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QUANTITY ESTIMATION OF UNDERGROUND MASS DEPOSIT USING MULTI-RESOLUTION TECHNIQUE

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Abstract

Multi-resolution analysis is a mathematical technique of analysing and interpreting discontinuities in transient signals. It is known for its efficiency in analysing gravity data. This study aims at computational estimation of the quantity of underground mineral deposit using multi-resolution technique with the objective of acquiring and processing the earth terrestrial gravimetric data using 2-Dimensional Wavelet Transform. The acquired data are gravity anomalies of points over an area of $64km^2$, sub-divided into square grids of 500m interval. These were analyzed using 2-Dimensional Discrete Wavelet Transform (2D-DWT). Also, they were decomposed and thresholded to separate the gravity anomalies into two components (regional and residual gravity anomalies). The Regional Residual Ratio (RRR) was estimated to be 52.6767dB with a Root Mean Square Error (RMSE) of 5.0074mgal. The quantity of the underground mineral deposit, as estimated using the Gauss' theorem and multi-resolution analysis, was found to be 2.0786792×10^7 tonnes. The statistical test carried out, on the results obtained, showed that there is no significant difference between the results obtained from the technique used in this study and the gold standard results obtained from the combined Least Square Collocation and Fourier analysis techniques which produced RMSE and RRR of 6.0072mgal and 19.33dB respectively at 95% confidence level. Therefore, it can be inferred that the Multi-Resolution analysis technique, with lower RMSE, has been found to have produced better estimate of the quantity of underground mineral deposit.

KEYWORDS: Gravity Anomaly Separation, Residual Gravity Anomalies, Multi-Resolution Analysis, Quantity Estimation, and Underground Mass Deposit.

1.0 INTRODUCTION

Gravimetric technique of geophysical exploration determines the varying effect of the earth gravity and takes advantage of the non-uniformity of the gravity force to interpret and predict the physical parameters of the earth contents and underground structures. The varying gravity field is caused by the heterogeneous nature of the earth materials [15]. This is basically explained by Newton's law of gravitation [18][19]. This in essence means that the parameters (i.e., density, mass, depth and shape) of underground minerals could be estimated by the measurement of gravity field around the locations of interest [17].

The challenge of geophysical technique of gravimetric geodesy generally lies in the non-satisfactory separation of gravity anomalies into regional components (regional gravity anomalies) and local components (residual gravity anomalies). This is because this challenge usually makes the analysis of the two gravity anomaly components difficult or near impossible thereby leading to non-satisfactory results. Efforts on gravimetric geophysical technique have been focused on how to improve on the processing technique. These include applications of Stokes Integral and Modified Stokes Integral, Least Square Collocation [12], Fourier analysis [1] and [2], Fast Fourier Transform, and Preferential Filtering [10], and Multi-Resolution Wavelet Transform technique [11], [3], and [9]. The computation of underground mineral deposit by double Fourier analysis provides a good time-frequency analysis [1]. However, the efficiency of this technique has been found to be less satisfactory in processing non-stationary signals [7][22][20]. Hence, there is the need for a more suitable analysis technique for processing stationary and non-stationary signals for the determination of more acceptable quantity of underground mineral deposit.

A multi-resolution analysis technique is a time-frequency technique that analyses signals in both scale and dilation. It is found to be efficient at processing stationary and non-stationary signals which are functions that change with time [22][16]. In other words, it is efficient at performing vector analysis which has a good time-frequency localization and better variable resolution technique for processing gravity data. Therefore, it is the objective of this study to carry out computational estimation of the quantity of underground mineral deposit using multi-resolution technique.

