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# Production - Inventory Policies for Deteriorating Items with Imperfect Production Process when Demand is Quadratic *(pp. 1-15)*

[1]Nita H. Shah , [2]Ajay S. Gor and [3]Chetan A. Jhaveri
[1]Department of Mathematics, Gujarat University, Ahmedabad-380009
[2]Pramukh Swami Science & H. D. Patel Arts College, Kadi, Gujarat
[3]S. L. Institute of Business Administration, Ahmedabad-380009, Gujarat, India
Correspondence E-mail: nitahsh@gmail.com

**Abstract:** This study is aimed at analysing integrated production - inventory policies for deteriorating items in supply chain when demand of product is subject to increase with time. The model assumes partial backlogging, imperfect production process and multiple deliveries. The elapsed time until the production process shifts is assumed to be exponentially distributed. It is observed that the joint decision lowers the total cost compared with an independent decisions made by the manufacturer and the retailer with the help of numerical data. The sensitivity analysis is carried out to study the variation in the decision variables and the total integrated cost.

**Key words:** integrated production-inventory policy, deterioration, imperfect processes, multiple deliveries, quadratic demand.

## NOTATIONS

| | |
|---|---|
| | **Retailer's demand rate (units/unit time) = $a(1 + bt + ct^2)$, where a is fixed demand, b and c are linear and exponential rate of change of a demand with respect to time respectively with $a > 0$, $a >> b, c$ and $0 < b, c < 1$** |
| **P(t)** | $= \gamma R(t)$ ; production rate(units/unit time), where $\gamma > 1$ |
| **B** | Fraction of retailer's demand back-ordered |
| **N** | The number of delivery per order (a decision variable) |
| **$T_1$** | The production period (a decision variable) |
| **$T_2$** | The non-production period (a decision variable) |
| **$T_3$** | The period for which a retailer has the stock (a decision variable) |
| **$T_4$** | The retailer's stock-out period.(a decision variable) |
| **T** | $( = T_1 + T_2 = n(T_3 + T_4) )$; the length of the cycle time (a decision variable) |
| **$I_{rm}(t_1)$** | Raw material's inventory level at any time $t_1$, $0 \le t_1 \le T_1$ |
| **$I_{mfi}(t_i)$** | Manufacturer's finished goods inventory level at any time $t_i$ , $0 \le t_i \le T_i$ , i = 1,2 |
| **$I_{ri}(t_i)$** | Retailer's inventory level at any time $t_i$, $0 \le t_i \le T_i$ , i = 3, 4 |
| **$A_{mo}$** | Manufacturer's ordering cost per order cycle ($/cycle) |
| **$A_{ms}$** | Manufacturer's set-up cost per production cycle ($/cycle) |
| **$A_r$** | Retailer's ordering cost per order cycle ($/order) |
| **$C_{mr}$** | Manufacturer's raw material's per unit cost ($/unit) |
| **$C_{mrw}$** | Manufacturer's finished goods per unit rework cost ($/unit) |

| | |
|---|---|
| $C_{mf}$ | Manufacturer's finished goods per unit cost (\$/unit) |
| $C_r$ | Retailer's purchase cost per unit (\$/unit) |
| $h_{mr}$ | Manufacturer's raw material's per unit holding cost per unit time(\$/unit/unit time) |
| $h_{mf}$ | Manufacturer's finished goods per unit holding cost per unit time (\$/unit/unit time) |
| $h_r$ | Retailer's per unit holding cost per unit time (\$/unit/unit time) |
| $\pi_{rB}$ | Retailer's per unit backlogged cost per unit time (\$/unit/unit time) |
| $\pi_{rL}$ | Retailer's per unit shortage cost for lost sales (\$/unit) |
| $MI_m$ | Manufacturer's finished goods maximum inventory level |
| $MI_r$ | Retailer's maximum inventory level |
| $Q_{rm}$ | Raw material's order quantity per order |
| $Q_m$ | Manufacturer's finished goods production quantity per production |
| $Q_r$ | Retailer's maximum purchase quantity per delivery from the manufacturer |
| $\theta$ | Deterioration rate of units in inventory ($0 < \theta < 1$) |
| **X** | Random elapsed time until production process shift |
| **f(X)** | Probability density function of X |
| **D** | Percentage of defective items produced once the system is in the out-of-control state |
| $TC_{rm}$ | Total cost of the raw material per unit time |
| $TC_m$ | Total cost of the manufacturer per unit time |
| $TC_r$ | Total cost of the retailer per unit time |
| **TC** | Total joint cost of an inventory system per unit time |

## 1 INTRODUCTION

The researchers are engaged in the development and analysis of inventory models from retailer's point of view only. With the globalization of business, the integrated/compromising decision is advantageous to the players of the supply chain. Also, due to alertness of the customers, the enterprises are tending to develop optimal decision that can respond quickly and efficiently to customer's needs with maximum service level and minimum stock. The cooperation between the manufacturer and the retailer is considered to be a key factor to improve the quality, quantity and satisfaction of the customer in the inventory control.

