Journal of Spatial Information Sciences

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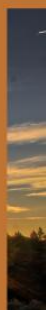

COMPARATIVE ANALYSIS AND ACCURACY ASSESSMENTS OF TERRAIN HEIGHTS DETERMINATION TOWARDS EFFECTIVE AND ACCURATE HEIGHTS MEASUREMENTS IN CONSTRUCTION WORKS

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DOI: https://doi.org/10.5281/zenodo.15667280

Abstract

Engineering and construction works require precise height or elevation determination, and providing such essential information is in the purview of geomatics engineers. Oftentimes, civil engineers are less concerned and mostly oblivious of the choice of method employed in the acquisition of such data but are more particular about the accuracy, time limitation and cost implication whereas, some of such engineering and construction works may require lesser accuracy. This study therefore attempts a comparative analysis and accuracy assessment of height determination by trigonometric heighting, stadia and tangential tacheometric methods relative to reference data acquired by spirit levelling operation. Thirty fixed points were selected and their elevations were determined using the afore-mentioned methods. Statistical analytical tools were employed for the comparative analytical process. The results analysis indicates that the three (3) methods — trigonometric heighting, stadia and tangential tacheometric methods — gave 0.077, 0.172 and 0.117 standard deviations and 0.017, 0.038, and 0.029 RMSE respectively. Thus, it can be inferred that Trigonometric heighting is more accurate relative to the other two (2) methods. However, a one-way ANOVA test shows P-value of 0.1102 as against F-Critical value of 3.1013. This reveals that there is no significant difference in the mean of the heights obtained by the three methods. Hence, the major mitigating factors which should influence the choice of any of the three (3) methods are time limitation and cost implication but not necessarily accuracy.

Keywords: Elevations, Terrain, Levelling, Trigonometry, Tacheometry, Engineering Works

Introduction

The physical surface of the earth called landform is a feature on Earth's surface that is part of the terrain which contains mountains, hills, plateaus, and plains as its four major types [1] but its main problem as a reference surface for position, dimension, shape, determination is its irregularity [2] and such irregularity is a function of non-uniformity in elevation / heights of points on the surface of the earth. In academic studies and engineering works, it is required to determine height differences between points or the height of points itself [3] and data obtain can be used for the designing of roads, canals, sewers, bridges, and other facilities having grad line that best suit existing topography, calculate volume of earth work and other materials, create maps that depicts general ground configurations [4]

Elevation is the distance measured along a vertical line from a vertical datum to a point or object [5] but from the engineering point of view, it is frequently useful in the case of inshore or offshore works to have the elevations related to the physical component with which the engineer is concerned [6]. Therefore, ground survey methods for the production of elevation models include spirit levelling, tacheometry, and trigonometric surveys (using total stations) [7] but GPS heighting is considered as an alternative to classical terrestrial height determination methods [8]. Adopting such alternative is necessary because the traditional methods of extending orthometric height such as terrestrial survey methods, can be time-consuming, expensive, and tedious [9]. In comparison to these, levelling offers a versatile yet simple, accurate and inexpensive field procedure for measuring heights [10] and this is the reason for its continued use on construction sites in competition with other methods.

The difference in elevation between two points can be determined by measuring (1) the inclined or horizontal distance between them and (2) the zenith angle or the altitude angle to one point from the other [5]. For all methods of trigonometrical heighting, it has already been noted that the accurate measurement of zenith (or vertical) angles is critical. In fact, under normal atmospheric conditions, angular errors are the most serious source of error in measured height differences [10]. With the introduction of an EDM instrument, and particularly the total station, the speed and reliability of making angle and distance observations have increased greatly [11] and have therefore becoming increasingly convenient to observe elevation differences using trigonometric methods.

Stadia tacheometry can be used to locate the position of a point and its elevation, and as such it enables contouring to be carried out in a manner similar to that using total stations [6]. The principle of this form of tacheometry is that the parallactic angle remains fixed and the staff intercept S varies with distance, D [6] by which the horizontal distances and the differences in elevation are determined indirectly using intercepts on a graduated scale and angles observed with a transit or the theodolite [12]. However, horizontal distances obtained by tacheometric observations do not require slope corrections, tension correction, etc. [13].

On the other hand, tangential tacheometry system is such that the stadia hairs are not essential but two readings are to be taken at the two targets at a fixed distance 'S' apart in the staff [12] with full meter values on the staff generally taken to avoid the decimal part and also for simplification of computations [14].