2.0 MATERIALS AND METHODS

The acquired gravity values were reduced to the geoid, the equivalent normal gravity values to the reduced gravity values and the gravity anomalies were computed at the observed gravity stations. The Discrete Wavelet Transform is functional shown in equation 1 [8]. It is a form of multi-resolution analysis known for its adaptive resolution and low computational complexity [22][23][6][21]. It was used to separate the gravity anomalies into regional and residual gravity anomalies as well as estimating the quantity of underground mineral deposit using residual gravity anomalies.

$$q(x) = \sum_{m \in Z} \sum_{n \in Z} d_n^m \Psi_{m,n}^{(\lambda_0,x_0)}(x) \tag{1}$$

Where,

$$d_n^m = \langle q(x), \Psi_{m,n}^{(\lambda_0,x_0)} \rangle = \sum_n \mathrm{q(x)} \Psi_{m,n}^{(\lambda_0,x_0)}(x)$$

$$\Psi_{m,n}^{(\lambda_0,x_0)}(x) = \lambda_0^{-m/2} \Psi(\lambda_0^{-m} x - n x_0)$$

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d_n^m are detailing coefficients, $\Psi_{m,n}$ is the wavelet function generated from the original mother wavelet function $\Psi \in L^2(\Re), \lambda_0$ is the scale space parameter, x_0 is the translation space parameter, m is the scale or level of decomposition integer, and n is the shifting or translation integer [8].

2.1 Data Acquisition

The study site, as abstracted from [12], is a land size of about 64km^2 on an approximate latitudes of 10° 04′ 48.87″ N and 10° 08′ 51.76″ N and longitudes 10° 35′ 23.1″ E and 10° 39′ 30.68″ E. It falls within block 809 of profile gravity survey carried out by Shell Nigeria Exploration Company in 1995. It is an oil prospecting area, located in the middle belt of Nigeria, between Gombe, Bauchi and Plateau States of Nigeria. The extract of the data is presented in Table 1. Columns 1, 2, 3, and 4 of the table show the serial number, station ID, northing coordinates, easting coordinate, elevations, and gravity anomalies respectively. The study site was selected due to the availability of reports of similar research works by [1], [2], and [5], in the area that could serve as gold standard results to verify the results obtained in this study.