Goyal and Nebebe (2000) formulated economic production and shipment policy of a product from a vendor to a buyer. Woo et al. (2001) analyzed an integrated inventory system where a manufacturer buys and processes the raw materials and delivers the finished product to multiple retailers. Deterioration is defined as decay, decomposition, evaporation of the product and thereby loosing 100% utility of its efficiency. For articles on deteriorating inventory models see review articles by Nahmias (1982), Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001). Most of the models exhibited in the review article's are derived from retailer's point of view. This decision may not be advantageous to the manufacturer. Rau et al. (2003) discussed a multi-echelon inventory model for deteriorating items and optimized joint total cost from an integrated perspective among the supplier, the producer and the retailer. Yang and Wee (2003) derived an integrated inventory model when units in inventory are subject to constant rate of deterioration and multiple deliveries. Hans et al. (2006) discussed a new method to obtain the joint economic lot-size in distribution system with multiple shipments. In above articles, deteriorated units were considered to be a complete loss for the decision maker. However, theses deteriorated units can be sold off at lower rate and hence, the total cost can be reduced. Shah et al. (2008) developed the optimal strategy for an integrated vendor-buyer inventory system for deteriorating items. The salvage value is incorporated to theses deteriorated units in the development of the model. Shah and Gor (2009) formulated an integrated economic lot-size for vendor-buyer supply chain when buyer receives units randomly due to worker's strike, failure of electricity, over/under production etc. at the manufacturer end. There is a possibility of the imperfect production due to age of the machine or faulty machine or quality of raw material and equipment or mishandling by incapable trainees. Kim and Hong (1999) obtained the optimal production run time in deteriorating production process. Salameh and Jaber (2000) and Goyal et al. (2002) derived an economic production-inventory model for items with imperfect quality. They assumed that poor-quality items are sold as a single batch by the end of the 100% screening process. Chang and Hou (2003) formulated a model to compute an optimal cycle time for a deteriorating production system when shortages are allowed. Papachristos and Konstantaras (2006) developed economic ordering quantity model for items with imperfect quality.

The above stated articles assumed demand to be constant. It is observed that demand of seasonal goods like air-condition, freeze, water cooler, room heaters, fashion clothes, air-tickets during week ends and vacations, etc increases with time. In this proposed study, an integrated production-inventory model under imperfect production processes and partial backlogging is developed when the demand increases with time. Here demand is assumed to be quadratic. The total joint cost of an inventory system is minimized to obtain the optimal production and replenishment schedule.

## 2 ASSUMPTIONS

The development of the mathematical model is carried out under the following assumptions:

1. The inventory system deals with a single item.
2. Single-manufacturer and single-retailer are considered.
3. The replenishment rate is proportional to demand rate. The demand rate is quadratic and increasing function of the time.
4. Shortages are not allowed at the manufacturer end. Replenishments are instantaneous.
5. Partial backlogging is allowed at the retailer end. It is cleared when fresh stock-arrives in the next delivery.
6. Multiple deliveries per order are considered. The planning horizon is infinite and cycles during the planning horizon are continuous. The units of the first delivery are ordered in the previous cycle.

7. In the beginning of each production cycle, the production process is in an in-control state producing quality items.
8. During a production run, the production may shift from an in-control state to an out-of-control state. An elapsed time is exponentially distributed with the known mean. Once the production process shifts to an out-of-control state, the shift can not be detected until the end of a production cycle and a percentage of the produced items are defective.
9. All defective items are detected at the end of each production cycle and there is rework cost for defective items.
10. A constant fraction; $\theta$ of units deteriorate in an inventory system. There is no repair or replenishment of deteriorated units during a cycle time.

## 3 MATHEMATICAL MODEL

We discuss the two stages of manufacturer-retailer joint venture. The first stage is the manufacturer's production system. The manufacturer's purchases raw material from outside supplier to produce the items. The second stage consists of the retailer's inventory status. The retailer procures the fixed amount of units in multiple deliveries at a fixed-time interval.

For the raw material's inventory system see Fig. 1. The manufacture purchases $Q_{mr}$ - units in the beginning to produce items. During the $T_1$ - time period, the inventory level of the raw-material decreases due to the manufacturer's demand and the deterioration of raw material. The rate of change of the manufacturer's raw material's inventory level at any instant of time $t_1$ ($0 \le t_1 \le T_1$) can be described by the differential equation:

$$\frac{dI_{rm}(t_1)}{dt_1} = -P(t_1) - \theta I_{rm}(t_1), \quad 0 \le t_1 \le T_1$$

or

$$\frac{dI_{rm}(t_1)}{dt_1} + \theta I_{rm}(t_1) = -\gamma R(t_1), \quad 0 \le t_1 \le T_1 \qquad (1)$$