None of the methods under consideration is free from different kind of errors – while trigonometric leveling is affected by Atmospheric Refraction (AR) and Earth’s Curvature (EC) [5], Stadia method is often affected by the verticality of the leveling staff [15] while tangential method is very sentsitive to anguar errors [5].

JOURNAL OF SPATIAL INFORMATION SCIENCES ISSN: 2354-3361
VOL. 2, ISSUE 2, PP 19–29, 2025 **DOI:** https://doi.org/10.5281/zenodo.15667280
PUBLISHED 15-05-2025
www.journals.unizik.edu.ng/jsis

Each of the three methods has its limitations ranging from compulsory clear line of sight for all the three methods making them unsuitable in heavily wooded or built-up areas [10]. Height determination by stadia tacheometry is limited by low height precision as a result of limitations in estimating stadia intervals accurately [16] and inability to maitain perfect verticality of leveling staff during fied observation [15] and only effective over short to moderate distances (typically <300 meters); accuracy decreases with distance [17]. Tangential method on the other hand requires more time for both field and off-field work as two different anguar observation must be taken [10]. It has poor visibility in long distance observation as specific values must be aimed at on the leveling staff.

Having options as to methods of obtaining height is good but oftentimes, the user of the products such as civil engineers are less concerned and mostly oblivious of the choice of method employed in the acquisition of such data but are more particular about the accuracy, time limitation and cost implication. This study therefore attempted to compare three common methods of elevation determination which are Trigonometric Levelling, Stadia and Tangential Tacheometry Methods.

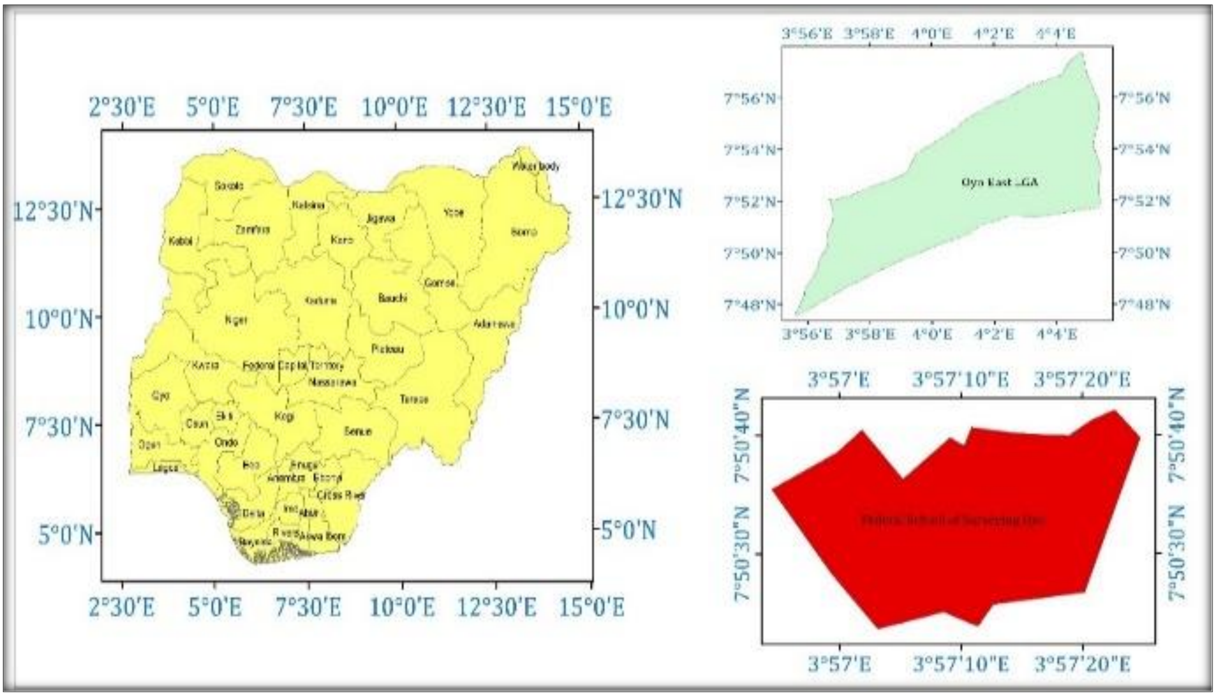

Figure 1. Diagram of the Study Area

The study area is Federal School of Surveying, Oyo Campus, Oyo, Oyo-East Local Government Area (LGA) of Oyo State, Nigeria. The geographical location of the study area lies between longitude range 03^{0}56’55”E to 03^{0}57’25”E and latitude range 07^{0}50’20”N to 07^{0}50’43”N (See Figure 1).

