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A technique of performance prediction of capacitor motors based on cross field concept

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Abstract

A Modified equivalent circuit has been developed using thevenin's theorem to predict the performance of a capacitor motor based on cross - field theory; An illustration of the validity of the resultant Two-Equation Model has been carried out using motor constants obtained from design considerations. Theoretical results obtained are shown graphically and compares favourably with those obtained from other known techniques.
Keywords: Prediction of capacitor motors; Modeling; Simulation; Cross-field theory

Nomenclature

a.	=	ratio of effective conductors in the aux. to the main wdg.
I_{1a}, I_{1n1}	=	aux./main wdg. Current
I_{2a}/I_{2m}	=	current in cross-field/main Axes of rotor,
Y_m	=	effective resistance in magnetizing branch to account for core-loss in mains
S	=	ratio of rated speed to synchronous speed.
V_{sc},V_{sm}	=	voltage induced in cross-field/main axes of rotor,
X_1,X_2	=	leakage reactance of main wdg/main axis of rotor,
X_m	=	effective magnetizing Reactance of main wdg.
Z_1,Z_2	=	primary/secondary leakage impedance of main wdg,
Z_{1a},Z_{2a}	=	primary/secondary leakage reactance of aux. Wdg,
Z_m,$Z_{ma}(a^2Z_m)$	=	effective impedance of magnetizing branch of main/aux.wdg.

1. Introduction

The theoretical development to 'predict the performance of a capacitor motor makes use of an equivalent network based upon either the revolving-field theory or the cross-field theory. Two simultaneous equations are required for the later (Torrey and Guru, 1997; Ilochi, 1993; Trckey, 1957). A procedure arranged in a very systematic manner for routine calculations in both cases have been developed (Ilochi, 1993).

Trckey (1957) devoted his entire paper to arrange the equations for hand calculation while Torrey and Guru (1997) made use of a complex subroutine to the job, both on the basis of cross-field theory. Thus it goes without saying that the revolving-field approach involves lesser unknowns as compared to the cross-field approach. Meanwhile, it is of little consequence if high speed computers are used to perform such calculations. Any number of simultaneous equations can be solved by making use of available computer library subroutines for matrix solutions, if computing time and cost are of secondary importance.

However, due to the unavailability of such computers and lack of the required expertise in developing nations, the purpose of this paper is to show that a conventional equivalent: circuit representation of a Capacitor motor on the basis of cross-field theory can be reduced to another representation by a suitable transformation. The new equivalent network helps calculate such unknowns as the currents in different parts of the motor and its performance characteristics with comparatively little expenditure of time and effort since it deals with only two simultaneous equation instead of four otherwise needed for a conventional network.

The rotor currents, that is, the currents in the main and the cross-field axe of the rotor, are determined first and then substituted in simple expressions to obtain the main and the auxiliary winding currents.

2. Modelling and analysis

In this analysis, the auxiliary winding is in space quadrature with respect to the main winding. The core-loss is represented by equivalent resistances in shunt with the magnetizing branches of the main and auxiliary windings and then transformed to simple series circuits. The impedance, current and voltage in the cross- field axis of the rotor are all referred to the auxiliary winding. Thus Z, being the secondary impedance of the main axis with reference to the main winding, a^2Z_2 is then the secondary impedance of cross-field axis. Losses such as stray load loss, friction, windage and rotor surface loss are all assumed to vary as the square of the speed and are considered simply as mechanical drag on the rotor. These losses are, therefore, subtracted from the output.

The effects of harmonics in the air gap, eddy currents and skin effects are neglected since the present analyses are intended for small induction motors. These assumptions and clarifications are necessary in understanding the formulation of the equivalent circuit and the ensuing performance equations.