Using the boundary condition, $I_{rm}(T_1) = 0$ ,the solution of the differential equation is

$$I_{rm}(t_1) = \gamma a[\{\frac{1+bT_1+cT_1^2}{\theta} - \frac{(b+2cT_1)}{\theta^2} + \frac{2c}{\theta^3}\} e^{\theta(T_1 - t_1)} - \frac{(1+bt_1+ct_1^2)}{\theta} + \frac{(b+2ct_1)}{\theta^2} - \frac{2c}{\theta^3}] \\ , 0 \le t_1 \le T_1 \qquad (2)$$

Also, $I_{rm}(0) = Q_{rm}$ gives the maximum inventory level of raw material as

$$Q_{rm} = \gamma a[\{\frac{1+bT_1+cT_1^2}{\theta} - \frac{(b+2cT_1)}{\theta^2} + \frac{2c}{\theta^3}\} e^{\theta T_1} - \frac{1}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}] \qquad (3)$$

The cost-components of raw material inventory system are as follows:

The ordering cost; $OC_{rm}$ of raw material per order is

$$OC_{rm} = A_{mo} \tag{4}$$

The purchase cost; $PC_{rm}$ of raw material is

$$PC_{rm} = C_{mr} Q_{rm} \tag{5}$$

The manufacturer stocks the raw material for $T_1$ time. Hence, inventory holding cost of the raw material is

$$IHC_{mr} = h_{mr} \int_0^{T_1} I_{rm}(t_1) dt_1 \tag{6}$$

Thus, for the raw material, total cost per time unit is

$$TC_{rm} = \frac{1}{T}[OC_{rm} + PC_{rm} + IHC_{mr}] \tag{7}$$

Next, we analyze the manufacturer's finished goods inventory system, See Fig 2.
The manufacturer has two phases viz. production phase and non-production phase. The production phase starts at $t_1 = 0$ and continues till maximum inventory $MI_m$ is produced at $t_1 = T_1$. During $[0, T_1]$ the retailer's inventory depletes due to demand during production phase and deterioration of units at a constant rate. At $T_1$, production stops and depletion of inventory level is due to the retailer's demand and deterioration of units. The inventory level reaches zero at $t_2 = T_2$. The inventory level without the delivery can be described by the following differential equations:

$$\frac{dI_{mf_1}(t_1)}{dt_1} = P(t_1) - R(t_1) - \theta I_{mf1}(t_1) = (\gamma - 1)R(t_1) - \theta I_{mf1}(t_1), \quad 0 \le t_1 \le T_1 \tag{8}$$

and

$$\frac{dI_{mf_2}(t_2)}{dt_2} = - R(t_2) - \theta I_{mf2}(t_2), \qquad 0 \le t_2 \le T_2 \tag{9}$$

Using the boundary conditions $I_{mf1}(0) = 0$ and $I_{mf2}(T_2) = 0$, the solution of the differential equations (8) and (9) can be given by

$$I_{mf1}(t_1) = (\gamma - 1)a[\frac{(1 + bt_1 + ct_1^2)}{\theta} - \frac{(b + 2ct_1)}{\theta^2} + \frac{2c}{\theta^3} - (\frac{1}{\theta} - \frac{b}{\theta^2} + \frac{2c}{\theta^3}) e^{-\theta t_1}], \; 0 \le t_1 \le T_1 \tag{10}$$

and

$$I_{mf2}(t_2) = a[\{\frac{1+bT_2+cT_2^2}{\theta} - \frac{(b+2cT_2)}{\theta^2} + \frac{2c}{\theta^3}\} e^{\theta(T_2-t_2)} - \frac{(1+bt_2+ct_2^2)}{\theta} + \frac{(b+2ct_2)}{\theta^2} - \frac{2c}{\theta^3}]$$
$$, 0 \le t_2 \le T_2 \qquad (11)$$

respectively. Using $I_{mf2}(0) = MI_m$; the maximum inventory of manufacturer's is

$$MI_m = a[\{\frac{1+bT_2+cT_2^2}{\theta} - \frac{(b+2cT_2)}{\theta^2} + \frac{2c}{\theta^3}\} e^{\theta T_2} - \frac{1}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}] \qquad (12)$$

The production quantity during $[0,T_1]$ is

$$Q_m = P(T_1)T_1 = a\gamma T_1[1+bT_1+cT_1^2] \qquad (13)$$

The cost components of manufacturer's inventory system are as follows:

The initial production set-up cost per cycle is

$$OC_m = A_{ms} \qquad (14)$$

The inventory is carried out during $T_1$ and $T_2$ time periods. If this system does not consider the retailer, all of the holding costs are incurred by the manufacturer, which are first two terms in equation (15). If this system considers the retailer, the inventory holding cost of the items those are supplied to the retailer is to be subtracted from that of the manufacturer which is the third term in equation (15). Hence, the Manufacturer's inventory holding cost is

$$IHC_m = h_{mf} \int_0^{T_1} I_{mf1}(t_1)dt_1 + h_{mf} \int_0^{T_2} I_{mf2}(t_2)dt_2 - h_{mf} \int_0^{T_3} I_r(t_3)dt_3 \qquad (15)$$