Materials and Methods

Both primary and secondary data were used for the research. Heights of Eleven (11) benchmarks were obtained from the SIWES and Practical Unit of Federal School of Surveying, Oyo, Oyo State, Nigeria. Within the study area, thirty (30) points were selected and marked with iron rods and concrete base for firmness. The selection of the stations was such that each is intervisible from at least one of the eleven benchmarks whose heights have been obtained earlier.

Data Acquisition

Geomax Zipp 02 Digital Theodolite and South NL-C32 Dumpy Level instruments was adopted for data acquisition due to their accuracy and versatility for long duration surveys. The equipment were tested to be in good working conditions and the heights of the benchmarks were checked for any insituability error. The error obtained were within the limits allowed. The stadia constants (additive and multiplying constants, C and K) of the theodolite were determined by observation of upper and lower stadia values over a distance of 10m and 50m respectively and repeatedly for 10times to have refined results. The values of K and C obtained were 100.444 and 0.383 respectively.
Spirit levelling operation was carried out to determine the heights of the thirty selected points which served as the reference data for the research. The digital theodolite was used to acquire data for Trigonometric Levelling, Stadia and Tangential Tacheometric observations. In each of the three methods, the benchmarks were used as instrument stations and the thirty selected stations as the target stations.
For Trigonometric levelling, vertical angles were measured on both faces (Left and Right) because according [11 nin trigonometric leveling, a minimum of a face I and face II reading should always be taken. The Electronic Distance Measuring component of the digital theodolite was used to measure distance between instrument station and the target (circular prism). Also, the instrument and target heights were measured and recorded (Figure 2).

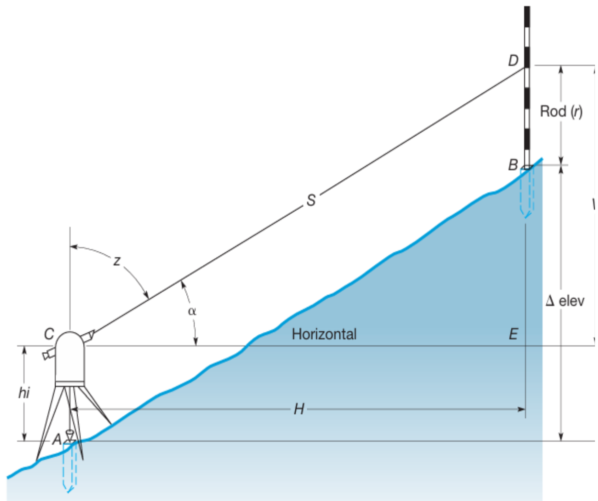

Figure 2. Trigonometric Levelling – Short Lines

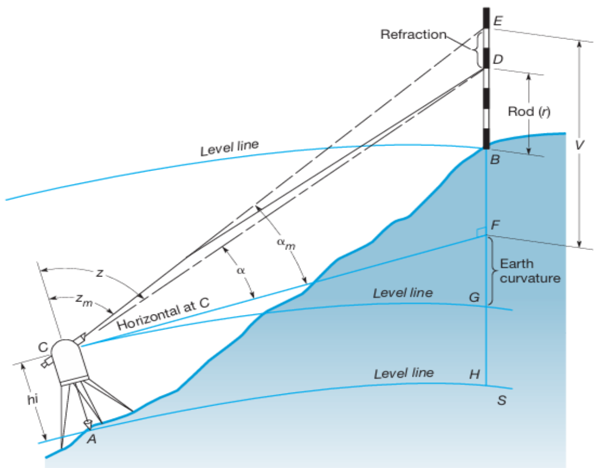

Figure 3. Trigonometric Levelling – Long Lines

In the case of stadia tacheometry observation, the target was replaced by levelling staff of 4m height. Stadia intercepts (Lower, Middle and Upper readings on the levelling staff) were measured and recorded.
The vertical circle readings were also measured in each case. In this method, the vertical circle was fixed while taking the staff intercept the values corresponding to the horizontal cross hairs (lower, middle and upper) were recorded. No distance was measured directly and the observation was on single face (face left). Also, the height of the theodolite was also measured.