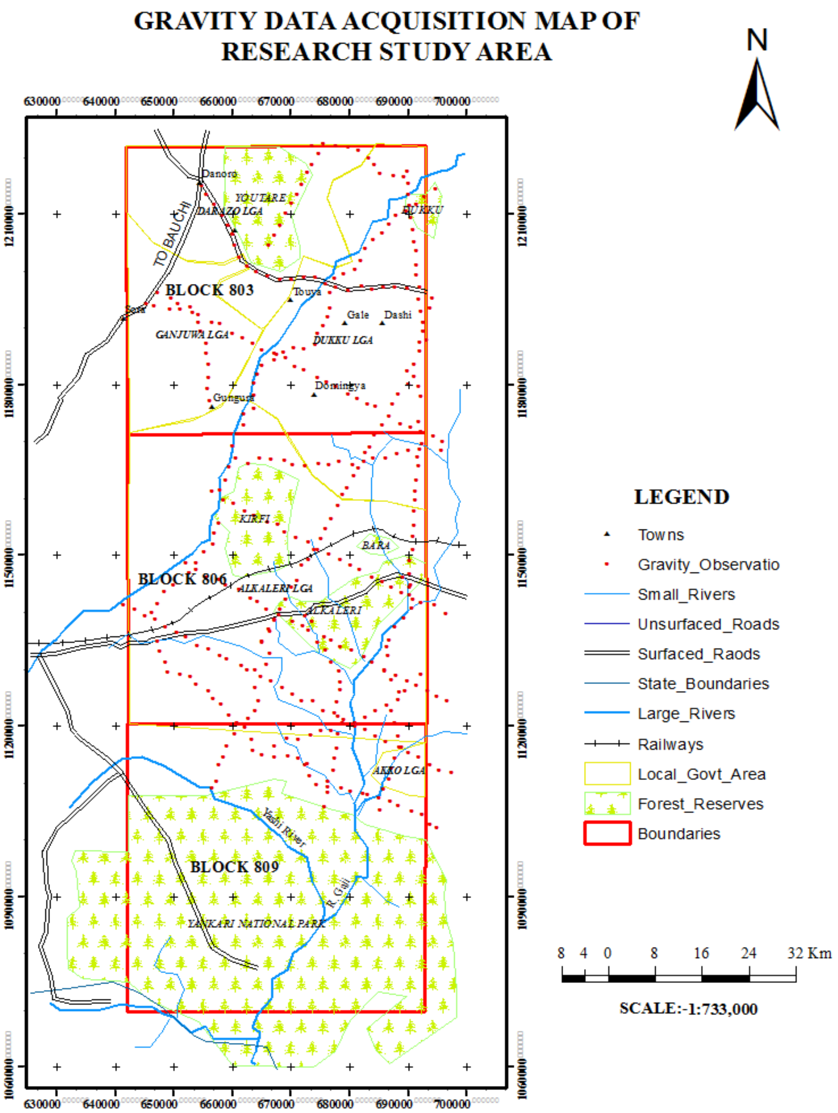

Figure 1: Gravity Acquisition Blocks of 803, 806 and 809 [12]

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Table 1: Sample Data of Gravity Anomalies

S/N	Station ID	Northings (metres)	Eastings (metres)	Elevations (metres)	Gravity Anomalies (mGal)
1	94V0201300	1122861	658742	428.4	-4.464
2	94V0201320	1122362	658731	426.6	-4.722
3	94V0201340	1122153	659104	422.3	-5.004
4	94V0201360	1121932	659539	420.6	-5.785
5	94V0201380	1121694	659972	416.6	-6.492
6	94V0201400	1121310	660276	411.9	-7.014
7	94V0201420	1120872	660497	406.5	-7.646
8	94V0201440	1120545	660872	402.5	-8.367
9	94V0201460	1120165	661165	398.5	-8.961
10	94V0201480	1120016	661635	394.6	-9.881

Source: [12]

2.1 Data Processing

There are three main steps involved in processing of gravity anomalies data. These are: interpolation of the obtained point data to form square grids, separation of the gridded gravity anomalies into regional and residual components, and computations of the quantity of underground mineral deposit.

2.2.1 Data Interpolation

The gridding of the gravity anomalies at grid points of 500 meters interval was done using Kriging method. Kriging is a group of Geostatistical techniques that interpolate for the value of a random field at unobserved locations. This technique is mathematically presented as shown in equations 2 and 3 [1][2].

$$\sigma_k^2(x_0) = Var[Z(x_0) - Z(x)] \tag{2}$$

$$= \sum_{i-1}^{n}\sum_{i-1}^{n} \omega_i(x_0)\omega_j(x_0)c(x_i x_j) + Var[Z(x)] - 2\sum_{i-1}^{n} \omega_j(x_0)c\left(x_i x_j\right)$$

The Kriging variance given as;

$$Var\hat{Z}(x_o) = Var\left(\sum_{i-1}^{n} \omega_i Z(x_i)\right) = \sum_{i-1}^{n}\sum_{i-1}^{n} \omega_i \omega_j c(x_i x_j) \tag{3}$$

Where, $\hat{Z}(x_o)$ is the Kriging predictor.

2.2.2 Separation of Residual Gravity Anomalies

This involves the identification of points of discontinuities in a function (this case, gravity anomalies). The Discrete Wavelet Transform used, at this stage, transformed the gravity anomalies into high and

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low frequencies of approximate and detailed (horizontal, vertical and diagonal) components, represented in Equations 4-7.

$$c_n^m = \sum_{l_1 \in z} h_{l_1} - 2n_1 \sum_{l_2 \in z} h_{l_2} - 2n_2 c_l^{(m-1)} \tag{4}$$

$$d_n^{Hm} = \sum_{l_1 \in z} g_{l_1} - 2n_1 \sum_{l_2 \in z} h_{l_2} - 2n_2 c_l^{(m-1)} \tag{5}$$

$$d_n^{Vm} = \sum_{l_1 \in z} h_{l_1} - 2n_1 \sum_{l_2 \in z} g_{l_2} - 2n_2 c_l^{(m-1)} \tag{6}$$

$$d_n^{Dm} = \sum_{l_1 \in z} g_{l_1} - 2n_1 \sum_{l_2 \in z} g_{l_2} - 2n_2 c_l^{(m-1)} \tag{7}$$

Where, c_n^m is the approximate coefficient, d_n^{Hm} is the horizontal detail coefficient, d_n^{Vm} is the vertical detail coefficient, d_n^{Dm} is the diagonal detail coefficient, h and g are known as low pass and high pass filter, respectively.

