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Modeling of creep in raffia plant fibre reinforced plastics

C.C. Ihueze

Department of Industrial and Production Engineering, Nnamdi Azikiwe University, Awka

Abstract

Larson-Miller, Sherby-Dorn and Manson-Hefered had used parametric methods to analyze creep in metals**.** This study used experimental and numerical methods with parametric method as mixed method. Multiple linear regression and the general Power law equations were employed to model the Raffia fibre composites creep limit responses. The creep limit estimated with numerical methods correlated with the experimental data and results of parametric methods, so that multiple linear regression model and power law model are good fits for creep limit of Raffia fibre composites working within temperature ranges of 30 ^oC to100^oC.The values 0.0062, 0.96926 and 0.9845 evaluated for standard error, coefficient of determination and correlation coefficient respectively support the conclusion that multiple linear regression model represents an excellent fit of creep function of raffia fibre composites. This paper therefore successfully presented two numerical models and parametric charts for raffia fibre reinforced plastic composites creep limit design.

1. Introduction

Viscoelastic materials show a time dependent response to applied stress. The creep limit of plastics needs to be established because of involvement of plastics in most recent designs such as in multi-layer moldings, design of snap fits, design of ribbed sections and in design of light weight structures, in every day use.

One of the major objectives of this paper is to establish whether some of the models established for metal creeps can be applied for plastics creep. Study also aims at obtaining the creep limit for Raffia fibre reinforced polyester matrix composites.

The influence of high temperature on tensile strength, yield strength and elongation of materials was reported by (Black and Adams, 1981). The maximum temperature in steam and gas turbine castings is limited to about 538^OC, although experimental value is in the range of 760 ^OC to 871 ^OC (Belyaev, 1979).

In addition to loss of strength at high temperature, steel and other metals exhibit the phenomenon of creep, which is the gradual elongation of the entire member at high temperatures over a long period of time .High temperature and high stresses increase the creep rate so that at high temperatures the part will not elongate (Shigley and Mischke.,1989) reported that a part may fail with a load that induces stresses though the load may lie between the yield strength and the tensile strength of the material. Classical reports show that creep can occur at low temperature in aluminum and polymeric composites (Crawford, 1998).

This study used experimental and numerical methods of multiple linear regression approach and the general power law modeling method in conjunction with parametric methods to model the Raffia fibre composites creep limit response.

2. Theoretical background

Andrade in 1910 was the first to establish relationships between creep strain and time as in (Benham and Warnock,1981).Though,Larson-Miller,Sherby-Dorn and Manson-Hefered had used parametric methods to analyze creep in metals no such indebt analysis had analysed creep in polymeric composites (local fibre composites) (Muben, 2003). Creep-strain equations for plastics were reported in (Crawford, 1998).

2.1. Creep-strain equations

Creep-strain equations for plastics were reported in (Crawford, 1998) as

$$\varepsilon = a + bt^c \text{ (parabolic)} \quad (1)$$

$$\varepsilon = a + bt^{1/3} + ct \text{ (power)} \quad (2)$$

$$\varepsilon = at/1+bt \text{ (Hyperbolic)} \quad (3)$$

$$\varepsilon = 1/1+at^{b} \text{ (Hyperbolic)} \quad (4)$$

where, ε = Creep strain, t = Creep time

2.2. Multiple linear regression model for modeling

The Multiple Linear Regression Model for modeling of nonlinear responses is expressed in (Canale and Chapra, 1998; Zill and Cullen, 1996; Ihueze, 2005; 2007;2008)

$$P_m(x) = a_0 + a_1x_{1i} + a_2x_{2i} \quad (5)$$

Following minimization of sum of squares of residuals as applied in polynomial regression.

$$E_i = p_m(x) - y_i = \text{Residual} \quad (6)$$

$$S_r = \sum((a_0 + a_1x_{1i} + a_2x_{2i}) - y_i) \quad (7)$$

By differentiating with respect to polynomial coefficients, a_0, a_1 and a_2

$$\frac{\partial s_r}{\partial a_0} = 2\sum((a_0 + a_1x_{1i} + a_2x_{2i}) - y_i) \quad (8)$$

$$\frac{\partial sr}{\partial a_1} = 2\sum x_1((a_0 + a_1x_{1i} + a_2x_{2i}) - y_i) \quad (9)$$

$$\frac{\partial s_r}{\partial a_2} = 2\sum x_{2i}((a_0 + a_1x_{1i} + a_2x_{2i}) - y_i) \quad (10)$$