JOURNAL OF SPATIAL INFORMATION SCIENCES ISSN: 2354-3361
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The tangential tacheometry observation was done with the same set of equipment and target as stadia but observation procedure differs. The staff intercepts were made constant and as whole numbers (2m and 3m respectively) while recording the corresponding vertical circle readings. The observations were made on both faces. No distance was measured directly in this method as well but the instrument height was measured.

Data Processing

The acquired data were processed by means of programming a Microsoft Excel Spreadsheet. The vertical angles, θ, were obtained by taking the absolute of the difference between the vertical circle reading and Ninety degrees. This was so because vertical angle is the deviation from the horizontal either above (elevation) or below (depression).

$\theta = Absolute\ (90 – Vertical\ Circle\ Reading)$ *Eqn.1*

For each of the three methods, the vertical angles were obtained through eqn. 1.
In order to determine the height of each point from Trigonometric Levelling data, the vertical component V was determined through eqn. 2.

$V = H \tan\theta$ *Eqn.2*

While the height of each point Hn was computed through;

$Hn = Hp + V + Hi + Ht$ *Eqn.3*

Where H is the horizontal distance, θ is the vertical angle, Hp is the height of the benchmark on which the instrument was set to make observation to such point. Also, Hi and Ht are the heights of instrument and target respectively and V is the vertical component.
The stadia tacheometry data was also computed to obtain stadia interval (S) from eqn. 4, vertical component (V) from eqn. 5 and the heights (Hn) of the selected points from eqn. 6.

$S = U – L$ *Eqn.4*

$V = 1/2\ K\ .S \sin2\theta + C \sin\theta$ *Eqn.5*

$Hn = Hp + V + Hi – M$ *Eqn.6*

Where U and L are the upper and lower stadia readings respectively, θ is the vertical angle and K and C are the multiplying and additive constants of the theodolite used whose values were pre-determined to be 100.444 0.383 respectively in section 2.1. V is the vertical component, Hn and Hp are the heights of the unknown point and the benchmark respectively. Also, Hi is the heights of instrument and M is the middle reading of the staff intercept.
Also, the tangential tacheometry data were also computed to obtain the stadia interval (S) from eqn.4, the horizontal distance (D) from eqn. 7 (elevation) and eqn. 8 (depression), vertical component (V) from eqn. 9 and the heights (Hn) of the selected points from eqn. 10.

$D = S/((\tan q - \tan t))$ *Eqn.7*

$D = S/((\tan t – \tan q))$ *Eqn.8*

$V = D \tan t$ *Eqn.9*

$Hn = Hp + V + Hi – L$ *Eqn.10*

Where D is the horizontal distance between the instrument and target stations, q is the upper vertical angle, t is the lower vertical angle, L is the lower staff intercept and other terms remained as mentioned earlier.
From the processes discussed, four (4) sets of heights of the 30 points were obtained through Spirit levelling, Trigonometric leveling, Stadia and Tangential tacheometric data. Using the spirit levelling data as reference, misclosure of each of the remaining three data sets from the reference data were computed.

The misclosures were analysed. Standard deviation and Root Mean Square Error (RMSE) of each set of misclosures were computed while One-Way ANOVA test was done to arrive at a reasonable conclusion. Also, charts were used to graphically represent the results.

Results

Table 1. Misclosures from Reference Data

Items	Stadia Tach.	Tangential Tach.	Trigonometric Levelling
Min.	-0.643	-0.429	-0.211
Max.	0.641	0.121	0.319
Mean	-0.025	-0.066	0.006
Range	1.284	0.550	0.530
95% Error	0.964	0.908	0.992
Stand. Dev.	0.172	0.117	0.077
RMSE	0.038	0.029	0.017