Thresholding is performed at this level to separate the discontinuous signals (residual anomalies) from the continuous signals (regional anomalies) by estimating a threshold value from the detailed coefficients. The steps adopted for this, as fully discussed in [22], are summarized below.

i. selection of mother wavelet and its wavelet function was Debauches (db3) and decomposition level was 3 [13][14];
ii. decomposition of 2D discrete wavelet transform (2D DWT) to transform the signal from time domain to frequency domain at the specified decomposition level;
iii. soft thresholding of the wavelet coefficients; and,
iv. inverse 2D discrete wavelet transform (2D IDWT) was performed on the thresholded detail coefficient result and the initial approximate coefficient at same level of decomposition to yield the regional gravity anomalies, devoid of the residual gravity anomalies.

2.2.3 Estimation of quantity of deposited mineral

The residual gravity anomalies were used to compute the excess and actual masses of the mineral deposit. To compute the excess mass, the Gauss' theorem (Equation 8) as explained by [4] was used.

$$\int g_M \,.\, dS = -\left(\frac{GM}{R^2}\right).4\pi R^2 = -4\pi GM \tag{8}$$

Where, g_M is the residual gravity anomaly, M is the excess mass of the anomaly, G is the gravitational constant (6.673x10^{-11} $m^2/kg.s^2$), $4\pi R^2$ is the surface area considering the body as a sphere with radius R. The negative sign indicates that g_Z is in opposite direction to the outward pointing normal.

The value of the excess mass was determined by further simplification of equation (8) as discussed in [1].

$$M_T = \frac{1}{2\Delta G}\sum_{i=1}^{N}\sum_{j=1}^{M}\Delta g(x,y)\Delta x\Delta y \;\; = 23.9 \;\; \sum_{i=1}^{N}\sum_{j=1}^{M}(\Delta g\Delta a) \;\; (in\ tonnes) \tag{9}$$

Where, Δg is the residual gravity anomalies representing a grid square, and Δa is the area of each grid square.

To compute the actual mass (M_a), the actual mass was independently obtained by decomposing the residual gravity anomalies and integrating the approximate and detailed coefficients as given in Equation (10).

$$M_a = \left(\frac{1}{2\pi G}\right) \sum_q \sum_s c_K \, \Delta g_{\,q}(x) + \sum_q \sum_s d_K^H \, \Delta g_{\,q}(x) + \sum_q \sum_s d_K^V \, \Delta g_{\,q}(x) + \sum_q \sum_s d_K^D \, \Delta g_{\,q}(x) \qquad (10)$$

Where, g_q is the residual gravity anomalies, c_K is the approximate coefficient, d_K^H, d_K^V, d_K^D

3.0 RESULTS PRESENTATION AND ANALYSIS

Table 2 contains the sample of the gravity anomalies separation results. Table 3 shows the quality of the gravity anomalies separation results. Figures 2, 3 and 4 show the surface plots of the separation results (gravity anomalies, regional and residual gravity anomalies surface plots). The Regional to Residual Ratio (RRR) of the separation result was estimated to be 52.6767decibels. Decibels (dB) are unit of measurement used to express the quality of signals (sound, energy levels, light intensity, image signals etc.).

