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DETERMINATION OF THE DEFLECTION OF THE VERTICAL USING A GRAVIMETRIC APPROACH WITHIN THE FEDERAL UNIVERSITY OF TECHNOLOGY AKURE, NIGERIA

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Abstract:

Deflection of the vertical is the angle between the true vertical (the direction of gravity) at a point on the Earth's surface and the normal to the reference ellipsoid (the idealized mathematical model of the Earth's shape). It arises due to the Earth's irregular mass distribution, which causes local variations in the direction of gravity. This research investigates the deflection of the vertical at the Federal University of Technology Akure using a gravimetric approach to determine geoidal undulation. Gravity data from 44 geodetic control stations were observed. The geoidal height and deflection of the vertical were calculated through the discrete wavelet decomposition method in MATLAB, with comparisons to ICGEM data. Geoidal heights at each station were also computed using the gravimetric (Stokes integral) approach, with results compared between methods. The study further evaluated high-resolution global geoid models, including EGM 2008, GECO, SGG-UGM-1, SGG-UGM-2, and XGM 2019e_2156. The result reveals that EGM 2008 has the lowest standard deviation of 0.3197m and SGG-UGM-2 with the lowest root mean square error (RMSE) of 0.4787m. For lower-resolution models (GOCO06S, GOSG01S, IGGT_R1, GGM05G, EIGEN 5C), the standard deviation and RMSE differences were also minimal, with EIGEN5C at 0.3180m and IGGT_R1 at 0.3137m. A z-test indicated significant differences between geoidal heights derived from the gravimetric and wavelet methods, leading to the rejection of the null hypothesis. Therefore, discrete wavelet decomposition should be adopted as an alternative method for computing the deflection of the vertical and geoidal heights when using a gravimetric approach.

Keywords: Geoidal height, Deflection of the verticals, Global Geoid Model, Discrete wavelet decomposition method.

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1.0 INTRODUCTION

Deflection of the vertical is the angular components between the true zenith nadir curve (plumb line and the normal vector to the surface of the reference ellipsoid chosen to approximate the Earth's sea-level surface) [1,17]. The deflection of the vertical which represents the angular separation between the geodetic normal and the vertical to the geoid, can also be represented as the slope of the geoid with respect to the ellipsoid [8,3].

Gravity is a fundamental force of nature that causes objects with mass to attract one another. In geophysical terms, it is the force due to the integrated mass of the whole Earth, which acts on the mechanism of a measuring instrument. Measurements are usually made at the surface of the Earth, in aircraft, or on ships. They may also be made in mines or on man-made structures. The gravity field in space may be inferred from the orbit of a satellite. The measuring instrument may be a very precise spring balance, a pendulum, or a small body falling in a vacuum [1].

Calculating the components of the deflection of the vertical at a point situated on the earth's surface from gravity anomalies which are known on the same surface is one of the classical problems in physical geodesy since the day of C.G. Stokes [4]. The well-known formula that was derived by Stokes for this purpose is theoretically beautiful but unfortunately, it cannot be used in its original form at present because of our rather poor knowledge concerning the gravity anomaly distribution all over the world, which is essential for the application of that method [15].

The applicability of spectral techniques for the computation of deflections of the vertical shows that an accuracy better than 1′ can be achieved, and that gravimetric methods can replace the time-consuming astro-geodetic methods for many purposes [8]. In practice, the vertical deflection can be determined in two ways, from astronomical observations, astronomical latitude (ϕ) and longitude (λ) and coordinates of a geodetic network, geodetic latitude (B) and longitude (L), or from the gravity data. It is conventionally divided into two perpendicular components; a north-south meridional component (ξ) and an east-west prime vertical component (η) [17]. If in a geodetic network, an astronomical position (ϕ, λ) and geodetic position (B, L) are determined at a geodetic station, thus the components of vertical ξ and η can be easily determined. If we have gravity data with sufficient density and accuracy, the deflection of vertical can be computed from the Vening-Meinesz integral [6]. The second approach for evaluating the discrete Vening-Meinesz integral is to use the fast Fourier transform (FFT) method [5]. In addition to its speed, this technique allows for the evaluation of the discrete Vening-Meinesz integral for all the points on a regular grid using all the data available in a large region simultaneously.

[16,4,12,9,13] all used the astro-geodetic method to determine the deflection of the vertical at Bavarian Alps and Switzerland. The results were found to be better than the Earth Gravity Model (EGM) 2008 result. All these research works were based on astro-gravimetric data.

Several researchers [18,14,12,10,11] compute geoidal height and components of deflections of the vertical, using Stokes integral, Fast Fourier transform and least square method, the need to improve the already existing methods and establishing the reliability of the new models for the computation poses the motivation for this study.

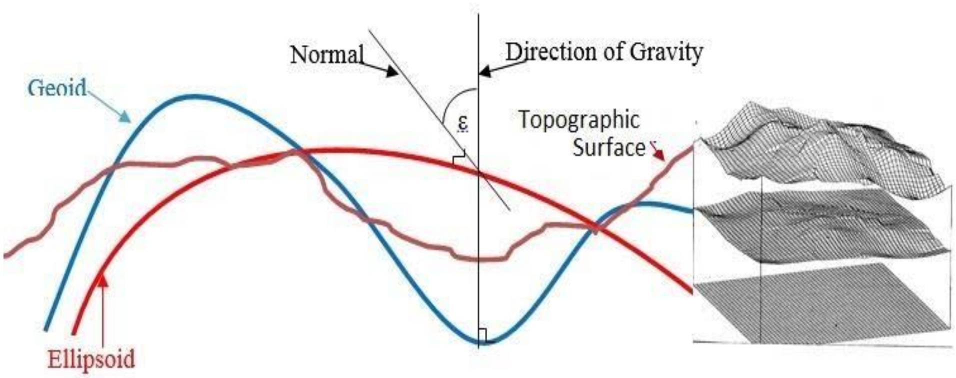

Figure 1: Reference surface [18]

Studies over the years have proven the positive contributions of computer algorithms and technology applications in solving geodetic integration problems and providing optimum results from time to time. However, there are some identified gaps in support of this research. It was established that the Wavelet Transform (WT) performs better than the Fast Fourier Transform but most of the research works above only used the FFT instead of the WT. [7] on the other hand used the Daubechies4 (db4) Discrete Wavelet Transformation (DWT) to compute the local geoid. Moreover, the deflections of the vertical at the study area have never been determined using the gravimetric method. This research will be using the wavelet transformation technique to perform Stoke's Integral function for the determination of the deflections of the vertical within the Federal University of Technology Akure Campus.