2.1. Two-equation development

A capacitor motor is illustrated schematically in Fig.1 with its equivalent network representation according to the cross-field theory. The equivalent circuit is broken down into two parts. The first is the main winding and the main axis of the rotor while the second part is the auxiliary winding and the cross-filed axis of the rotor. The two parts are treated separately in writing the network equations except for the consideration of the induced speed voltages in the main winding and the cross-field axis of the rotor due, to its rotation. At any given time, the direction flow of currents and the polarities of the applied voltages are as an indication that the current in the auxiliary winding is ahead of the current in the main winding *curved* arrow in Fig.1.

The specification of direction of rotation helps to fix the signs of j-terms for the speed voltages induced in the main and cross-field axis of the rotor.

With the assumed direction of currents in different parts of the network, the speed voltages in the main and the cross-field axes of the rotor are:

$$V_{sm} = jI_IaZ_m \quad S_a + jI_{2a} \ (jx_2 + Z_m) S_a \qquad (2.1)$$

$$V_{sc} = jI_{Im} Z_m \ S_a - jI_{2m} (jX_2 \_+ Z_m) S_a \qquad (2.2)$$

The direct application of the network of fig.1 yields four simultaneous equations in terms of four such unknowns as the main winding current, the auxiliary winding current, the current in the main axis of the rotor, and the current in the cross-field axis of the rotor. However, if the rotor currents can be expressed in terms of the main winding currents or vice versa mathematically, the number of the equations can be reduced to two. In this section, by the application of thevenin's theorem, the existence of such a possibility is shown whereby the winding currents are stated in .terms of rotor currents. The statements of the thevenin's theorem is as follows:

> "The thevenin' s theorem states that an electric network can be replaced by an emf in series with impedance. The emf is the open circuit voltage of the network. The series impedance is the impedance measured back into the network with all the emf sources set to zero, but the internal resistance of the generator is left in circuit for this measurement".

The straight forward application of the thevenin's theorem to the equivalent circuit of fig.1 transforms it into another equivalent circuit as shown in fig.2, where

$$V_m = VZ_m/(Z_1 + Z_m) \qquad (2.3)$$

$$V_a = VZ_{ma}/(Z_{ma} + Z_c + Z_{1a}) \qquad (2.4)$$

$$Z_{1m} = Z_1Z_m/(Z_1+Z_m) \qquad (2.5)$$

$$Z_{1a} = (Z_{1a} + Z_c) Z_{ma}/(Z_{1a} + Z_c + Z_{ma}) \qquad (2.6)$$

It is however, obvious that the equivalent network of fig. 2 involves only the currents in the main and cross-field axes of the rotor, if the provision are made to eliminate the winding currents from the induced speed voltages as given in equations. (2.1) and (2.2). From fig.1, the following mesh equations can be written:

$$I_{1m} = (V + I_{2m}Z_m)/(Z_1 + Z_m) \qquad (2.7)$$

$$I_{1a} = (V + I_{2a}Z_{ma})/(Z_{1a}+Z_c + Z_{ma}) \qquad (2.8)$$

Using equations (2.7) and (2.8), equation (2.1) and (2.2) can be re-written as:

$$V_{sm} = (-j (V + I_{2a}Z_{ma})/Z)(Z_mS_a) + jI_{2a} \ (jX_2 + Z_m)S_a$$

Expressing Z_{ma} in terms of Z_m, the equation reduces to

$$V_{sm} = (-jVS_aZ_m/Z-jI_{za} \ a^2Z_{2m}S_a/Z$$
$$= j (jX_2 + Z_m)S_aI_{2a}.$$

And adding $jI_{2a} \ a^2Z_m \ S_a/Z$ to either sides of the R.H.S. gives;

$$V_{sm} = (-jVS_aZ_m/Z) - I_{2a} (x_2-jZ + ja^2Z_{2m}/Z)S_a \qquad (2.9)$$

Similarly,

$$V_{sc} = (jVS_aZ_m/(Z_1+Z_m)) \ -1_{2m} \ (-X_2 + jZ_m - jZ_{2m}/(Z_I + Z_m))S_a \qquad (2.10)$$