By setting (8) - (10) to zero, the following system of linear equations was obtained.

$$na_0 + a_1\sum x_{1i} + a_2\sum x_{2i} = \sum y_i \quad (11)$$

$$a_0 x_{1i} + a_1\sum x_{1i}^2 + a_2\sum x_{1i}x_{2i} = \sum x_{1i}y_i \quad (12)$$

$$a_0\sum x_{2i} + a_1\sum x_{2i} + a_2\sum x_{1i}x_{2i} = \sum x_{2i}\, y_i \quad (13)$$

Putting (Eqs. 11 – 13) in matrix form

$$\begin{bmatrix} n & \sum x_{1i} & \sum x_{2i} \\ \sum x_{1i} & \sum x_{i1}^{2} & \sum x_i\, x_{2i} \\ \sum x_{2i} & \sum x_{i1}\, x_{2i} & \sum x_{2i}^{2} \end{bmatrix} = \begin{bmatrix} a_o \\ a_1 \\ a_2 \end{bmatrix} \begin{bmatrix} \sum y_i \\ \sum x_{1i}\, y_i \\ \sum x_{2i}\, y_i \end{bmatrix} \quad (14)$$

Where x_1 = temperature, x_2 = time,t

2.3. General power law-multiple regression model

The general power law-multiple regression model for the modeling of nonlinear responses is expressed in (Canale and Chapra, 1998) as

$$Y = a_0x_{1i}^{a1}x_{2i}^{a2}x_{3i}^{a3}\ldots\ldots\ldots x_{mi}^{am} \quad (15)$$

By linear transformation of Eq(15)

$$\log y = \log a_0 + a_1\log x_1 i + a_2\log x_{2i}\ldots\ldots.a_m\log x_m i \quad (16)$$

By comparing Eq(5) and Eq(16)

$$\log a_0 = a_0, \log x_{1i} = x_{1i}, \ \log x_{2i} = x_{2i} \quad (17)$$

so that the constants a_o and a_1, a_2 can be solved with matrix

$$\begin{bmatrix} n & \sum \log x_{1i} & \sum \log x_{2i} \\ \sum \log x_{1i} & \sum (\log x_{1i})^2 & \sum \log x_{1i}\, \log x_{2i} \\ \sum \log x_{2i} & \sum \log x_{1i}\, \log x_{2i} & \sum (\log x_{2i})^2 \end{bmatrix} x \begin{bmatrix} \log a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum \log y_i \\ \sum \log x_{1i}\, \log y_i \\ \sum \log x_{2i}\, \log y_i \end{bmatrix} \quad (18)$$

2.4. Classical parametric methods

Sherby-Dorn, Larson-Miller and Manson-Herfered had developed master curves for the prediction of creep following the linearization of the rate equation (Muben, 2003).

$$t = c \exp(E/RT) \quad (19)$$

By linearization of Eq. (19),

$$\ln t = c + E/RT \quad (20)$$

where

E = activation energy
R = gas constant
T = absolute temperature

c= a constant and is a function of stress.
Two possibilities may occur with the graphics of (1/T, lnt)

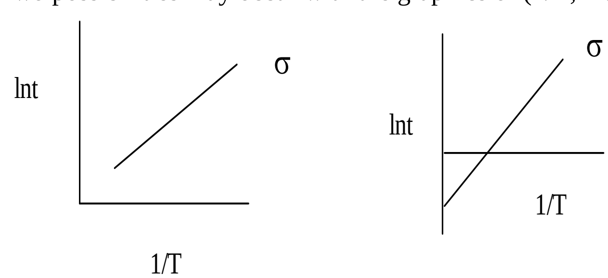

Fig. 1. E/R is the slope of the line.