2.0 THE STUDY AREA

Akure is the host town of the study area 'Federal University of Technology, Akure (FUTA)' a protected academic area. It is geographically geo-referenced on coordinate lines of latitude 7° 17' 42.617" N to 7° 18' 55.15" N and longitude 5° 07' 01.57" E to 5° 08' 30.486" E of the WGS84 Zone 31N coordinate system on the Eastern flank of meridians. The University is one of the Federal Universities of Technology established by the Federal Government of Nigeria in 1981 in a quest for technological development. Down to the south is the Aule Community; up in its north is Ipinsa Community Lands interposed by the Akure Ilesa Expressway, on the west Ilara and Ibule settlements while to the east is the southern part of Akure Metropolis in Akure South Local Government Area in Ondo State, Nigeria; all these areas are made up of FUTA environs. The elevation range is between 356m to 395m. Observations are to be carried out on some coordinated points within the school campus from Obakekere to Obanla area.

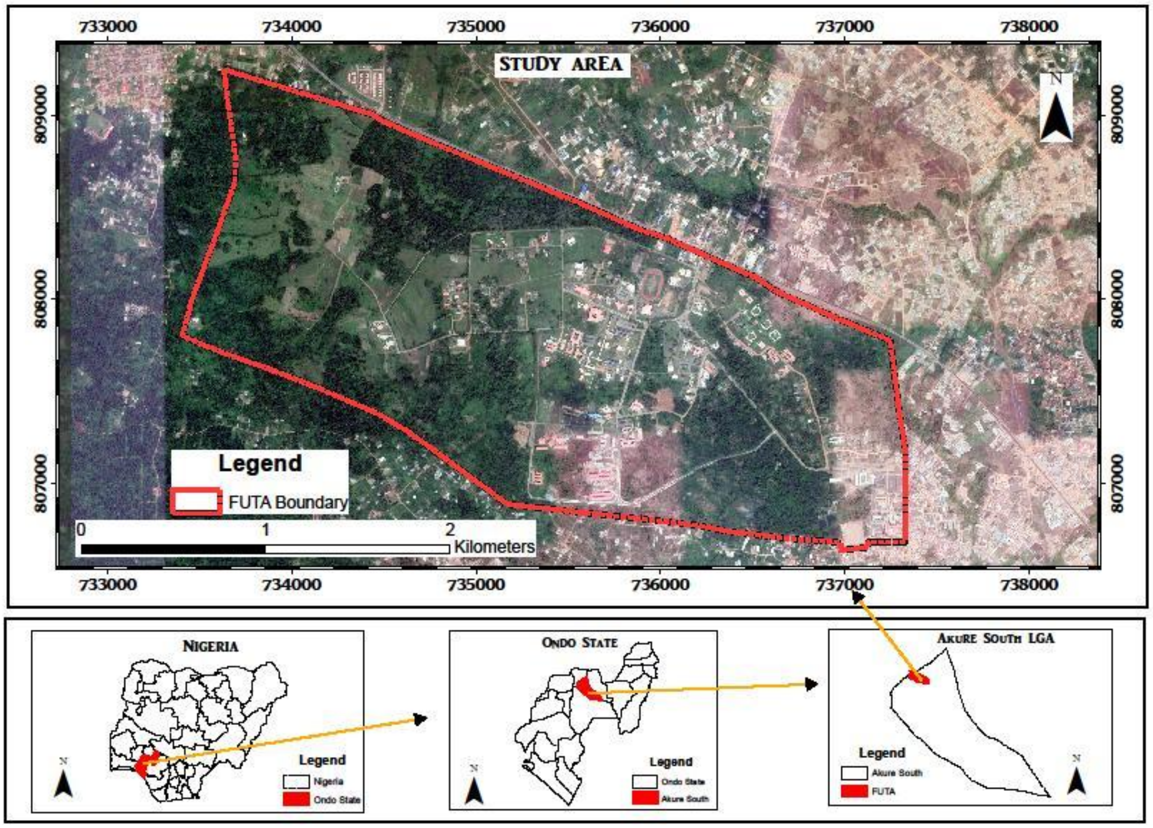

Figure 2: Map showing the study area
Source: [19].

3.0 MATERIALS AND METHOD

This research aims to determine the deflections of the vertical using a Gravimetric approach with a view to determining geoid height within The Federal University of Technology Akure campus. Gravity data for 44 geodetic control stations within the University campus was acquired by the Department of Surveying and Geoinformatics Federal University of Technology, Akure. Gravity anomaly, Δg, is the difference between the observed gravity value (g) reduced to the geoid, and a normal gravity value (γ_0) at the mean earth ellipsoid. The gravity anomaly Δg of the observed stations was computed for each geodetic station using Equation 4, the geoid height using Equation 5, and the deflections of the vertical of each of the geodetic stations were computed from the gravity anomalies using the discrete wavelet decomposition method. The normal gravity of each of the gravity stations was computed using the international gravity formula of the geodetic system 1930 for normal gravity computation as given by Tata and Ono, (2018). The normal gravity (γ_0) was computed using equation 3.
The computations were done in MATLAB software environment and values of the corresponding deflection of the verticals and geoid height were compared with ICGEM data for the analysis. Figure 3.1 shows the flowchart of the research methodology.

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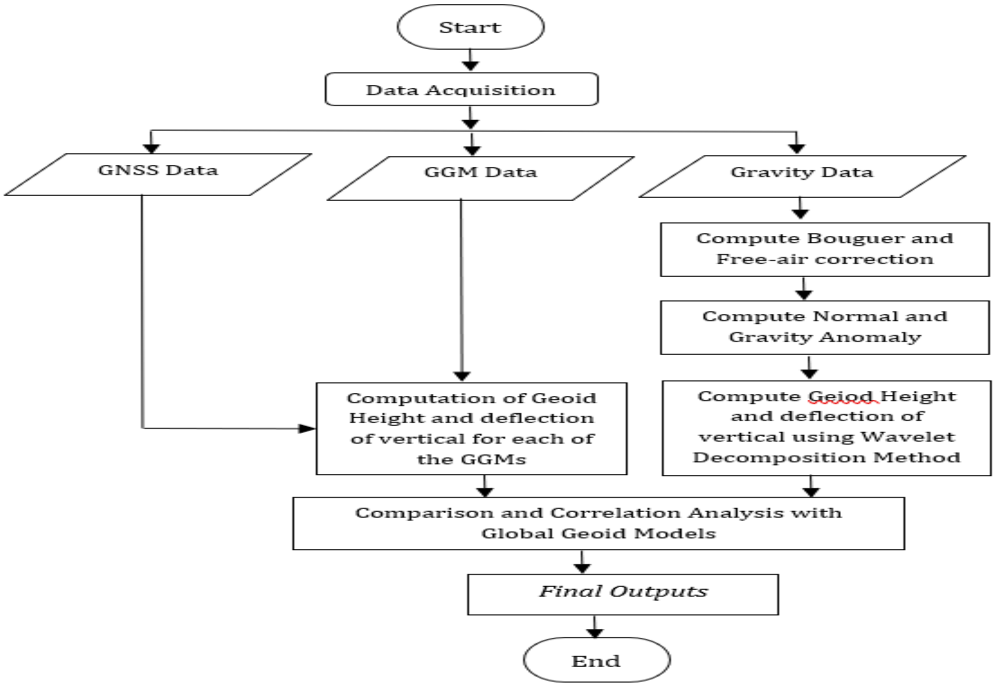