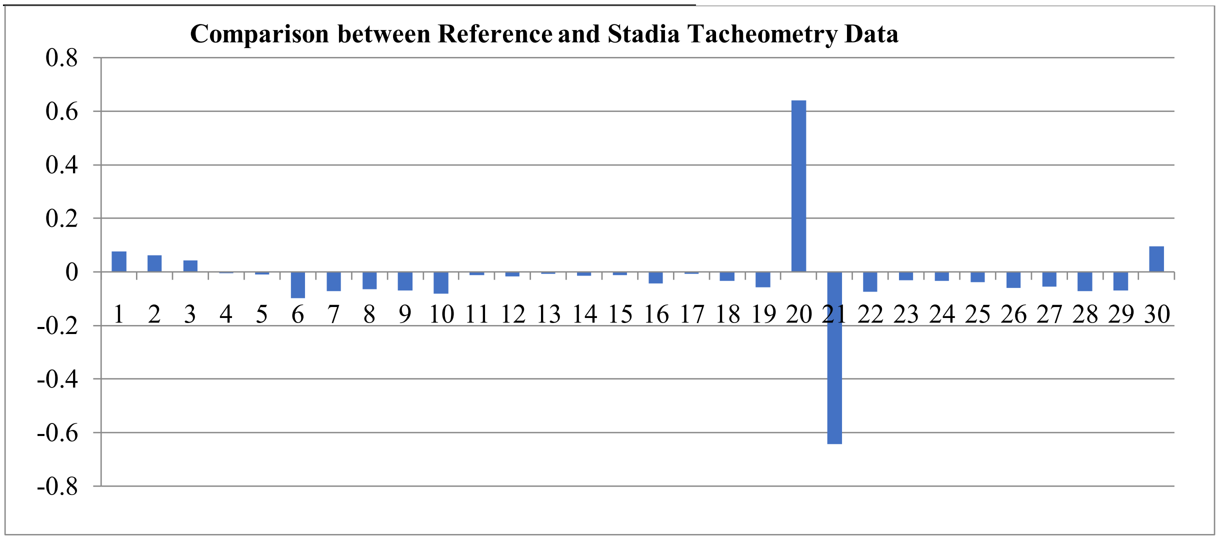

Figure 4: Misclosures on Stadia Tacheometry Data

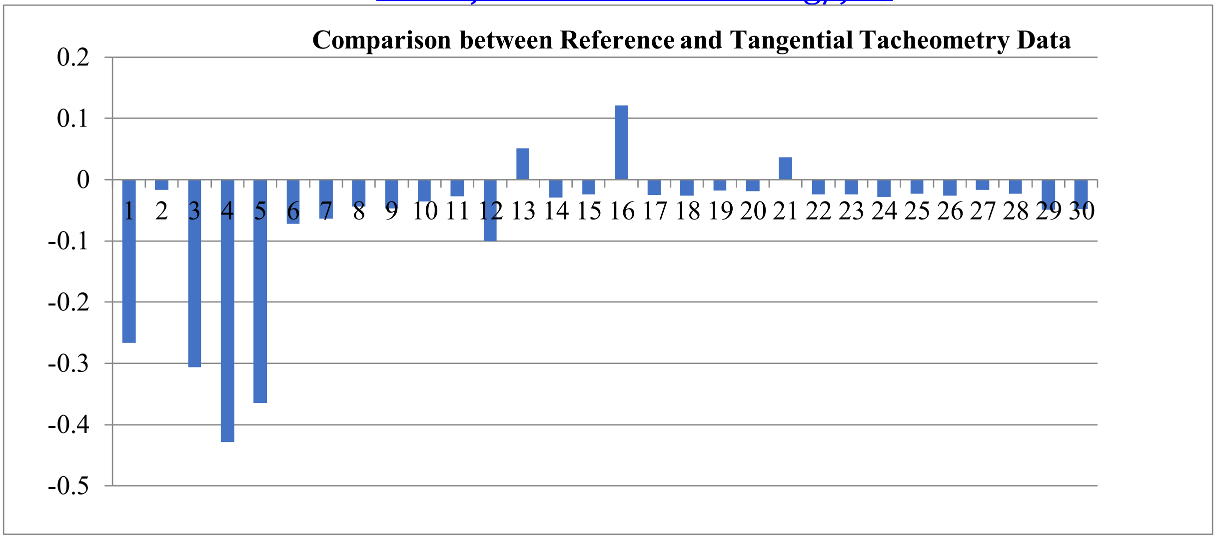

Figure 5. Misclosures o Tangential Tacheometry Data

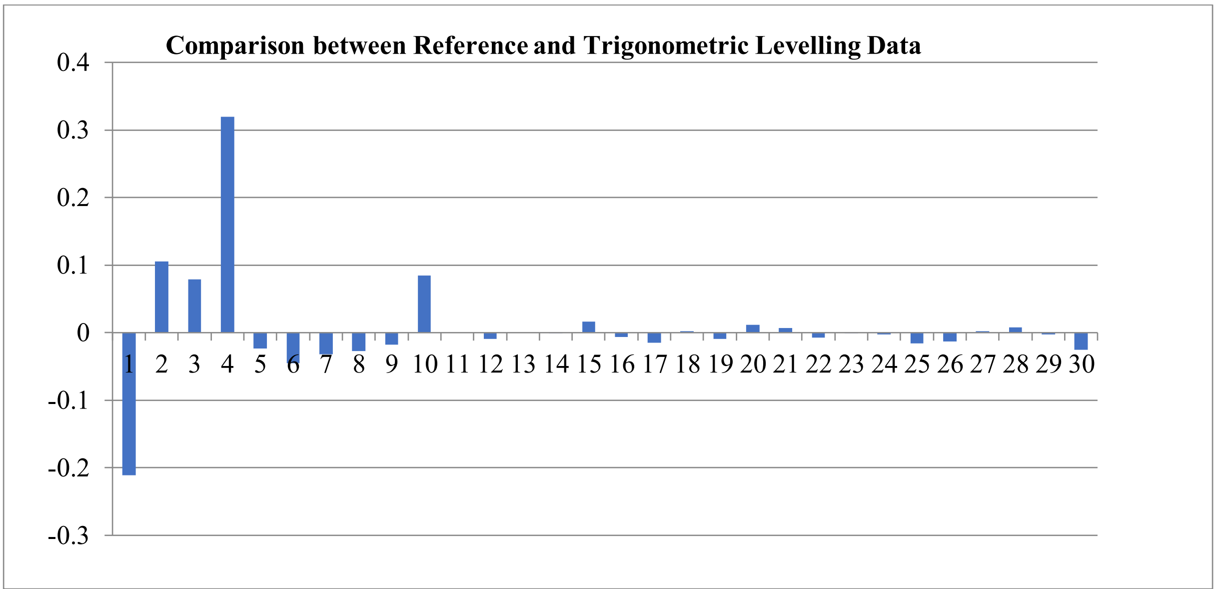