2.5. Sherby-Dorm parameter

By employing Eq. (20) and Fig.1 in which E is a constant, c a function ofσ, $f_1(\sigma).f_1$ (σ) is known as the Sherby-Dorn parameter and is expressed as:

$$c = f_1(\sigma) = \ln t - E/RT = \ln t - m/T \quad (21)$$

where
m = E/R = a constant ,c is determined by performing quick creep tests at several stress levels, the lnt vs 1/T lines are plotted, their slopes determined and ,master curve prepared by plotting lnσ as function of $f_1(\sigma)$.

2.6. Larson-Miller Parameter

When Eq. (20) is used and ln σ is plotted as function of f_2 (σ)

$$c = \ln t - m/T \quad (22)$$

$$T(\ln t - c) = E/R = m \quad (23)$$

$$m = E/R, \text{ a constant} \quad (24)$$

where f_2 (σ) is the larson-Miller parameter taken to be equal to m ie m = f_2 (σ). The Larson-Miller master curve is obtained after short experiments to obtain 1/T for several stress levels for determination of f_2 (σ) and m = f_2 (σ).

3. Methodology

The mixed-methodology of this work consists of three parts, Hand-lay up method to form Raffia fibre composites, Testing or Experimentation to obtain creep failure data and Analytical or computational method to model and analyse creep data.

3.1. Hand-lay up method

Raffia fibre composites are formed. The full details of the processes involved are found in (Ihueze, 2005). Replicated samples of composites are formed for creep tests following standard procedures.

3.2. Experimentation and method

Creep testing equipment was used on replicated samples of Raffia composite considering temperatures of 30^{o}C, 50^{o}C and 100^{o}C and deformation response data recorded as presented in Table1. The full details of the equipment and procedures used are found in Anambra State University mechanical engineering laboratory, (Enendu et al., 2006) and (Oreko et al., 2006).

Table 1
True stress responses at elevated temperatures

	30^{o}C	50 ^{o}C	100 ^{o}C
Time (minutes)	σ_{30} (MPa)	σ_{50} (MPa)	σ_{100} (MPa)
5	0	0	8.506
10	9.1163	0	8.5086
15	9.1191	0	8.5113
20	9.122	0	8.5139
25	9.1248	8.7848	8.5165
30	9.1276	8.7862	8.5178
35	9.1304	8.7916	8.5178
40	9.1346	8.797	8.5178
45	9.1388	8.8011	8.5178
50	9.1416	8.8011	
55	9.1416	8.8011	
60	9.155	8.8011	
65	9.1416		

Source: (Enendu, 2006)

Table 2
Creep deformation response of raffia composites

Time (minutes)	n_{R30} (mm/mm)	n_{R50} (mm/mm)	n_{R100} (mm/mm)
5	0.0002		0.0003
10	0.0005		0.0006
15	0.0008		0.0009
20	0.0011		0.0012
25	0.0014	0.0002	0.0015
30	0.0017	0.0003	0.0017
35	0.0022	0.0009	0.0017
40	0.0026	0.0015	0.0017
45	0.0029	0.002	0.0017
50	0.0029	0.002	
55	0.0029	0.002	
60	0.0029	0.002	
65	0.0029		

Source: (Enendu, 2006)

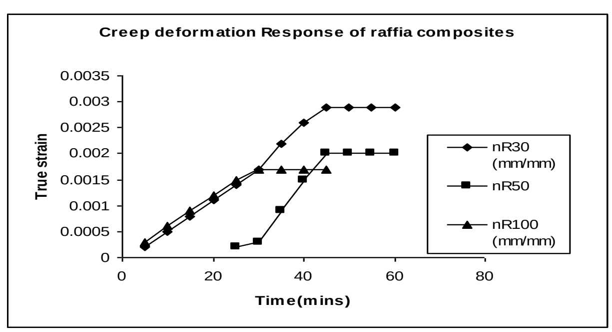

Fig. 2. Creep deformation response of raffia composites.

4. Computational modeling of creep data

Creep limit and strain are time dependent responses and are functions of two variables, temperature and time. This work considered three temperature levels, 30°C, 50°C and 100°C. Table 1 shows that at 30°C, the creep limit was 9.1416 MPa at time 65mins and at 50°C, creep limit is 8.8011 MPa at time 60 mins and that at 100°C creep limit was 8.5178 MPa at time 45 mins.These information were correlated with multiple linear regression and general power law models as well as with parametric methods using Table 3.