Figure 3: Methodology Flow Chart.

The gravity data obtained for this research was sourced from a secondary source, the data set contains the longitudes, latitudes, elevations, gravity values, and time. Five high and low resolution Global Geoid Models (EGM2008, XGM2019e_2159, SGG-UGM-1, SGG-UGM-2, GECO,GOCO06S, GOSG01S, IGGT_R1, GGM05G, EIGEN 5C) were chosen for the analysis. The geoidal height and the deflection of the verticals of these selected controls were computed for each model and were compared with the corresponding discrete wavelet decomposition method. Additionally, geoid height obtained from the gravimetric approach (stoke integral) was then compared with the discrete wavelet decomposition method.

Forty-four existing control points within the study area were subjected to GPS observation. The Tersus Differential Global Positioning System (DGPS) was utilized in static mode for one hour to determine the positions and ellipsoidal heights of each point. The observed data were processed using a Tersus GPS processor to obtain the latitude and longitude of the control points.

3.1 Global Geoid Models

The web-based gravity data for ten (10) Global Geopotential Models (GGMs)were obtained from the International Centre for Global Earth Models (ICGEM) website. Five of the Models were of high resolution while Five models were of low resolution. Tables 1 and 2 shows the five high-resolution GGMs and Five low-resolution GGMs respectively.

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The following are the high-resolution Global Gravity Models (GGMs) that were used during the research for the comparison of the deflection of the vertical components. These models provide spatial resolution sufficient for detailed studies in geodesy, oceanography, and Earth sciences.

Table 1: High-resolution Global Gravity Models that were used during the research.

S/N	Models	Degree and Order	Spatial Resolution (Approx.)	Year
1.	SGG-UGM-2	2190	9km	2020
2.	XGM 2019e	2190	9km	2019
3.	SGG-UGM-1	2159	9.2km	2018
4.	GECO	2190	9km	2015
5.	EGM 2008	2190	9km	2008

The following are the low-resolution Global Gravity Models (GGMs) that were used during the research for the comparison of the deflection of the vertical components. These models address broad-scale phenomena with reduced computational complexity and noise, they can also serve as baselines in the absence of detailed data.

Table 2: Low-resolution GGMs that were used during the research.

S/N	Models	Degree and Order	SpatialResolution (Approx.)	Year
1.	GOCO06S	300	133km	2019
2.	GOSG01S	220	182km	2018
3.	IGGT_R1	240	167km	2017
4.	GGM05G	240	167km	2015
5.	EIGEN 5C	360	111km	2008

The Global Geoid Models (GGMs) were downloaded from the International Center for Global Gravity Field Models (ICGEM) (Potsdam Germany) website (http://icgem.gfz-potsdam.de). On this website, several GGMs are available in the form of fully normalized spherical harmonic coefficients that can be used to compute geodetic and Earth's gravity field quantities. The procedure for downloading is as follows:

Open your browser and type the URL http://icgem.gfz-potsdam.de. After the page has been fully loaded Click on the static model. It will display a table of several earth models. Then navigate to each of the Global Geoid Model (EGM2008, XGM2019e_2159, SGG-UGM-1, SGG-UGM-2, GECO,GOCO06S, GOSG01S, IGGT_R1, GGM05G, EIGEN 5C) respectively.

Then click on Show to open another page. Then zoom the map on the grid selection to your project location or your area of interest. Then input the coordinates of the study location in the space provided, longitude and latitude. Choose your reference ellipsoid WGS 84 and check all its parameters to avoid blunder. Radius is used interchangeably with semi-major axis in this context which is 6378137m and Flattening is 1/298.257223563.

Then select geoid and deflection of vertical under functional selection. Then click on Start Computation to compute the geoid height and deflection of vertical for each of the gravity stations. After the computation is done, then click on the download grid to download the result.

3.2 Gravity Reductions

The observed gravity values acquired from the secondary source were subjected to Free-Air, Bouguer corrections. The Free-Air and Bouguer corrections were added to and subtracted from the observed gravity values respectively using the equation 3.1 and 3.2 respectively given by Hofmann-Wellenhof and Moritz, (2005):

$$FC = -\frac{dg}{dh} H \approx -\frac{dg}{dh} H_p \approx +0.3086Hp \quad 1$$

$$BC = 2\pi G\rho H \approx 0.1119\, HP\, [mGal] \quad 2$$

3.3 Normal Gravity Computation

The normal gravity of each of the gravity stations was computed using the international gravity formula of the geodetic system 1930 for normal gravity computation as given by Tata and Ono, (2018). The normal gravity (γ_0) is computed as

$$\gamma_0 = 9.78049(1 + 0.0052884\sin^2\varphi - 0.0000059\sin^2 2\varphi)\ \mathrm{ms}^{-2} \quad 3$$

3.4 Gravity Anomaly Computation

The gravity anomaly, Δg, is the difference between the observed gravity value (g) reduced to the geoid, and a normal gravity value (γ_0) at the mean earth ellipsoid. The gravity anomaly Δg of the observed stations is determined as

$$\Delta g = g - \gamma_0 \quad 4$$

3.5 Computation of Geoid Height (N) Using Gravimetric Approach (stoke integral)

Geoid height of the gravity stations was computed using the gravimetric approach.

$$N^{gravimetric} = h - H \quad 5$$

H and h are the orthometric and ellipsoidal heights of points, respectively. The former is obtained from the GPS observation. This computation was done using Microsoft Excel.