Where $Z = Z_{ma} + Z_{Ia} + Z_c$

The mesh equations for the modified circuit of fig. 2.2 are

$$V_m = I_{2m}(Z_{Im} + Z_2) + V_{sm} \quad (2.11)$$

$$V_a = I_{2a}(Z_{1a} + Z_{2a}) + V_{sc} \quad (2.12)$$

Substituting for V_{sm} V_{sc}, and after some rearrangement, the above mesh equation can be written in the general form as:

$$V_{m1} = I_{2m}Z_{II} + I_{2a}Z_{12}S \quad (2.13)$$

$$V_{m2} = I_{2m}Z_{21}S + I_{2a}Z_{22} \quad (2.14)$$

Where

$$Z_{II} = Z_{1m} + Z_{2a}$$
$$Z_{22} = Z_{1a} + Z_{2a}$$
$$Z_{12} = (-X_2 + jZ_m - ja^2Z^2{}_m/Z)a$$

$$Z_{21} = (X_2 - jZ_m + jZ_m{}^2)/(Z_I + Z_m)a \quad (2.15)$$

Furthermore,

$$V_{mI} = V_m + jVS_aZ_m/Z$$
$$V_{m2} = V_a - jVS_aZ_m/(Z_I + Z_m) \quad (2.16)$$

V_m and V_a are the modified voltage sources as given by equation (2.3) and (2.4) respectively.

The solutions to the simultaneous equations (2.3) and (2.4) in terms of the impedances and voltages defined above are: From (2.3),

$$V_m - I_{2m}Z_{12}S = I_{2m}Z_{II}$$

$$\therefore I_{2m} = V_{mI} - I_{2a}Z_{12}S/Z_{II}$$

Substituting into (2.4)

$$((V_{m1} - I_{2a}Z_{12}S)/Z_{II})Z_{21}S + I_{2a}Z_{22} = V_{m2}$$
$$V_mZ_{12}S - I_{2a}Z_{12}Z_{21}S^2 + I_{2a}Z_{22}Z_{II} = V_{m2}Z_{II}$$
$$I_{2a} = (V_{m2}Z_{11}V_{mI}Z_{21}S)/Z_{11}Z_{22} - Z_{12}Z_{21}S^2$$
$$I_{2m} = (V_{mI}Z_{22} - V_{m2}Z_{12}S)/Z_{II}Z_{22} - Z_{12}Z_{21}S^2$$

A simple substitution of equivalent circuit parameters of the capacitor motor into the above mesh equation will yield the rotor circuit, current in the main and cross-field axes. The main and the auxiliary winding current can be calculated from equations (2.1) and (2.2) respectively. This method eliminates the necessary of a complex computer subroutine to solve a matrix. The other performance parameters such as the net torque developed, input and output powers, efficiency etcetera may be obtained using standard relations.

3. Simulation of models and interpretation of result

In the preceding section, effort was made to develop a new technique of performance prediction of a capacitor motor by deriving appropriate equivalent circuit using the concept of cross-field theory. The simulation technique of the development model is enunciated here. Due to the complex nature of the resultant equation, the whole process was programmed on an IBM AT machine using Fortran 90. To understand the steps taken in the software development effort, the modular programme design diagram is shown in modem programming, the Warnier Orr axiom is used. In this description, every system exhibits a hierarchy such that each level of the hierarchy, the elements of the system are ordered in terms of a sequence, or an alteration/selection, or a repetitive or a concurrence, or a recursion (Ilochi, 1996; Torrey and Guru, 1997). In using this axiom, a modular flow diagram is developed for the program written to solve the two-equation technique as shown in fig.3.

In our solution, motor constants obtained from design considerations and experiments (Trckey, 1957) for a 0.2Hp, capacitor-start induction motor were applied to the derived models. Theoretical results realized are illustrated graphically in figs. 4 through 7.

These results were compared, with those obtain using revolving-field theory (Trckey, 1957) and were almost duplicated. In figure 4 through 7, are shown the variations of the current with the main and cross-field axes of the rotor, and the main and auxiliary winding currents with various values of slip. The current increases directly as slip value increases. Hence, as the slip approaches unity, there is an increase of the winding currents and consequently, the torques.