Figure 6. Misclosures on Trigonometric Levelling Data

ISSN: 2354-3361
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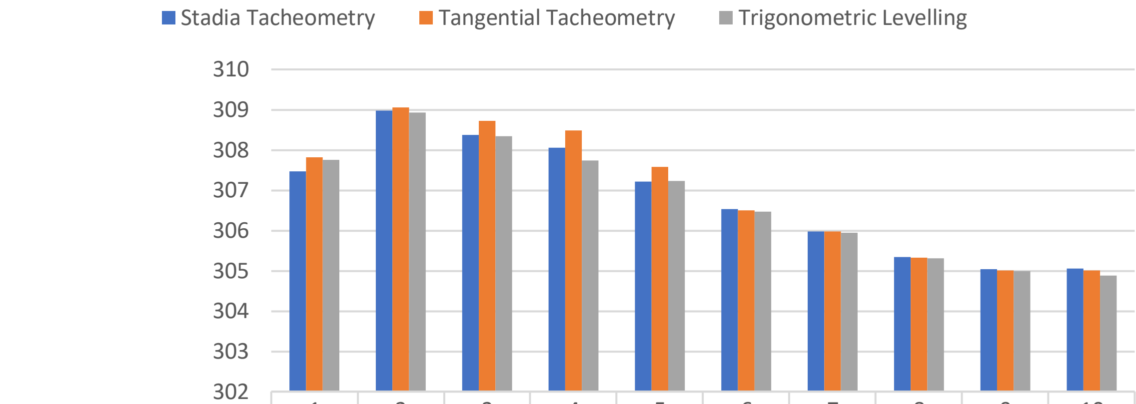
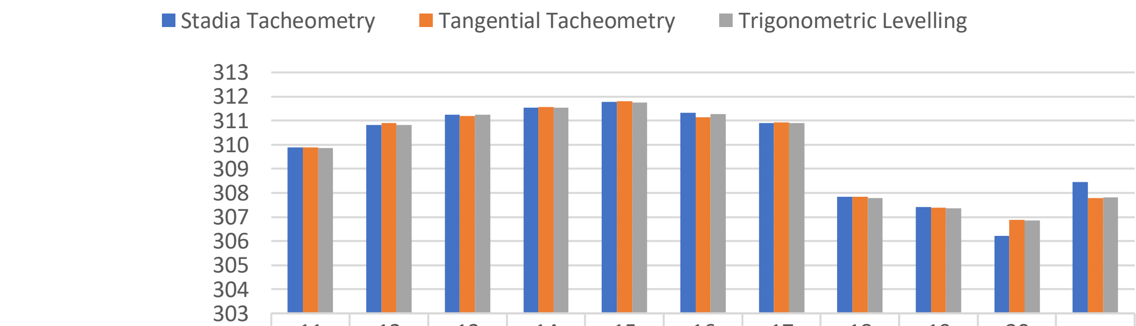