3.6 Computation of Deflection of Vertical and Geoid Height Using Discrete Wavelet Decomposition

The two integrals evaluated using the wavelet thresholding technique are the Stokes integral for the determination of geoid undulation and the VeningMeinesz for the orthogonal components of the deflection of the vertical (Heiskanen and Moritz, 1967). The Integrations of Stoke's and VeningMeinesz's were performed using a 2D Discrete Wavelet Transformation (2D DWT) function, an efficient signal processing function for solving boundary value problems other than the Fast Fourier Transformation (FFT) method.

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At first, the kernel functions of the Stoke's and the Veining Meinesz's Integrals were computed using equations 6, and 7 considering the planar coordinates of computation and data position are in singularity.

$$N_{(x_2,y_2)} = \frac{1}{2\pi\gamma} \iint \Delta g_{(x_1,y_1)} K_{N((x_1,\, y_1,\, x_2,\, y_2)} dxdy \qquad 6$$

$$\xi_{(x_2,y_2)} = \frac{1}{2\pi\gamma} \iint \Delta g_{(x_1,y_1)} K_{\xi((x_1,\, y_1, x_2, y_2)} dxdy \qquad 7$$

The kernel functions and the gravity anomalies were then arranged in a squared grid in the order of data acquisition. Both sets of data were integrated, but for the purpose of this research, they were transformed and decomposed into wavelet signals using the 2D Discrete Wavelet Transformation method. Thereafter, the wavelet coefficients of the two transformed and decomposed data were multiplied. Inverse 2D Discrete Wavelet transformation was performed on the data to yield the geoid height and the deflection of the vertical at the meridian.

A MATLAB script was developed to compute the geoid height and deflection of the vertical at the meridian. The step-by-step process for this computation is outlined as follows:

i. Start
ii. Input the gravity values and coordinates
iii. Compute bouguer correction
iv. Compute free-air correction
v. Compute Normal Gravity
vi. Compute Gravity anomaly
vii. Perform 2D Discrete wavelet transform of the gravity anomalies using **wavedec2**inbuilt function
viii. Compute the wavelet transform of the planar kernel for geoid height and deflection of vertical
ix. Extract and compute the approximate and detailed coefficients of the transformed data using **appcoef2** and **detcoef2** inbuilt function for geoid height and deflection of vertical
x. Compute the summation of dyadic intervals of the approximate and detailed coefficients for geoid height and deflection of vertical
xi. Compute the Inverse wavelet transform of the summation results using **idwt2**inbuilt function for geoid height and deflection of vertical.
xii. Print the geoid height values and deflection vertical values at each gravity stations
xiii. End

4.0 RESULTS AND DISCUSSION

Table 3: showing results of the gravity, normal gravity, and gravity anomaly of some gravity stations within the study area (an Extract).

Station	Latitude	Longitude	Orthometric Height	Gravity (mGal)	Normal Gravity (mGal)	Gravity Anomaly
T3	7.3007	5.1315	339.089	978049.212	978131.9862	151.0217
T2	7.2952	5.1310	337.944	978049.842	978132.0225	151.8796
T1	7.2965	5.1394	353.803	978056.002	978132.1131	161.5301
SVG1758	7.3025	5.1431	347.491	978049.687	978132.0655	152.4388
SVG1757	7.3020	5.1422	345.477	978050.038	978132.0611	151.0218
SVG1756	7.3035	5.1406	347.718	978050.085	978132.0632	150.1261
SVG1755	7.3046	5.1401	351.525	978051.234	978132.1381	154.0115
SVG1638	7.2930	5.1468	336.270	978051.155	978132.1063	151.6231
SVG1637	7.2947	5.1494	337.249	978052.355	978132.1289	154.0115

Table 4: Extract of 20 points from 44 GCPs for the study area showing Geoid Height and Deflection of Vertical computed using Discrete Wavelet Decomposition Method and stoke integral.

S/N	Point name	Longitude	Lattitude	Orthometric height (m)	Computed geoid height using stoke integral (m)	Computed geoid height using the wavelet method (m)	Deflection of vertical using wavelet method (arcseconds)
1	T3	5.132	7.301	339.090	25.166	24.259	0.162
2	T2	5.131	7.295	337.944	25.675	24.211	0.196
3	T1	5.139	7.296	353.804	29.292	24.877	0.250
4	SVG1758	5.143	7.302	347.491	27.758	24.612	0.259
5	SVG1757	5.142	7.302	345.478	27.129	24.527	0.258
6	SVG1756	5.141	7.304	347.718	27.846	24.622	0.251
7	SVG1755	5.140	7.305	351.525	28.871	24.782	0.258
8	SVG1638	5.147	7.293	336.271	23.422	24.140	0.248
9	SVG1637	5.149	7.295	337.249	23.939	24.181	0.253
10	SVG1636	5.145	7.297	338.553	23.971	24.236	0.259
11	SVG1634	5.146	7.264	335.305	23.841	24.100	0.256
12	SVG1633	5.145	7.296	333.369	23.659	24.018	0.255
13	SVG1630	5.142	7.300	340.502	25.176	24.318	0.252
14	SVG1629	5.141	7.301	343.882	28.034	24.460	0.247

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15	SVG1628	5.140	7.302	348.484	28.085	24.654	0.242
16	SVG1525	5.139	7.301	350.361	28.366	24.733	0.245
17	SVG1524	5.140	7.302	348.969	28.254	24.674	0.249
18	SVG1522	5.139	7.304	353.259	29.299	24.855	0.253
19	SVG1521	5.139	7.305	352.636	29.366	24.828	0.252
20	SVG1520	5.140	7.306	352.110	29.129	24.806	0.245