Table 2: Gravity Anomalies Separation Results

S/N	Northings (meters)	Eastings (meters)	Gravity Anomalies (mGal)	Regional Anomalies (mGal)	Residual Anomalies (mGal)
1	1114708	674208	-8.6604	-8.7394	0.079
2	1114708	674708	-8.5905	-8.6695	0.079
3	1114708	675208	-8.5514	-8.6304	0.079
4	1114708	675708	-8.5376	-8.6166	0.079
5	1114708	676208	-8.519	-8.44	-0.079
6	1114708	676708	-8.5355	-8.4565	-0.079
7	1114708	677208	-8.5176	-8.4386	-0.079
8	1114708	677708	-8.5748	-8.4958	-0.079
9	1114708	678208	-8.6843	-8.7515	0.0671
10	1114708	678708	-8.7012	-8.7684	0.0671

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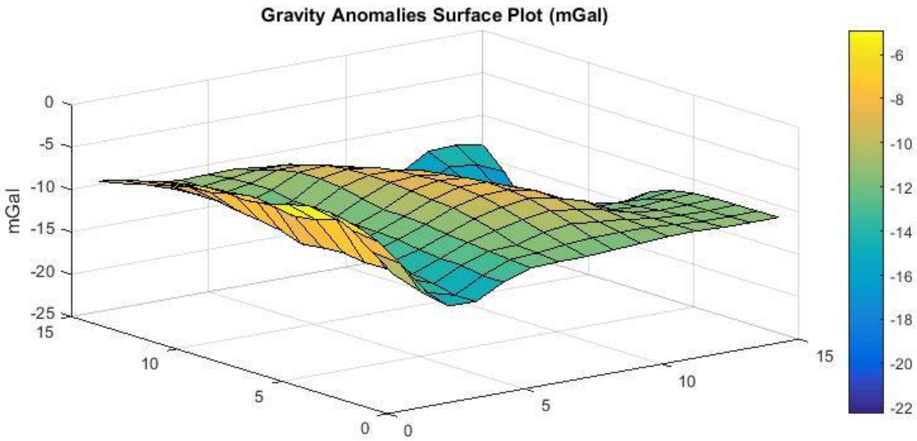