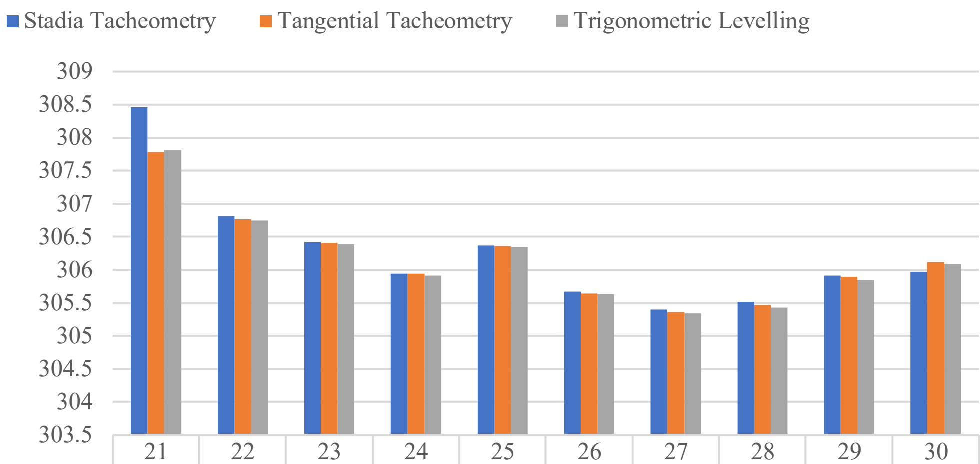
	1	2	3	4	5	6	7	8	9	10
Stadia Tacheometry	307.474	308.976	308.377	308.063	307.218	306.529	305.978	305.344	305.037	305.053
Tangential Tacheometry	307.818	309.054	308.725	308.487	307.573	306.503	305.971	305.323	305.015	305.007
Trigonometric Levelling	307.762	308.932	308.34	307.739	307.231	306.476	305.939	305.306	304.986	304.886

Figure 7. The three datasets for Points 1 to 10



	11	12	13	14	15	16	17	18	19	20	
Stadia Tacheometry	309.878	310.815	311.242	311.538	311.785	311.313	310.899	307.834	307.414	306.222	308.458
Tangential Tacheometry	309.894	310.899	311.184	311.552	311.798	311.149	310.916	307.826	307.375	306.882	307.778
Trigonometric Levelling	309.867	310.808	311.235	311.524	311.758	311.276	310.906	307.798	307.366	306.851	307.808

Figure 8. The three datasets for Points 11 to 20



	21	22	23	24	25	26	27	28	29	30
Stadia Tacheometry	308.458	306.817	306.412	305.945	306.37	305.674	305.397	305.51	305.909	305.97
Tangential Tacheometry	307.778	306.767	306.406	305.94	306.355	305.641	305.359	305.462	305.888	306.113
Trigonometric Levelling	307.808	306.75	306.383	305.915	306.348	305.628	305.34	305.431	305.842	306.09

Figure 9. The three datasets for Points 21 to 30

Discussions and Conclusion

Table 1 shows that the minimum misclosures recorded for stadia tacheometry, tangential tacheometry and trigonometric levelling data were -0.643, -0.429 and -0.211 respectively while their corresponding maximum misclosures were 0.641, 0.121 and 0.319. Also, their corresponding standard deviation were 0.172, 0.117 and 0.077 while the RMSE were 0.038, 0.029 and 0.017 respectively.

Figure 4 shows the graphical representation of the misclosure of the stadia tacheometric data from the reference data. It shows that only point 20 and 21 have misclosure of ±0.6m while other 28points has misclosure of less than ±0.1m. In similar manner, figure 5 shows graphical representation of the misclosure of the tangential tacheometric data from the reference data. Points 1. 3, 4, 5, and 16 have more error above or below ±0.1m while others were within that. Also, in the case of trigonometric levelling data, figure 6 shows that only points 1, 2 and 4 has ±0.1m misclosure. Figures 7, 8 and 9 show the comparison of the three data sets for all the 30 points in bar chats.

