane forces cannot develop in the plate, and as a result, for the analysis of such structures at ultimate limit state, small deflection plate theory can be conveniently employed.

Also, considering the behaviour of cantilever members in the structure, the same behaviour was observed. For the cantilever subjected to a block work load of 10.41KN at the free end,a deflection of 1.2667mm was observed at the free end. This result was actually less than the exact solution (1.2928mm) by -2.060%. The displacement from large deflection analysis at 10 iteration steps was found to be 2.243mm for the service load. This was 42.36% higher than the exact solution. The displacement from large deflection theory at ultimate load was observed to be 3.115mm. This was found to be 27.99% higher than the result from the service load. However, a little consideration will show that that this is still very consistent with results from small deflection theory.

5.0 CONCLUSION

From the above results, we can conclude that the method of isogeometric analysis is very efficient for the analysis of structures, and presents very interesting concepts when compared with the famous finite element analysis method. Also, the use of small deflection theory has been shown to be sufficient for the analysis of buildings in erosion prone regions, especially in Nigeria. This is so because the normally used thickness for slabs in Nigeria is 150mm, and for slabs with small and moderate spans, this is quite satisfactory for most ultimate and serviceability limit state requirements. From the study, geometric non-linearity cannot be the major cause of failure of structures but issues like loss of equilibrium, mass wasting, and loss of bearing capacity of foundations can be seen to be more prominent in causing failure of buildings at erosion sites.

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Table 4: Summary of the Field Inspection

BUILDINGS	**IDENTIFICATION**	**CRACKS**	**DEFORMATION**	**OBSERVATIONS/COMMENTS.**
EB1	Fence and draining structures caving into gullies at Ekwulobia	Yes	Yes	A large cracks and large deformation was observed
EB2	Building curving into gully at Ekwulobia	–	Yes	Deformation about to enhance crack wasn't observed due to access
OB1	Building near erosion threatened gully at Oko	No	No	Gradually washing away of the building surrounding by flood was observed.
OB2	Erosion threatened building at Oko	No	No	The building is under treat Although No crack or deformation was observed.
OB3	Building which measures about 50m away from gully	Yes	No	Cracks measuring about 5m in length negligible width was observed but no deformation.
EB3	Student Hostel which is about 250m away from the erosion site	No	NO	No cracks and deformation was observed.

EB4	A 3 bed room flat captioned at Oko near erosion prone region	No	No	This building near a flood way but no crack and deformation which may be as a result of quality construction.
OB4	A hall Oko near an erosion threatened region	Yes	No	A negligible crack was observed but No deformation.
EB5	Floor of compound near a gully	Yes	No	A crack was encountered L=2m W=0.5mm Measured with the help of a tape & a broom stick
OB5	Hostel built beside a gully	No	No	No cracking and deformation due to the presence of a retaining wall.
OB6	A residential building built beside a swamp at Oko	No	No	No crack and deformation due to the presence of pile foundation and retaining wall.
EB6	Fence of an incomplete building built along a moving flood	No	No	No cracks was observed because they made use of a retaining wall with a good concrete mixture.