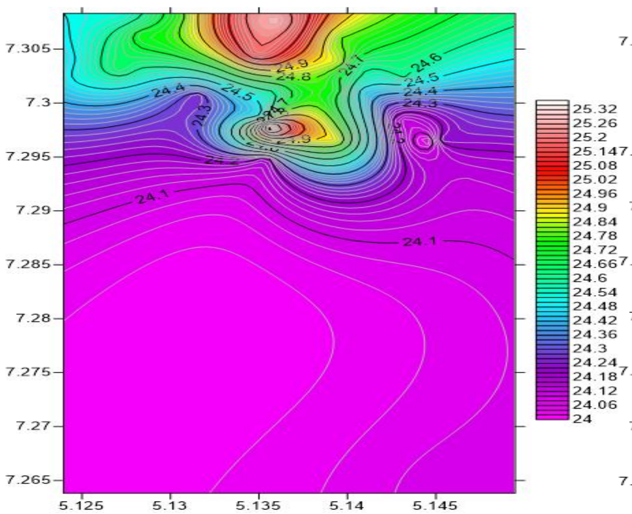

Figure 4a: Contour plot of geoid height by wavelet method

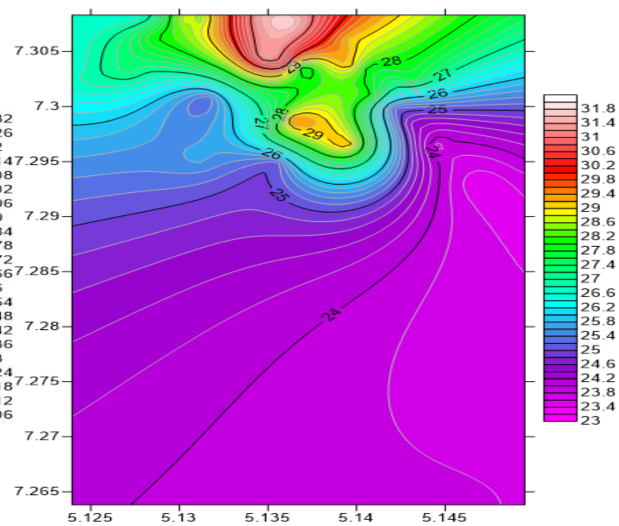

Figure 4b: Contour plot of geoid height by stoke integral.

Figure 5 shows the graphical representation of the Geoid height values computed using stoke integral and discrete wavelet decomposition. From Figure 4.1, there is a high disparity in the geoid height values computed using the stoke integral and those computed using the discrete wavelet method at some gravity stations while at some gravity stations, there is little disparity in the geoid height values. Also from Figure 5, the discrete wavelet decomposition method shows a fair representation of the terrain.

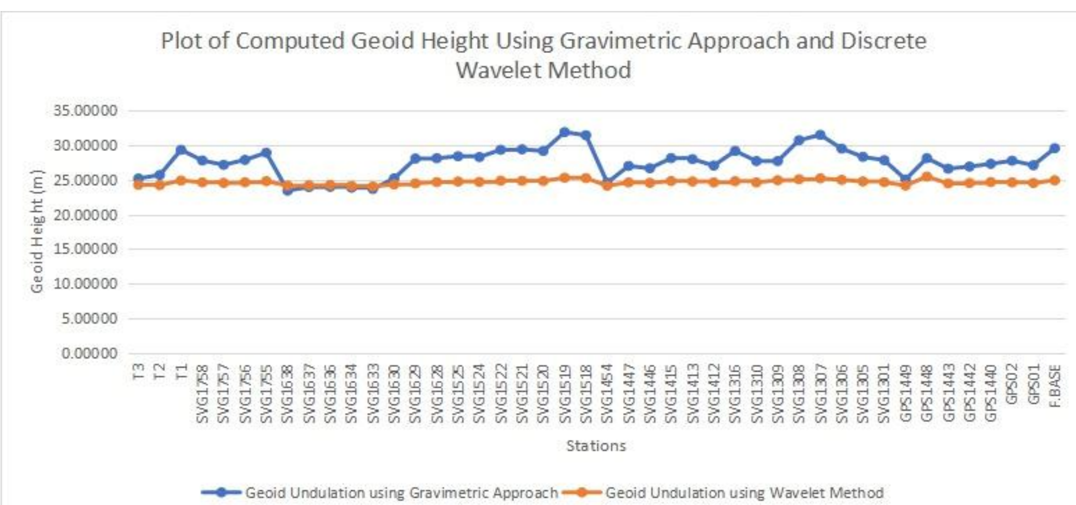

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Figure 5: Graphical representation of Computed Geoid Height Using Stoke Integral and Discrete Wavelet Method.

Table 5: ICGEM Result for Geoid Height and Deflection of Vertical for High Resolution Global Geoid Models

Coordinates		EGM 2008		GECO		SGG-UGM-1		SGG-UGM-2		XGM2019e_2156	
Lat.	Long.	Geoid Height (m)	Defl. of Vertical (arcseconds)	Geoid Height (m)	Defl. of Vertical (arcseconds)	Geoid Height (m)	Defl. of Vertical (arcseconds)	Geoid Height (m)	Defl. of Vertical (arcseconds)	Geoid Height (m)	Defl. of Vertical (arcseconds)
7.301	5.132	25.297	0.201	25.042	0.184	24.997	0.200	24.995	0.196	25.115	0.198
7.295	5.131	25.278	0.204	25.026	0.187	24.979	0.203	24.977	0.200	25.096	0.202
7.297	5.139	25.281	0.203	25.027	0.187	24.983	0.202	24.980	0.199	25.101	0.200
7.303	5.143	25.300	0.200	25.043	0.183	25.002	0.199	24.998	0.196	25.120	0.196
7.302	5.142	25.298	0.200	25.042	0.183	25.000	0.199	24.997	0.196	25.118	0.196
7.304	5.141	25.304	0.199	25.047	0.182	25.006	0.198	25.003	0.195	25.123	0.196
7.305	5.140	25.308	0.198	25.051	0.182	25.009	0.198	25.006	0.194	25.127	0.195
7.293	5.147	25.267	0.206	25.013	0.189	24.970	0.204	24.967	0.201	25.089	0.200
7.295	5.149	25.272	0.205	25.016	0.188	24.975	0.203	24.971	0.200	25.094	0.199

Figure 6 shows the graphical representation of the Geoid height values computed using the discrete wavelet decomposition method and those derived from ICGEM for high resolution. From the figure, there is a high disparity between the geoid height values computed from the discrete wavelet method and the geoid height values computed for each high-resolution model. SGM-UGM-1 is closer to the wavelet method at some gravity stations. There are little differences between geoid height values computed for each of this Global Geoid Model.