The production cost of the manufacturer is

$$PC_m = C_{mf} Q_m \qquad (16)$$

The number of defective items; N in a production cycle is given by

$$N = \begin{cases} 0 & , \text{when } X \ge T_1 \\ dP(T_1)(T_1 - X) & , \text{when } X < T_1 \end{cases} \qquad (17)$$

Then, the expected number of defective items during a production cycle is

$$E(N) = \int_0^{T_1} Nf(X)dX \tag{18}$$

Under the assumption that an elapsed time shift is exponentially distributed with a mean of $\frac{1}{\mu}$, the rework cost is

$$RW = C_{mrw} dP(T_1)\mu \int_0^{T_1} (T_1 - X)e^{-\mu X} dX \tag{19}$$

Therefore, the total cost per time unit of manufacturer inventory system is

$$TC_m = \frac{1}{T}[OC_m + PC_m + IHC_m + RW] \tag{20}$$

Next, the retailer's inventory system is exhibited in Fig. 3.
With the assumption that the retailer orders at $t_3 = 0$, the back-order is cleared leaving a balance of $MI_r$ – units in the retailer's inventory system. During $T_3$ - time period, the inventory depletes due to the quadratic demand and deterioration of units. At $T_3$ - time, retailer's inventory level reaches zero. During the $T_4$ – time period, part of the shortages is back-logged and part of it is lost sales. Only the back-logged units are replaced by the next arrival. There are n -deliveries in cycle time T (= $T_1 + T_2$). The retailer's inventory level at any instant of time can be described by

$$\frac{dI_{r_3}(t_3)}{dt_3} = -R(t_3) - \theta I_{r3}(t_3), \quad 0 \le t_3 \le T_3 \tag{21}$$

And

$$\frac{dI_{r_4}(t_4)}{dt_4} = -BR(t_4), \quad 0 \le t_4 \le T_4 \tag{22}$$

Using the boundary conditions $I_{r3}(T_3) = 0$ and $I_{r4}(0) = 0$, the solutions of the differential equations are

$$I_{r3}(t_3) = a[\{\frac{(1+bT_3+cT_3^2)}{\theta} - \frac{(b+2cT_3)}{\theta^2} + \frac{2c}{\theta^3}\} e^{\theta(T_3-t_3)} - \frac{(1+bt_3+ct_3^2)}{\theta} + \frac{(b+2ct_3)}{\theta^2} - \frac{2c}{\theta^3}], \quad 0 \le t_3 \le T_3 \tag{23}$$

and

$$I_{r4}(t_4) = -Ba[t_4 + \frac{b}{2}t_4^2 + \frac{c}{3}t_4^3)], \quad 0 \le t_4 \le T_4 \tag{24}$$

Using $I_{r3}(0) = MI_r$ , the retailer's maximum inventory is given by

$$MI_r = a[\{\frac{(1+bT_3+cT_3^2)}{\theta} - \frac{(b+2cT_3)}{\theta^2} + \frac{2c}{\theta^3}\} e^{\theta T_3} - \frac{1}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}] \tag{25}$$

Hence, the quantity to be received per delivery by the retailer is

$$Q_r = MI_r - BR(T_4)T_4 \tag{26}$$

The cost components of the retailer for n - deliveries are as follows:

Ordering cost for n-orders is $OC_r = nA_r$ (27)

Purchase cost for n-deliveries is $PC_r = nC_rQ_r$ (28)

Inventory holding cost for n-deliveries is $IHC_r = nh_r \int_0^{T_3} I_{r3}(t_3)dt_3$ (29)

Shortages cost for n-orders is $SC = n\pi_{rB} \int_0^{T_4} -I_{r4}(t_4)dt_4$ (30)

Lost sales cost for n-orders is $LS = n\pi_{rL} \int_0^{T_4} (1-B)R(t_4)dt_4$ (31)

Hence, the retailer's total cost per time unit for n-orders is

$$TC_r = \frac{1}{T}[OC_r + PC_r + IHC_r + SC + LS] \tag{32}$$

Using equation (7), (20) and (32) the total joint cost per unit time of an inventory system is

$$TC = TC_{rm} + TC_m + TC_r \tag{33}$$

The joint total cost; TC is a function of discrete variable; n and continuous variables; $T_1$, $T_2$ , $T_3$ and $T_4$. Since $T= T_1 + T_2 = n (T_3 +T_4)$, we have

$$T_3 = \frac{T}{n} - T_4 \tag{34}$$

Using the continuity of the functions $I_{mf1}(t_1)$ and $I_{mf2}(t_2)$ at $T_1$, the relation between $T_1$ and $T_2$ is

$$T_1 \approx \frac{1}{\gamma-1}\left(T_2 + \frac{(b+\theta)T_2^2}{2} + \frac{(c+\theta)T_2^3}{3}\right) \tag{35}$$

Substituting (34) and (35) in (33), the total joint cost is function of discrete variable n and continuous variables $T_2$ and $T_4$.