4. Conclusion

A two-equation model has been developed for performance assessment of a capacitor motor by making use of thevenin theorem on the basis of cross-field theory.

The application of the two-equation method to any capacitor motor is simply straight forward and involves little more than the knowledge of simple network analysis. This approach has been found very useful to reduce the computer memory space and consequently the computing time and cost since it has eliminated a complex subroutine otherwise required to solve a 4 x 4 matrix.

A demonstration of the differences is computing time and cost was carried out and found that this method offers a reduction of as much as 45 percent. From the result comparism using both concepts the value are quite similar, suggesting that the performance of a single-phase motor can be predicted using either the double revolving-field or the cross-field concept. Hence, they supplement rather than contradict each other.

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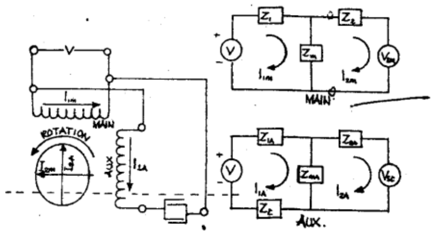

Fig. 1. Systematic diagram of a capacitor motor and its equivalent circuit representation.

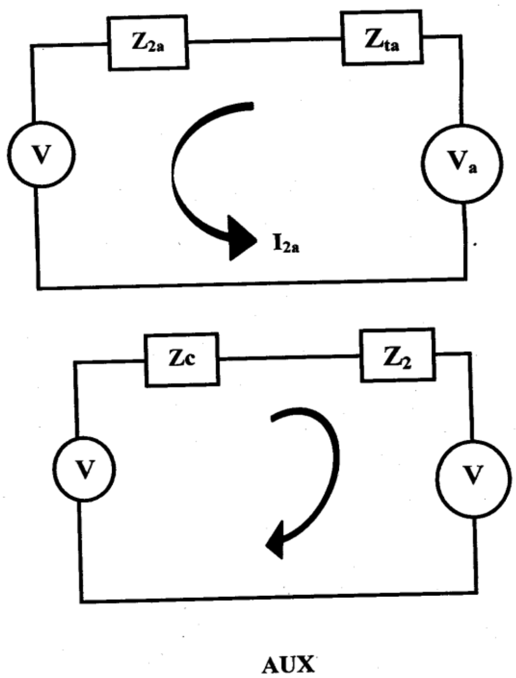

Fig. 2. Transformed equivalent circuit of a capacitor motor.

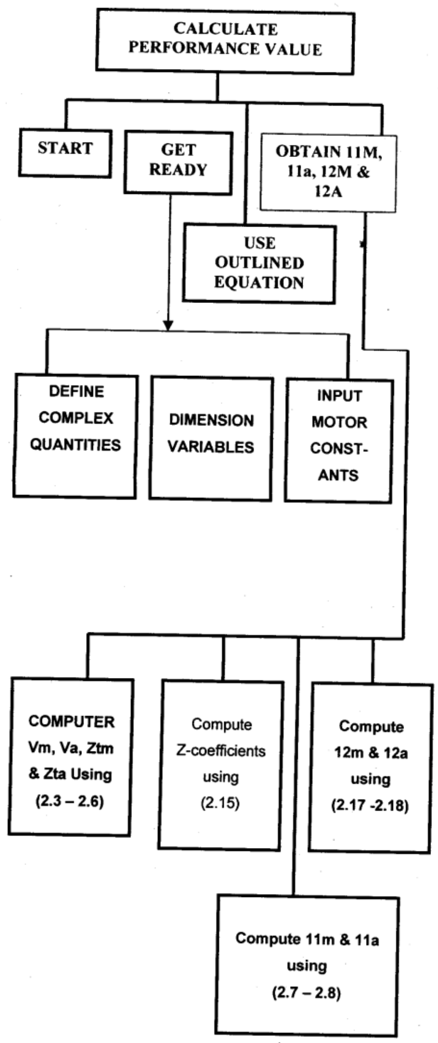

Fig. 3. Modular flow diagram of the cross-field analysis.