Figure 2: Surface plot of Gravity Anomalies

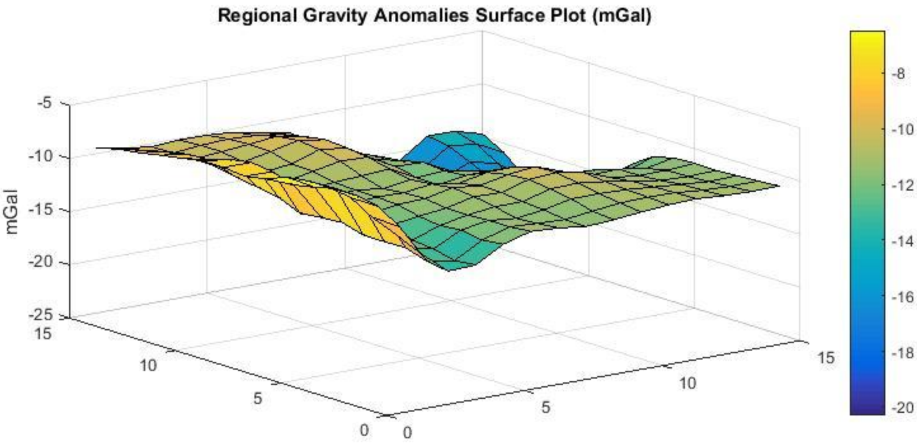

Figure 3: Surface plot of Regional Gravity Anomalies

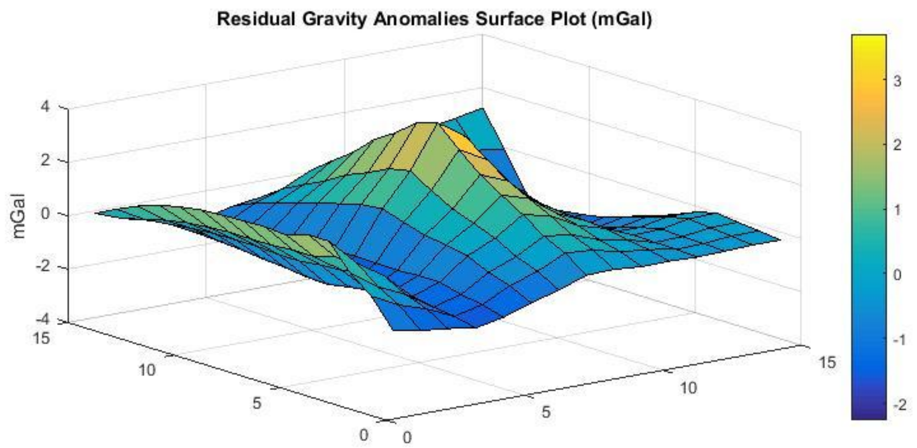

Figure 4: Surface plot of Residual Gravity Anomalies

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www.journals.unizik.edu.ng/jsis

Table 3: Quality of Separation Result

Quality	Value
RRR (dB)	52.6767
RMSE (mgal)	5.0074
Variance of Regional Gravity Anomaly (mgal)	0.21185
Variance of Residual Gravity Anomaly (mgal)	0.00041

The high value, 52.676dB, of RRR means that a substantial value of residual gravity anomaly was removed from the gravity anomaly leaving behind a smooth residual gravity anomaly. This leads to the considerable and satisfactory low values of the variances. Table 4 shows the estimated quantities of the mineral deposit obtained from the Multi-Resolution and double Fourier analysis techniques. The estimated quantity by Multi-Resolution analysis is less compared to the double Fourier analysis result. This could be attributed to the localization property of Wavelet Transform [22] which in turn could mitigate over hauling of estimated or computed quantities.

Table 4: Estimated Mass of Deposited Mineral

	Estimated Mass (Tonnes)	
Method	Multi-Resolution Analysis	Double Fourier Analysis *(Gold Standard)*
Actual Mass	2.0786792×10^{7} tonnes	7.19383934×10^{8} tonnes

A hypothesis F-test was conducted to investigate whether there is significant difference between the two results at 0.05 significant level. The variances obtained from the Fourier analysis and Multi resolution analysis techniques are 0.2108308mgal and 0.212616mgal respectively.

Null hypothesis($\boldsymbol{H_0}$) :

$$H_0: S_1^2 = S_2^2$$

Alternative hypothesis($\boldsymbol{H_1}$) :

$$H_1: S_1^2 > S_2^2$$

Table 5: Table of Statistical Analysis

Degree of freedom	v_1	255
Degree of freedom	v_2	143
Multi-resolution Analysis	S_1^2	0.21262
Fourier Analysis	S_2^2	0.21083
Computed F-Statistic	$\frac{S_1^2}{S_2^2}$	1.0085
Significance Level	(α)	0.05
Table F-Statistic	$F_{\alpha,255,143}$	1.282

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This is a one-tail hypothesis F-test where the null hypothesis is rejected when the value of computed F-Statistic is greater than the Table F-Statistic. The result of the test shows that the computed F-statistic is smaller than the Table F-statistic. That is, the null hypothesis is accepted. Therefore, it can be inferred that there is no significant difference between the computed masses of the same underground mineral deposit obtained from the two techniques.

4.0 CONCLUSIONS

In this study, multi-resolution analysis technique has been successfully used to estimate the quantity of underground mineral deposit. Gravity anomalies of points within the study area were acquired and predicted to obtain square grids where the distance of each grid is 500m. The gravity anomalies were separated into two (regional and residual) components. The result of separation was considered satisfactory with the high quality value of RRR (52.6767dB). The wavelet coefficients of the thresholded residual gravity anomalies were integrated and the quantity of the underground mineral deposit was estimated to be 2.0786792 x 10^7 tonnes.

The result of the earlier research on the same data, using double Fourier analysis technique revealed the estimated quantity of mineral deposit as 7.19383934 x 10^8 tonnes. The statistical test conducted to determine the reliability of the results obtained shows that there is no significant difference between the results obtained from the two techniques at 5% level of significance. However, due to the smaller value of RMSE and localization property of the Multi-Resolution analysis technique that mitigates over hauling of estimated quantities, the Multi-Resolution analysis technique is recommended for estimating the quantity of underground mineral deposits.

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