However, one factor ANOVA test on the datasets shown that the P-value was 0.1102 as against the F-critical value of 3.1013. Stadia tacheometry, tangential tacheometry and trigonometric levelling recorded correlation values of 0.996797, 0.0.998525 and 0.999365 respectively indicating that the four methods were strongly correlated. This is an indication that there is no significance difference between the heights obtained from the three sources as compared to the reference method (Spirit Levelling).

Considering the values of RMSE, the Trigonometric Levelling data with the minimum RMSE of 0.017 is considered more accurate than the other two. This was due to the fact that fewer field data (vertical angle, height of instrument and target) were required for Trigonometric Leveling while other methods require more field data. The more the number of field parameters, the more the source of errors. On the other hand, Tangential is more accurate than that of Stadia tacheometry. It must also be noted that the two methods with best results (Trigonometric levelling and tangential tacheometry methods) were advantaged during data acquisition as their vertical angles were determined on both faces of the theodolite and this must have contributed to having better results.

However, the ANOVA analysis and the values of correlations indicate that there is no significant different between the results. In view of that, it was concluded that the major mitigating factors which should influence the choice of any of the methods are time limitation and cost implication but not necessarily accuracy. The adoption of any of the methods is hereby recommended when low order accuracy of height determination is required.

References

[1] Rutledge, K., McDaniel, M., & Teng, S. (2025, 03 02). *Landforms*. National Geographic: https://education.nationalgeographic.org/resource/landform/ adresinden alındı

[2] Agajelu, S. I. (2018). *Geodesy: The Basic Theories – Classical & Contemporary.* . Enugu: El 'Demak Publishers.

[3] Ceylan, A., Inal, C., & Sanlioglu, I. (2005). Modern Height Determination Techniques and Comparison of Accuracies . *FIG Working Week 2005 and GSDI-8 themed From Pharaohs to Geoinformatics* (s. 21 - 28). Cairo, Egypt.: FIG.

[4] Adewale, A., Emenari, U. S., & Uwaezuoke, I. C. (2014). Comparison of Height Differences obtained from Automatic, Digital and Tilting Level Instruments. *Journal of Environmental Issues and Agriculture in Developing Countries., 6*(1).

[5] Ghilani, C. D., & Wolf, P. R. (2012). *Elementary Surveying: An Introduction to Geomatics* (13 b.). Boston: Prentice Hall Publishers.

[6] Schofield, W. (2001). *Engineering Surveying* (5 b.). Oxford: Butterworth Heinemann Publishers.

[7] Kavanagh, B. F., & Mastin, T. B. (2014). *Surveying Principles and Applications* (9 b.). New Jersey: Pearson Publishers.

[8] Svabensky, O., & Weigel, J. (2004). Optimized Technology for GPS Height Determination. *FIG Working Week 2004* (s. 22-27). Athens: FIG.

[9] Raufu, I. O., Tata, H., & Olaosegba, S. O. (2023). Modelling local geoid undulations using unmanned aerial vehicles (UAVS): a case study of the Federal University of Technology, Akure, Nigeria. *Geodesy, Cartography and Aerial photography*, 63 - 75. doi:https://doi.org/10.23939/istcgcap2023.98.063

[10] Uren, J., & Price, B. (2010). *Surveying for Engineers* (5 b.). Hampshire: Palgrave Macmillan Publishers.

[11] Ghilani, C. D., & Wolf, P. R. (2006). *Adjustment Computations: Spatial Data Analysis* (4 b.). New-Jersey: John Wiley & Sons Inc.

[12] Roy, S. K. (1999). *Fundamentals of Surveying* (1st b.). New Delhi, India: Prentice Hall.

[13] Agor, R. (2011). *Surveying and Levelling* (11 b.). New Delhi, India: Khanna Publishers, Darya Ganj.

[14] Thakur, P. (2023, March 15). *Tacheometric Surveying – Purpose, Procedure, and Formula.* Civil Concept: https://civilconcept.com/ /tacheometric-surveying/. adresinden alındı

[15] Schofield, W., & Breach, M. (2007). *Engineering Surveying* (6th b.). United Kingdom: London.

[16] Ghilani, C. D. (2017). *Adjustment Computations: Spatial Data Analysis* (6th b.). Hoboken: John Wiley & Sons Inc.

[17] Bannister, A., Raymond, S., & Baker, R. (1998). *Surveying* (7th b.). Essex: Longman / Pretice Hall.