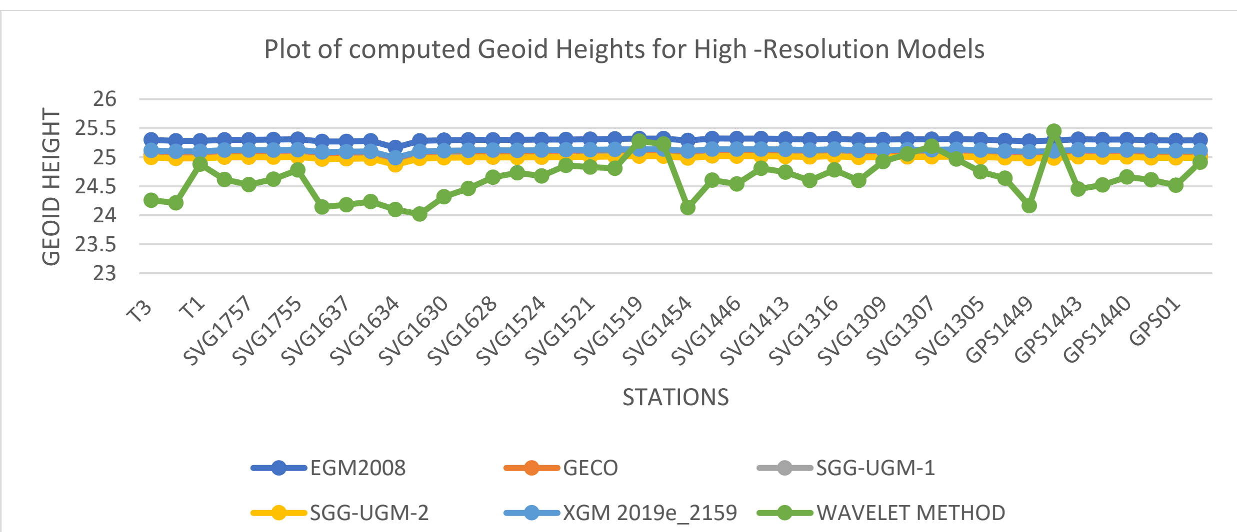

Figure 6: Graphical representation of Computed Geoid Heights for High-Resolution Models

Figure 7 shows the graphical representation of the deflection of vertical values computed using the discrete wavelet decomposition method and those derived from ICGEM for high resolution at the prime meridian. From the figure, there is a high disparity in the deflection of vertical values computed from the discrete wavelet method and the deflection of vertical values computed for each high-resolution model. The deflection of vertical values computed for each model shows slight variations.

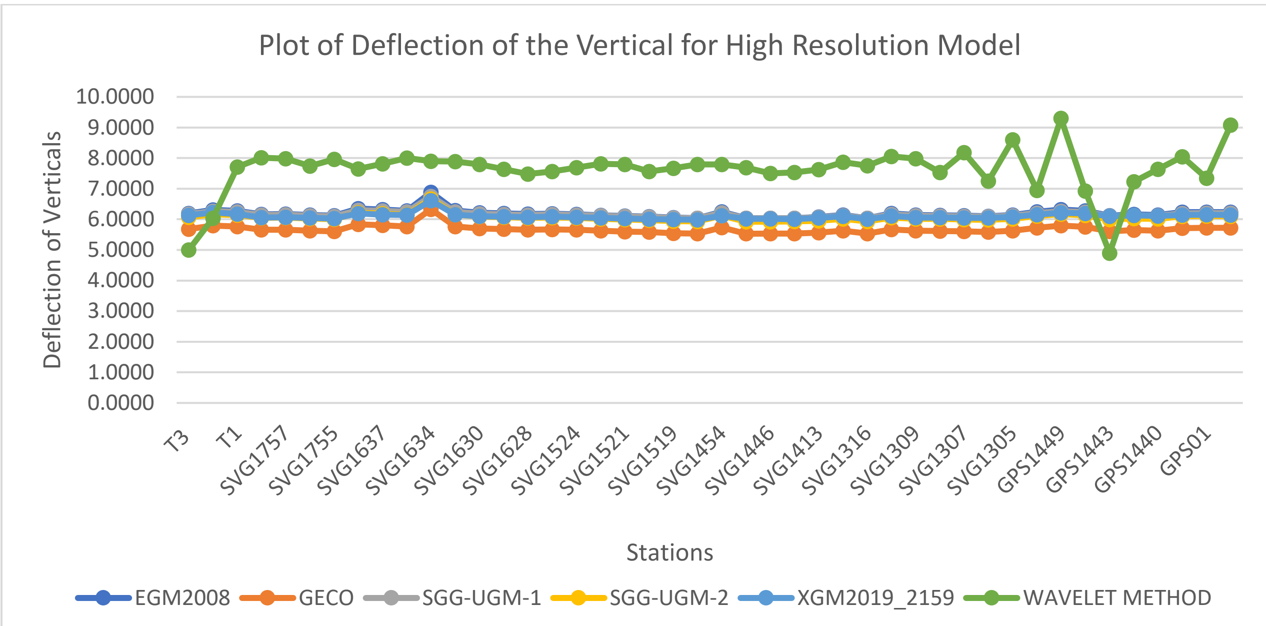

Figure 7: Graphical representation of Computed Deflection of Vertical for High-Resolution Models

Table 6: ICGEM Result for Geoid Height and Deflection of Vertical for Low-Resolution Global Geoid Models

Coordinates		**IGGT_R1**		**GOSG01S**		**GOCO06S**		**GGM05G**		**EIGEN 5C**	
Lat.	**Long.**	**Geoid Height (m)**	**Defl. of Vertical (arcseconds)**	**Geoid Height (m)**	**Defl. of Vertical (arcseconds)**	**Geoid Height (m)**	**Defl. of Vertical (arcseconds)**	**Geoid Height (m)**	**Defl. of Vertical (arcseconds)**	**Geoid Height (m)**	**Defl. of Vertical (arcseconds)**
7.301	5.132	24.604	0.229	24.806	0.230	24.865	0.187	24.974	0.218	25.221	0.224
7.295	5.131	24.611	0.228	24.812	0.229	24.870	0.187	24.980	0.218	25.227	0.223
7.297	5.139	24.626	0.226	24.831	0.227	24.886	0.187	24.998	0.217	25.253	0.221
7.303	5.143	24.618	0.228	24.819	0.229	24.876	0.186	24.987	0.217	25.233	0.222
7.302	5.142	24.617	0.227	24.819	0.229	24.876	0.186	24.987	0.217	25.236	0.222
7.304	5.141	24.617	0.227	24.820	0.228	24.877	0.187	24.988	0.217	25.238	0.222
7.305	5.140	24.630	0.226	24.833	0.227	24.888	0.186	25.000	0.216	25.252	0.221
7.293	5.147	24.625	0.227	24.828	0.228	24.883	0.186	24.995	0.217	25.246	0.221
7.295	5.149	24.628	0.226	24.832	0.228	24.886	0.186	24.999	0.217	25.250	0.221

Figure 8 shows the graphical representation of the Geoid height values computed using the discrete wavelet decomposition method and those derived from ICGEM for low resolution. The discrete wavelet decomposition can provide a multiresolution analysis, allowing data to be studied at different scales. This capability is particularly valuable for identifying global trends and localized anomalies in datasets such as geoid heights or gravity field models.From the figure, there is a little disparity in the geoid height values computed from the discrete wavelet method and the geoid height values computed for each low-resolution model. The geoid height values computed for each model exhibit only minor differences.