## 4 COMPUTATIONS AND ANALYSIS

To minimize (33) with respect to decision variables n, $T_2$ and $T_4$, proceed as follows:
Step 1: Start with n = 1.
Step 2: Compute $T_2$ and $T_4$ by solving

$$\frac{\partial TC(n,T_2,T_4)}{\partial T_2} = 0 \quad \text{and} \quad \frac{\partial TC(n,T_2,T_4)}{\partial T_4} = 0$$

Step 3: Compute TC from equation (33) by using values of $T_2$ and $T_4$ obtained in step 2.
Step 4: Increment n by 1 and repeat steps 2 and 3 until following condition is satisfied.
$TC(n-1, T_2(n-1), T_4(n-1)) \geq TC(n, T_2, T_4) \leq TC(n+1, T_2(n+1), T_4(n+1))$
Step 5: Obtain $T_1$, $T_3$, $Q_{rm}$, $Q_m$, $Q_r$, $TC_{rm}$, $TC_m$ and $TC_r$.

### 4.1 Numerical Example:

Considering following parametric values in appropriate units:
$[a, b, c, \gamma, B, A_{mo}, A_{ms}, A_r, C_{mr}, C_{mf}, C_r, C_{mrw}, h_{mr}, h_{mf}, h_r, \pi_{rB}, \pi_{rL}, d, \mu, \theta]$
= [750, 10%, 15%, 3, 80%, 90, 100, 80, 8, 10, 12, 10, 0.60, 0.80, 1, 10, 5, 5%, 0.001, 5%]
The computational results are shown in Table 1.

### 4.2 Sensitivity Analysis

In table 3, the parameters are varied by -10%, -5%, +5% and 10%. The variations in the decision variables and total cost are studied. The last column exhibits percentage savings in the total joint cost. PCI is defined as $\frac{TC_o - TC^*}{TC^*} * 100$. It is observed that the model is very sensitive to changes in a, $C_{mr}$, $C_{mf}$, $C_r$, marginal sensitive to B, $\pi_{rL}$, $\pi_{rB}$, $\theta$ and less sensitive to changes in the rest of the model parameters.

## 5 DISCUSSION OF RESULTS

The following conclusions are drawn from the numerical example. For the data taken in the numerical example, two orders by the retailer gives minimum total joint cost; TC = \$23520. For this minimum cost, manufacturer's production time $T_1$ = 0.127 units and non-production time $T_2$ = 0.249 units. While the retailer's stock-in-time $T_3$ = 0.047 units and stock-out-time $T_4$ =0.1415 units. The total cycle time is 0.376 units for the manufacturer and 0.1885 units for the retailer. The optimum purchase quantity of raw material, is $Q_{rm}$ = 289 units, the production quantity of manufacturer is $Q_m$ = 290 units and the purchase units of the retailer per order is $Q_r$ = 122 units. The convexity of the total joint cost TC with respect to $T_2$, $T_4$ and n exhibited in Fig. 4 and Fig. 5 respectively. From retailer's point, the optimal cost is $23580 for one order which is $ 60 more than that of the joint solution. If decision is to be taken from raw material's point, then the optimal solution is three orders by retailer with total cost $23528 which is $ 8 more than the integrated cost. For n=2, 3 and 1, the variation in different costs are computed in Table 2.

Increase in number of deliveries increases $T_1$ and $T_2$ and decreases $T_3$ and $T_4$. The reason is increase in demand and deterioration of units. For $n \geq 4$, $T_3$ is infeasible. Here, the retailer does not carry any inventory. The retailer's inventory system only allows partial backlogging which will be cleared by the arrival of next replenishment. When B=1, i.e. complete back logging, total joint cost is $24007.

## 6 CONCLUSION

A joint production-inventory model for deteriorating items is derived for the manufacturer and retailer's end. Demand is considered to be increasing with time. The elapsed time for the production process shifts to imperfect production is considered to be exponentially distributed. A numerical example is given to support the basis of the model. Sensitivity analysis is carried out to characterize sensitive parameters.

It is observed that the multiple orders reduce the total cost of an inventory system when compared with an independent decision by the manufacturer or the retailer. The future research to study problem with multi-buyers, multi manufacturer is in progress.