Table 3
Creep strengths extracted from Table 1

x_1(°C)	x_2(mins)	y (MPa)
30	65	9.1416
50	60	8.8011
100	45	8.5178

4.1. Multiple linear regression modeling

By evaluating terms of Eq (14) as

$n=3, \sum x_{1i} = 180, \ \sum x_{2i} = 170, \sum y_i = 26.4649,$

$\sum x_1 i^2 = 13400, \ \sum xi\, x_2 i = 9450, \ \sum x_{1i}\, yi = 1566.215,$

$\sum x_2 i = 170, \ \sum x_{2i}^2 = 9850, \ \sum x_2\, i\, yi = 1505.857$ and substituting in Eq(14) a system of 3x3 matrix equations is obtained as

$$\begin{bmatrix} 3 & 180 & 170 \\ 180 & 13400 & 9450 \\ 170 & 9450 & 9850 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 26.4649 \\ 1566.215 \\ 1505.857 \end{bmatrix} \quad (25)$$

The variable parameters a_0, a_1, a_2 were obtained by direct method of solving system of linear equations of Crout method (LU – decomposition) as a_0 = 26.4529, a_1 = -0.07514, a_2 = -0.23158. These parameters were substituted in Eq. (5) to obtain multiple linear regression creep model as

$$y = 26.4529 - 0.07514\, x_1 - 0.23158\, x_2 \quad (26)$$

4.2. Power law modeling

By evaluating terms of Eq.(18) as
$n = 3.0000, \sum \log x_1 i = 5.1761, \ \sum \log x_2 i = 5.2443,$

$\sum \mathrm{Log}\, yi = 2.8361, \sum(\log x_1 i)^2\ 9.0684,$

$\sum \log x_1 i \log x_2 i = 9.0053, \ \sum \log x_1 i\, \mathrm{Log}\, yi\ \ 4.8853,$

$\sum (\log x_2 i)^2 = 9.181,$

$\sum \log x_2 i \log yi = 4.960187\ 857$ and substituting in Eq.(18) system of 3x3 matrix equations is obtained as

$$\begin{bmatrix} 3.0000 & 5.1761 & 5.2443 \\ 5.1761 & 9.0684 & 9.0053 \\ 5.2443 & 9.0053 & 9.1816 \end{bmatrix} \begin{bmatrix} \log a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.8361 \\ 4.8853 \\ 4.960187 \end{bmatrix} \quad (27)$$

Similarly this equation is solved by LU – decomposition to obtain log a_0 = 1.446765, a_0 = Antilog 1.446765 = $10^{1.446765}$ = 27.97467. a_1 = -0.112965, a_2 = -0.1753288. When these constant coefficients were substituted in Eq.(15) the following power law model is obtained

$$y = 27.97467\, x_1^{-0.112965}\, x_2^{-0.17533} \quad (28)$$

4.3. Comparison and validation of models.

4.3.1. Comparison of models.

The constants of Eqs. (26) and (28) appear approximately equal supposing that predictions of the two equations may be similar. The validity of the two predictive models can be assessed by using x_1 = 30 mins and x_2 = 65 mins from Table 1 in Eqs. (26) and (28) respectively to obtain, y = 9.146 MPa and 9.16295 MPa. , for Eqs. (26) and (28).

Both estimates approximate experimental data, so that multiple linear regression model and power law model are good fits for creep limit of Raffia fibre composites working within temperature ranges of 30 °C to 100 °C.

4.3.2. Computation for error analysis of regression models.

By using $y^1 = \sum y / 3 = 8.8202$, a_o =26.4529 , a_1 = – 0.07514 , a_2 = – 0.23158 in Table7 the following were evaluated:

Standard error,

$$s_{y/x} = \sqrt{s_r / [n - (m+1)]} = 0.0062 \quad (29)$$

Coefficient of determination,

$$r^2 = [\sum (y-y^1)^2 - \sum s_r] / [\sum (y-y^1)^2 = 0.96926 \quad (30)$$

Correlation coefficient,

$r = \sqrt{r^2} = 0.9845.$

The values 0.0062, 0.96926 and 0.9845 evaluated for standard error, coefficient of determination and correlation coefficient respectively support the conclusion that Multiple Linear Regression Model represents an excellent fit of creep function of raffia fibre composites.