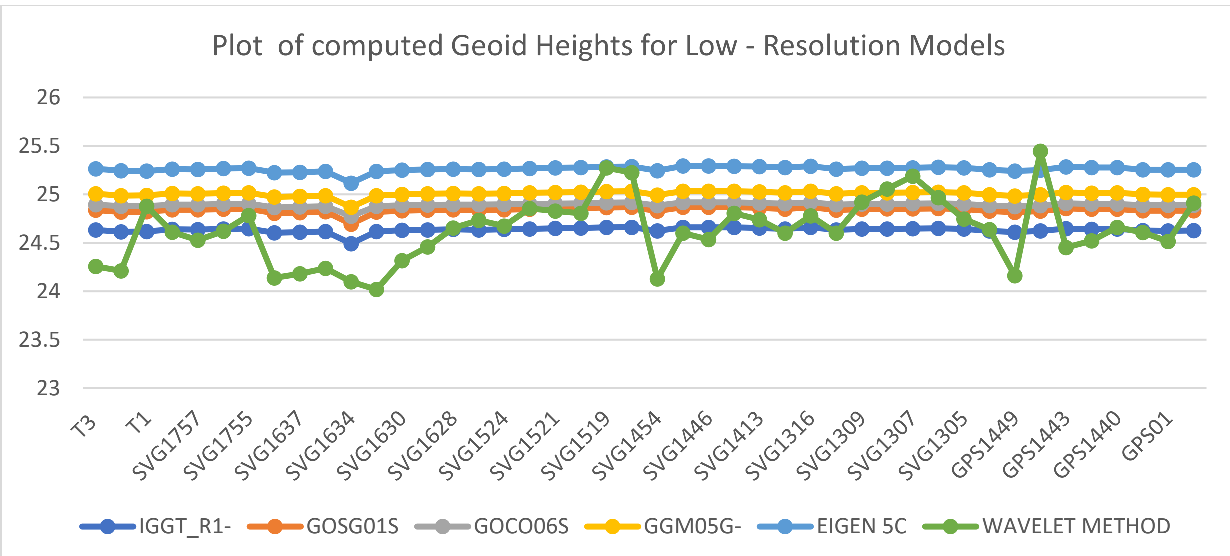

Figure 8: Graphical representation of Computed Geoid Heights for Low-Resolution Models

Figure 9 shows the graphical representation of the deflection of vertical values computed using discrete wavelet decomposition method and those derived from ICGEM for low-resolution. From the figure, there is little disparity in the deflection of vertical values computed from discrete wavelet method and the deflection of vertical values computed for each low-resolution model. GOSG01S is closer to the wavelet method when compared with other low-resolution models. There are minor differences between the deflection of vertical values computed for each model.

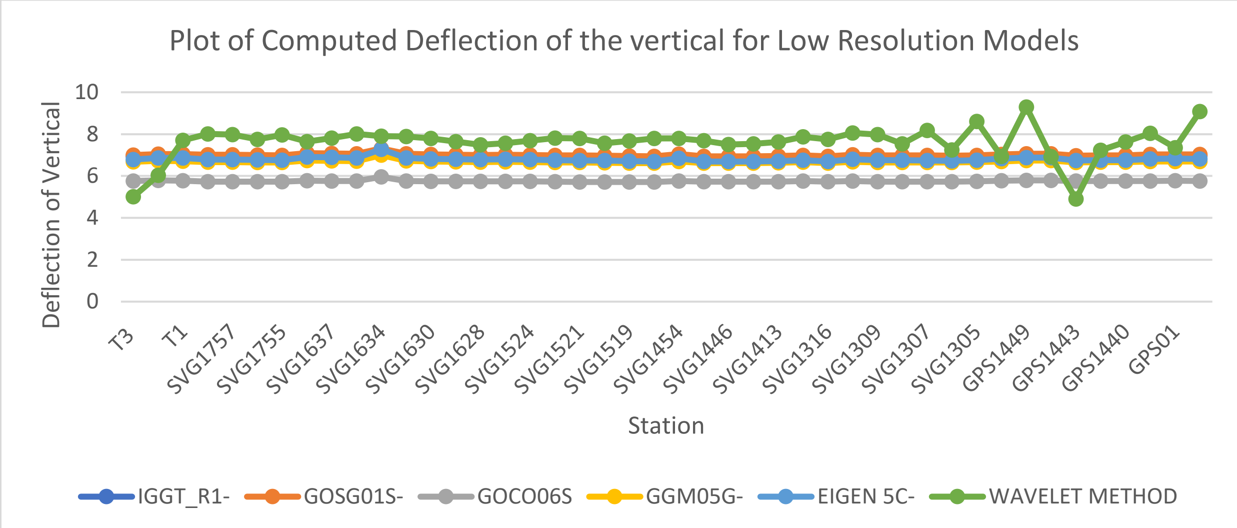

Figure 9: Graphical representation of Computed Deflection of Vertical for Low-Resolution Models

Table 4.5 presents the analysis of the z-test of Geoid height values computed using both the gravimetric approach (stoke integral) and the discrete wavelet decomposition method. The computed z-value (z-computed) for the geoid height value is 12.28055494, and the value for $z_{0.95}(44)$from the z-distribution table is 1.644853627. Since z-computed $>z_{0.95}(44)$, the null hypothesis is rejected and we can conclude that there is a significant difference between the means Geoid height values computed using both gravimetric approach (stoke integral) and discrete wavelet decomposition method.

Table 7: Z- Statistical Test for Geoid height values computed using both gravimetric approach (stoke integral) and discrete wavelet decomposition method

	Computed Geoid Height Using Stoke integral	Computed Geoid Height Using Wavelet Method
Mean	27.52440355	24.63562413
Known Variance	2.1029	0.3318
Observations	44	44
Hypothesized Mean Difference	0	
z computed	12.28055494	
P(Z<=z) one-tail	0	
z Critical one-tail	1.644853627	
P(Z<=z) two-tail	0	
z Critical two-tail	1.959963985	

4.3 Discussion

The gravity anomalies were determined from the computed normal gravity and observed gravity. The gravity anomaly was used in the discrete wavelet decomposition method to compute the deflection of vertical and geoid height of geodetic stations. The Geoid Height and Deflection Vertical of each of the geodetic stations were also computed for high-resolution and low-resolution Global Geoid Models from the International Centre for Global Gravity Field Models (ICGEM) website.