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**APPENDIX**

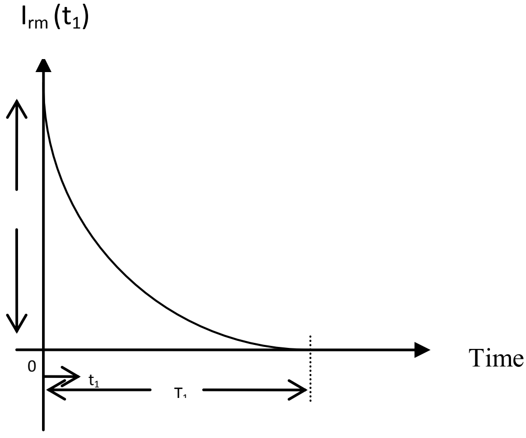


**Figure- 1 The raw material's**

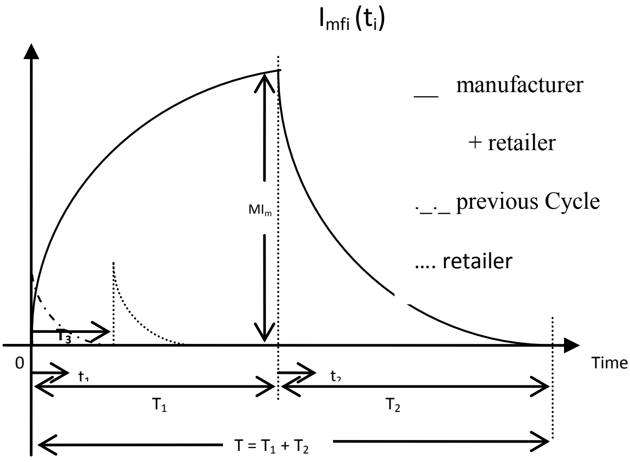


**Figure- 2 The manufacturer's inventory**

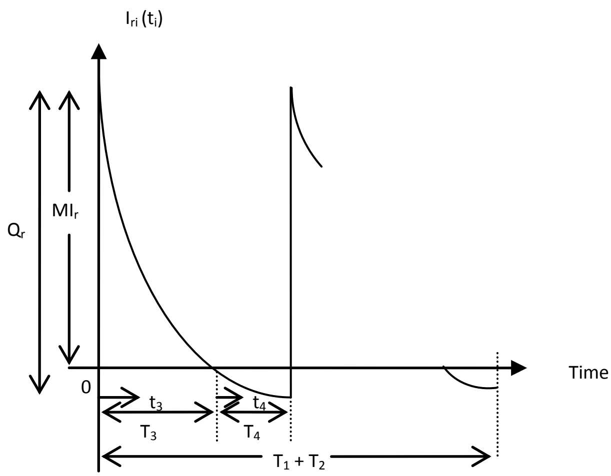


**Figure- 3 The retailer's inventory**

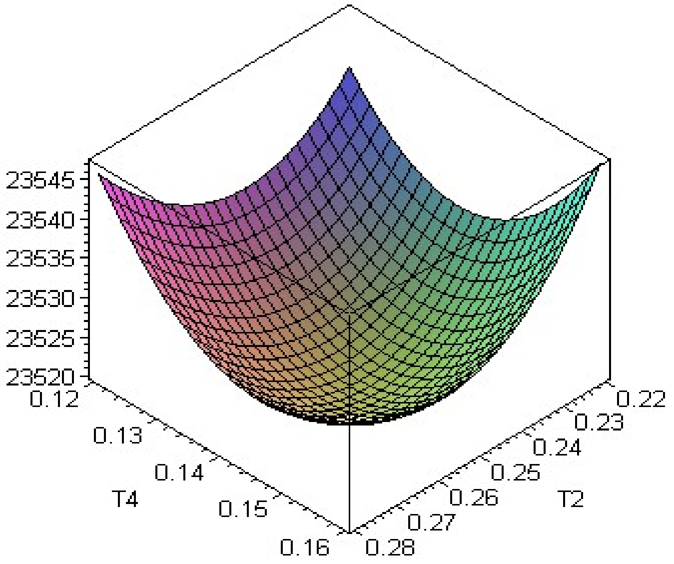


**Figure- 4 Convexity of TC w. r. t. $T_2$ and $T_4$ for n = 2**

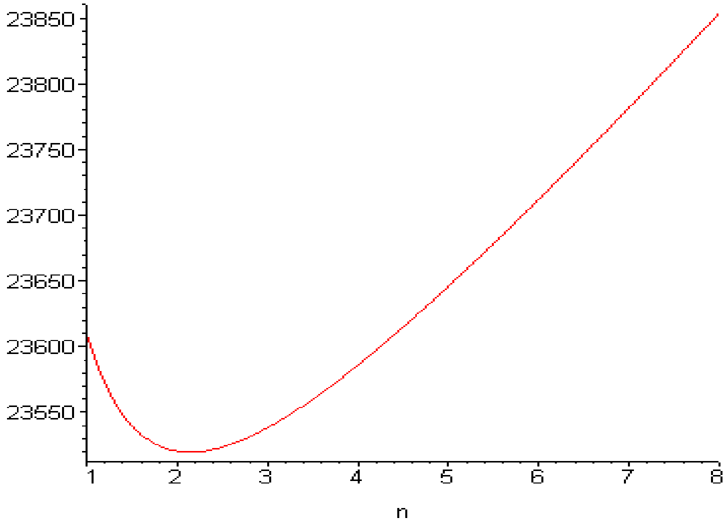


**Figure- 5 Convexity of TC w. r. t. n for obtained $T_2$ and $T_4$**

**Table 1: Computational Results**

| n | $T_{11}$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $Q_{rm}$ | $Q_m$ | $Q_r$ | $TC_{rm}$ | $TC_m$ | $TC_r$ | TC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.110 | 0.110 | 0.2167 | 0.1628 | 0.1640 | 249.99 | 250.96 | 224.21 | 6419.2 | 8025.9 | 9135.8 | 23580.9 |
| **2** | **0.127** | **0.127** | **0.2498** | **0.0469** | **0.1415** | **289.29** | **290.65** | **121.75** | **6405.5** | **8048.4** | **9066.6** | **23520.5** |
| 3 | 0.138 | 0.138 | 0.2710 | 0.004 | 0.1322 | 314.63 | 316.29 | 83.75 | *6401.5* | 8055.5 | 9071.3 | 23528.3 |
| 4 | 0.147 | 0.147 | 0.2878 | -0.018 | 0.1270 | 334.82 | 336.74 | 63.64 | 6401 | 8062.3 | 9092.8 | 23556.1 |

Note : Bold is the feasible total joint cost of an inventory system.