Table7
Computation for error analysis of multiple linear regression model

x_1	x_2	Y	$(y-y^1)^2$	$[y-(a_{o}+a_1x_1+a_2x_2)]^2$
30	65	9.1416	0.1o34	0.000019
50	60	8.8011	0.0004	0.000004
100	45	8.5178	0.0914	0.000016
	∑	26. 4605	0.1952	0.000039

4.4. Parametric modeling of creep data.

The creep limits at various temperatures were presented with time for the creep event to occur in Table 4. Creep parameters were determined following Sherby-Dorn and Larson-Miller methods as followings:

Table 4
Parametric analysis data evaluated from Table 1

T(k)	t (hrs)	y (MPa)	1/T	lnt
303	1.0833	9.1416	0.0033	0.08
323	1	8.8011	0.0031	0
373	o.7500	8.5178	0.0027	-0.288

The graphics of Table 4 to show relationship of data before analysis is in Fig. 3.

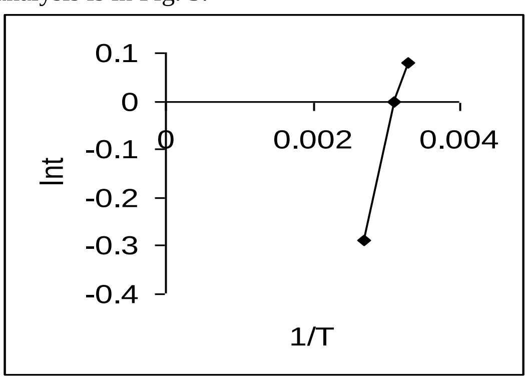

Fig. 3. lnt vs 1/T for different stress levels.

4.4.1. Sherby-Dorn analysis

For material to follow the Sherby-Dorn, the curve of lnt – 1/T must be straight. By employing Eq.(16) and Table 4, Sherby-Dorn parameter, c was estimated as:

$$\ln t = c + m/T$$

$$\ln 1.0833 = c + m/303 \quad (31)$$

$$\ln 1 = c + m/323 \quad (32)$$

By solving Eqs. (27) and (28).

$$303\ln 1.0833 = 303c + m$$
$$323\ln 1 = 323c + m \quad (33)$$
$$c = -1.21218$$

c is the Sherby-Dorn parameter. For value of m, substitute c = - 1.2118 in Eq. (28), m = 323* 1.2118, = 391.4114,
Since $f_1(\sigma_1) = c$, $f_1(\sigma_2) = c$, $f_1(\sigma\alpha_3) = c$
So that by Eq. (29),

f_1 (9.146) = ln (1.0833) – (391.4114)/303 = -1.2118

f_1 (8.8011) = ln1 – (391.4114)/323 = 0-1.2118

f_1 (8.5178) = ln 0.75 –(391.4114)/373 = -0.2877 – 1.0494 = -1.3371

The Sherby-Dorn parameters are presented in Table 5 while the excel graphics of Table 5 is in Fig. 4

Table 5
Sherby-Dorn master curve data

Y	f1(y)	lny
9.1416	-1.2118	2.2133
8.8011	-1.2118	2.2133
8.5178	-1.3371	2.142

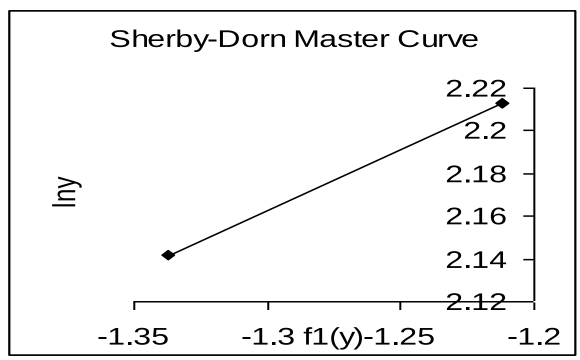

Fig. 4. Sherby-Dorn master curve.