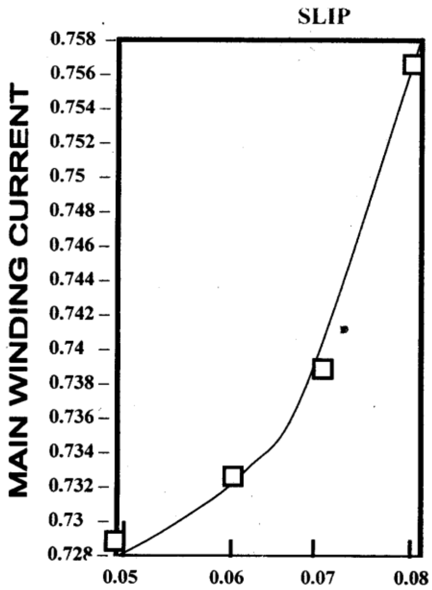

Fig. 4. Main winding current/slip characteristic curve.

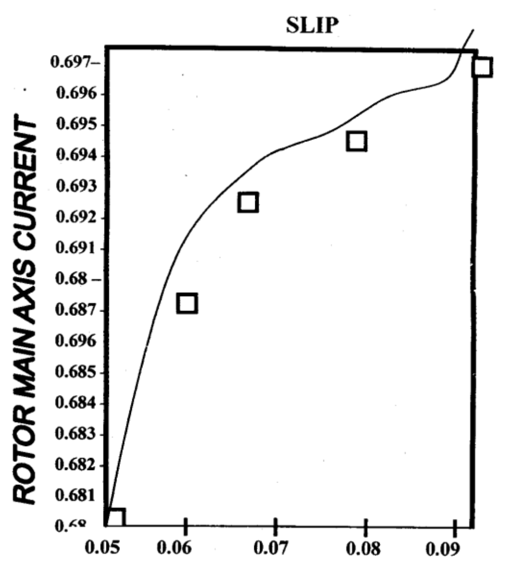

Fig. 6. Rotor main axis current/slip characteristic curve.

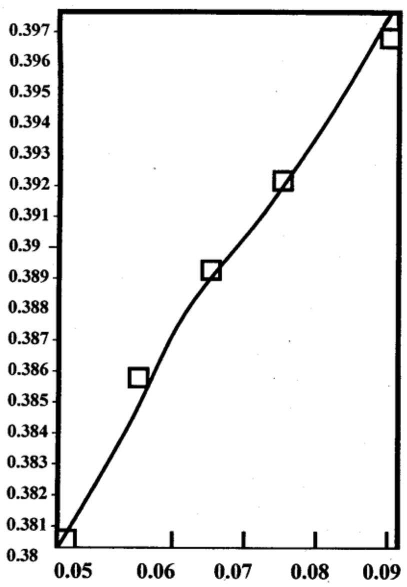

Fig. 5. Auxiliary current/slip characteristic curve.

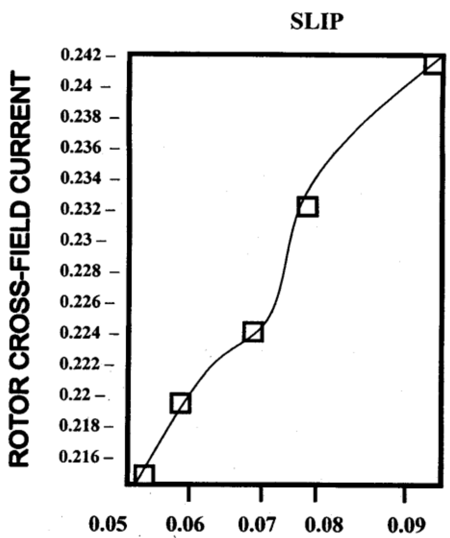

Fig. 7. Rotor X/fld, axis current/slip characteristic curve.