Blue(italics) solution is optimum from manufacturer end and green solution is optimum from raw material point.

**Table 2: Cost Variation for n=2, 3 and 1**

| Cost | (1) n=2 | (2) n=3 | (3) n=1 | **(2)-(1)** | **(3)-(1)** |
|---|---|---|---|---|---|
| T | 0.37 | 0.409 | 0.3268 | 0.04 | -0.043 |
| **Raw material's** | | | | | |
| OC | 90 | 90 | 90 | 0 | 0 |
| IHC | 11.07 | 13.04 | 8.28 | 1.97 | -2.79 |
| PC | 2314.28 | 2517.06 | 1999.9 | 203 | -314.4 |
| Total= | 2415.35 | 2620.05 | 2098.1 | 205 | -317.3 |
| TCrm | 6405.50 | 6303.2 | 6419.20 | -102 | 13.7 |
| **Manufacturer's** | | | | | |
| OC | 100 | 100 | 100 | 0 | 0 |
| IHC | 28.28 | 34.07 | 13.64 | 5.79 | -14.64 |
| PC | 2906.48 | 3162.92 | 2509.6 | 256 | -396.9 |
| RW | 0.009 | 0.011 | 0.0067 | 0 | -0.002 |
| Total= | 3034.76 | 3297.02 | 2623.26 | 262 | -411.5 |
| TCm | 8048.38 | 8055.51 | 8025.86 | 7.13 | -22.52 |
| **Retailer's** | | | | | |
| OC | 80 | 80 | 80 | 0 | 0 |
| IHC | 0.84 | 0.023 | 10.08 | -0.82 | 9.24 |
| PC | 1461.03 | 1005 | 2690.45 | -456 | 1229.4 |
| SC | 60.43 | 53.69 | 81.22 | -6.74 | 20.79 |
| LS | 107.02 | 99.89 | 124.21 | -7.13 | 17.19 |
| TC/order | 1709.35 | | 2986.05 | -1709 | 1276.7 |
| Total=n*TC/order | 3418.69 | 3712.76 | 2986.05 | 294 | -432.6 |
| TCr | 9066.60 | 9071.40 | 9135.8 | 4.8 | 69.2 |
| GTC= | 23520.5 | 23528.31 | 23580.9 | 7.81 | 60.4 |

**Table 3: Sensitivity Analysis**

| | **-10 % changed** | | | | | **-5 % changed** | | | |
|---|---|---|---|---|---|---|---|---|---|
| Parameter | $T_2^o$ | $T_4^o$ | $TC^o$ | PCI | | $T_2^o$ | $T_4^o$ | $TC^o$ | PCI |
| Γ | 0.2344 | 0.1414 | 23522.93 | 0.01 | | 0.2425 | 0.1414 | 23521.14 | 0.002 |
| A | 0.2683 | 0.1441 | 21257.63 | -9.621 | | 0.2588 | 0.1428 | 22390 | -4.806 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| B | 0.254 | 0.1439 | 23486 | -0.147 | | 0.2522 | 0.1427 | 23502.80 | -0.075 |
| C | 0.2507 | 0.1423 | 23516.50 | -0.017 | | 0.2503 | 0.1419 | 23518.60 | -0.0084 |
| $A_{mo}$ | 0.2453 | 0.1409 | 23496.60 | -0.10 | | 0.2476 | 0.1412 | 23508.60 | -0.0509 |
| $A_{ms}$ | 0.2447 | 0.1408 | 23493.23 | -0.116 | | 0.2473 | 0.1412 | 23506.91 | -0.0581 |
| $A_r$ | 0.2417 | 0.1404 | 23477.60 | -0.1827 | | 0.2457 | 0.1409 | 23498.70 | -0.0930 |
| $h_{mr}$ | 0.2504 | 0.1416 | 23517.10 | -0.0147 | | 0.2501 | 0.1415 | 23519.20 | -0.0059 |
| $h_{mf}$ | 0.2511 | 0.1418 | 23512.90 | -0.0326 | | 0.2505 | 0.1417 | 23516.60 | -0.0169 |
| $h_r$ | 0.2503 | 0.1413 | 23520.10 | -0.0020 | | 0.2501 | 0.1414 | 23520.30 | -0.0012 |
| $PI_{rB}$ | 0.2465 | 0.1497 | 23486 | -0.147 | | 0.2482 | 0.1455 | 23503.70 | -0.0718 |
| $PI_{rL}$ | 0.2413 | 0.1476 | 23461.60 | -0.2507 | | 0.2457 | 0.1446 | 23491.20 | -0.1249 |
| $C_{mrw}$ | 0.2498 | 0.1415 | 23520.10 | -0.0020 | | 0.2498 | 0.1415 | 23520.15 | -0.0020 |
| $C_{mr}$ | 0.25256 | 0.1419 | 22906.40 | -2.6112 | | 0.25117 | 0.14174 | 23213.20 | -1.3069 |
| $C_{mf}$ | 0.2542 | 0.1422 | 22749.50 | -3.2783 | | 0.2520 | 0.1419 | 23135 | -1.6393 |
| $C_{rf}$ | 0.2674 | 0.1282 | 22735.36 | -3.3384 | | 0.2590 | 0.13501 | 23130.30 | -1.6593 |
| B | 0.1727 | 0.1851 | 23030.30 | -2.0845 | | 0.2199 | 0.1634 | 23315.90 | -0.8702 |
| M | 0.2498 | 0.1415 | 23520.50 | -0.0003 | | 0.2498 | 0.1415 | 23520.30 | -0.0003 |
| D | 0.2498 | 0.1415 | 23520.10 | -0.002 | | 0.2498 | 0.1415 | 23520.30 | -0.0012 |
| Θ | 0.2516 | 0.1415 | 23511.80 | -0.0373 | | 0.2507 | 0.1415 | 23516.70 | -0.0164 |