The computed Geoid Height and deflection of vertical using the Discrete Wavelet decomposition method were compared with those derived from ICGEM for both high-resolution and low-resolution Global Geoid Models. Statistics of their differences in geoid height and deflection of vertical were computed in terms of mean deviation, standard deviation, and root mean square error.

The standard deviation of the differences between Geoid height computed using the discrete wavelet decomposition method and those computed from ICGEM for EGM 2008, GECO, SGG-UGM-1, SGG-UGM-2, and XGM 2019e_2156 is 0.3197, 0.3802, 0.3199, 0.3199, and 0.3201 respectively. The root mean square (RMSE) of the differences between Geoid height computed using the discrete wavelet decomposition method and those computed from ICGEM for EGM 2008, GECO, SGG-UGM-1, SGG-UGM-2 and XGM 2019e_2015 is 0.7323, 0.5146, 0.4806, 0.4787, and 0.8559 respectively. The result of the RMSE shows a little disparity between the geoid height computed from each of the GGMs and the geoid height computed using the discrete wavelet method. From the analysis, it is discovered that EGM 2008 has the least mean deviation and standard deviation while SGG-UGM-1 and SGG-UGM-2 have the least root mean square error values when compared with other high-resolution models. This implies that Model SGG-UGM-1 and SGG-UGM-2 are closer in value to the wavelet method than other high-resolution models.

Also, in terms of the deflection of vertical, the standard deviation of the differences between the deflection of vertical computed using discrete wavelet decomposition method and those computed from ICGEM for EGM 2008, GECO, SGG-UGM-1, SGG-UGM-2, and XGM 2019e_2156 is 0.777809, 0.776326, 0.7756, 0.774595, and 0.783599 respectively. The root mean square (RMSE) of the differences between the deflection of vertical computed using the discrete wavelet decomposition method and those computed from ICGEM for EGM 2008, GECO, SGG-UGM-1, SGG-UGM-2 and XGM 2019e_2156 is 1.625841, 2.091604, 1.647842, 1.736896, and 1.125982 respectively. The result of the RMSE shows a high disparity between the deflection of vertical computed from each of the GGMs and the geoid height computed using the discrete wavelet method. From the analysis, it is discovered that EGM 2008 has the least mean deviation, SGG-UGM-2 has the least standard deviation and XGM 2019e_2156 has the least root mean square error values when compared with other high-resolution models. This implies that Model XGM 2019e_2156 is closer in value to the discrete wavelet method than other high-resolution models.

From the statistics of differences in geoid height and deflection of vertical between computed using discrete wavelet decomposition method and those computed from ICGEM for low-resolution GGMs. The standard deviation of the differences in geoid height for IGGT_R1, GOSG01S, GOCO06S, GGM05G, and EIGEN 5C is 0.3189, 0.3184, 0.3207, 0.3189, and 0.3180 respectively. The root mean square (RMSE) of the differences between Geoid height computed using the discrete wavelet decomposition method and those computed from ICGEM for IGGT_R1, GOSG01S, GOCO06S, GGM05G, and EIGEN5C is 0.0.3137, 0.3748, 0.4086, 0.4868and .6999 respectively. The result of the RMSE shows a little disparity between the geoid height computed from each of the GGMs and the geoid height computed using the discrete wavelet method. From the analysis, it is discovered that EIGEN5C has the least mean deviation and standard deviation while IGGT_R1 has the least root mean square error values when compared with other high-resolution models. This implies that Model IGGT_R1 is closer in value to the discrete wavelet method than other low-resolution models.

Also, in terms of the deflection of vertical, the standard deviation of the differences between the deflection of vertical computed using discrete wavelet decomposition method and those computed from ICGEM for IGGT_R1, GOSG01S, GOCO06S, GGM05G, and EIGEN 5C 0.7670, 0.7710, 0.7779, 0.7744 and 0.7721 respectively. The root mean square (RMSE) of the differences between the deflection of vertical computed using discrete wavelet decomposition method and those computed from ICGEM for IGGT_R1, GOSG01S, GOCO06S, GGM05G, and EIGEN5C is 0.9690, 0.9765, 2.0196, 1.2136 and 1.1247 respectively. The result of the RMSE shows a little disparity between the deflection of vertical computed from each of the GGMs and the deflection of vertical computed using the discrete wavelet method. From the analysis, it is discovered that IGGT_R1 has the least mean deviation, standard deviation, and root mean square error values when compared with other low-resolution models. This implies that Model IGGT_R1 is closer in value to the discrete wavelet method than other low-resolution models.

Comparative analysis was done to investigate whether there were significant differences between the Geoid height computed using the two methods. Z- statistical test was performed to determine significant differences. From the analysis, the null hypothesis is rejected since z-computed $> z_{0.95}(44)$, and we can conclude that there is a significant difference between the means Geoid height values computed using both the gravimetric approach (stoke integral) and discrete wavelet decomposition method

5.0 CONCLUSION

This study aimed to assess the deflections of the vertical using a Gravimetric approach with a view to determining geoid height within the Federal University of Technology Akure using five high- and low-resolution Global Geoid Models. Geodetic positions (Latitude and Longitude) and Ellipsoidal Heights of 44 stations within the Federal University of Technology Akure were determined using South GNSS instruments. The observations were conducted in static mode, with each station observed for an hour to ensure accuracy. Five high- and low-resolution Global Geoid Models (EGM 2008, GECO, SGG-UGM-1, SGG-UGM-2 and XGM 2019e_2156, GOCO06S, GOSG01S, IGGT_R1, GGM05G, EIGEN 5C) were chosen for analysis. Deflection of the verticals and Geoid heights for the selected control points in the study area were computed from the gravity anomalies using the discrete wavelet decomposition method and also for each model. The geoid height computed using the gravimetric approach (stoke integral) was compared with those computed using the discrete wavelet decomposition method. Therefore, based on the results obtained in this study, the discrete wavelet decomposition method used in this research is hereby recommended as an alternative method to compute the deflection of vertical and geoid height of points when a gravimetric approach is considered.

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