4.4.2. Larson-Miller master analysis.

The Larson-Miller Parameters were also determined for the three true stress levels using Eq.(20) for evaluation of m as: lnt = c + m/T
From three stresses of Table 4,

$$\ln 1.0833 = c + m_1/303 \quad (34)$$

$$\ln 1.0000 = c + m_1/323 \quad (35)$$

By solving Eqs. (34) and (35) as usual

c = -1.2118, m = 391.4114, but Larson-Miller parameter, is

expressed as
$f_2(\sigma) = m = T(\ln t - c)$,so that for three stress levels,

$f_2(9.146) = 303(\ln 1.0833 + 1.2118), = 391.4192$

$f_2(8.8011) = 323(\ln 1 + 1.2118) = 391.4114$

$f_2(8.5178) = 373(\ln 0.75 + 1.2118) = 344.69599$

The Larson-Miller parameters are presented in Table 6 while the excel graphics of Table 6 is in Fig.5.

Table 6
Larson-Miller master curve data

y	$f_2(y)$	lny
9.1416	391.4192	2.2133
8.8011	391.4114	2.2133
8.5178	344.69599	2.142

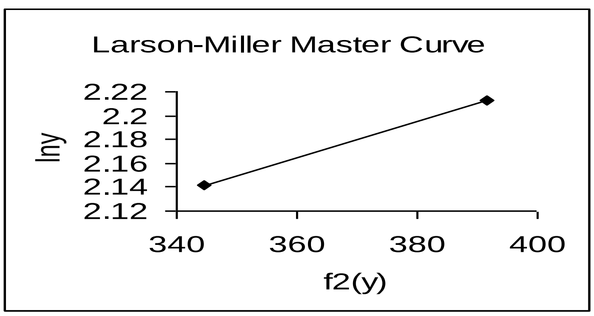

Fig. 5. Larson-Miller master curve.

4.3.2. Use of master curve

Once the creep time and temperature are known the creep parameter, f(y) =f (σ) is evaluated using either Sherby-Dorn or Larson-Miller equation to estimate the parameter. The parameter is then used in master curve Fig. 4 or Fig. 5, to estimate the creep limit, lny = ln σ.

5. Discussion of results

Table 1 and Table 2 describe the time dependence response of gradual deformation of Raffia fibre composite (creep) with time and temperature. Creep limits of Raffia fibre at temperatures of 30^0C, 50^0C and 100^0C were recorded in Table I as 9.1416MPa, 8.8011 MPa and 8.5178MPa.

Multiple linear regression model and power law model were used to establish models for design and manufacture of Raffia fibre composites serving within temperature up to 100^0C. The regression models were compared and the predictions of models gave creep limit as 9.146MPa and 9.16295MPa respectively.

Graphics were also established in the form of master curves. These curves follow the usual Sherby-Dorn and Larson-Miller master curves as shown in Figure 5 and Figure 6 respectively. These curves are applied when the relationship lnt vs 1/T is linear as shown in Figure4. Further comparison with Sherby-Dorn and Larson-Miller master curves gave similar results.

The master curve application involves estimation of parameters f(σ) and extrapolation of creep limit from the master curves. The major objective of this study which is to obtain appropriate creep models for design and manufacture of Raffia fibre composite parts and to ascertain whether some existing models could be applicable were met with Eqs. (22) and (24) .

The values 0.0062, 0.96926 and 0.9845 evaluated for standard error, coefficient of determination and correlation coefficient respectively support the conclusion that Multiple Linear Regression Model represents an excellent fit of creep function of raffia fibre composites.

6. Conclusions

Creep occurs in raffia fibre composites subjected to constant stresses with time and could be modeled by multiple linear regression equation and General power law equation. These models compared favourably with Sherby-Dorn and Larson-Miller parametric methods predictions and therefore could be used in structural and component design for economic manufacture. The values, 0.0062, 0.96926 and 0.9845 evaluated for standard error, coefficient of determination and correlation coefficient respectively support the conclusion that Multiple Linear Regression Model represents an excellent fit of creep function of raffia fibre composites. This paper therefore successfully presented two numerical models and parametric charts for raffia fibre reinforced plastic composites creep limit design.

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