| | **+5 % changed** | | | | | **+10 % changed** | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Parameter** | $T_2^o$ | $T_4^o$ | $TC^o$ | PCI | | $T_2^o$ | $T_4^o$ | $TC^o$ | PCI |
| Γ | 0.2563 | 0.1416 | 23519.90 | -0.003 | | 0.2622 | 0.1417 | 23519.30 | -0.005 |
| A | 0.2413 | 0.1403 | 24649.20 | 4.798 | | 0.2332 | 0.1392 | 25776.80 | 9.592 |
| B | 0.2475 | 0.1404 | 23536.90 | 0.069 | | 0.2452 | 0.1393 | 23554.10 | 0.1425 |
| C | 0.2494 | 0.1411 | 23522.40 | 0.0077 | | 0.2489 | 0.1408 | 23524.20 | 0.0154 |
| $A_{mo}$ | 0.2520 | 0.1418 | 23531.90 | 0.0481 | | 0.2542 | 0.1421 | 23544 | 0.0996 |
| $A_{ms}$ | 0.2523 | 0.1419 | 23533.33 | 0.0542 | | 0.2548 | 0.1422 | 23546.83 | 0.1116 |
| $A_r$ | 0.2537 | 0.1421 | 23541.70 | 0.0897 | | 0.2577 | 0.1426 | 23562.10 | 0.1765 |
| $h_{mr}$ | 0.2495 | 0.1415 | 23521.50 | 0.0039 | | 0.2492 | 0.1414 | 23523.20 | 0.0111 |
| $h_{mf}$ | 0.2491 | 0.1414 | 23524 | 0.0145 | | 0.2485 | 0.1412 | 23527.60 | 0.0298 |
| $h_r$ | 0.2496 | 0.1417 | 23520.50 | -0.0003 | | 0.2494 | 0.1418 | 23520.60 | 0.0000 |
| $PI_{rB}$ | 0.2513 | 0.1378 | 23535.40 | 0.0630 | | 0.2526 | 0.1342 | 23550.50 | 0.1272 |
| $PI_{Rl}$ | 0.2537 | 0.1385 | 23548.20 | 0.1174 | | 0.2574 | 0.1354 | 23575.30 | 0.2327 |
| $C_{mrw}$ | 0.2498 | 0.1415 | 23520.15 | -0.0020 | | 0.2498 | 0.1415 | 23520.15 | -0.0020 |
| $C_{mr}$ | 0.24847 | 0.14136 | 23827.30 | 1.3040 | | 0.2472 | 0.1412 | 24134.20 | 2.6088 |
| $C_{mf}$ | 0.2477 | 0.1413 | 23905.60 | 1.6369 | | 0.2456 | 0.14096 | 24291 | 3.2755 |
| $C_{rf}$ | 0.2397 | 0.1476 | 23905.20 | 1.6352 | | 0.2287 | 0.1532 | 24284.40 | 3.2475 |
| B | 0.2698 | 0.1200 | 23676.20 | 0.6616 | | 0.2831 | 0.0992 | 23796.90 | 1.1748 |
| M | 0.2498 | 0.1415 | 23520.50 | -0.0003 | | 0.2498 | 0.1415 | 23520.20 | -0.0016 |
| D | 0.2498 | 0.1415 | 23520.30 | -0.0012 | | 0.2498 | 0.1415 | 23520.30 | -0.0012 |
| Θ | 0.2489 | 0.1415 | 23524.26 | 0.0156 | | 0.2480 | 0.1415 | 23528.53 | 0.0338 |