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Computational approach for optimum compressive strengths of glass fibre reinforced polyester composites

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Abstract

This Paper presents finite difference method and method of separation of variables as computational approaches for optimization of compressive failure response of Glass fibre reinforced polyester (GRP). A two dimensional multiple linear regression model obtained by subjecting replicated samples of GRP composites to compressive failure was found to be a two dimensional Laplace function , through transformation to partial differential equation(PDE). By passing the Finite difference model of the function through nine interior grid or mesh points of a composite region a system of nine by nine linear equations was developed and solved by Leibman method (Gauss-Seidel iteration). The Leibman method algorithms result was optimized by Visual Basic (VB) language Programme. The separation of variables method was employed to obtain three product solution models that solve the approximate 2-D PDE (Laplace) function. While both the finite difference method and method 0f separation of variables gave the ultimate compressive strength as 154 MPa, the method of separation of variables gave the minimum strength as 100MPa.Both the finite difference method and method of separation of variables showed that the compressive failure response of GRP composites could be represented by either elliptical model or exponential model.

Keywords: Optimum; Compressive strengths; GRP; Homogeneous PDE; Finite difference; Reinforced plastics

1. Introduction

Composites with their high strength to weight ratio have become very important in many technological applications such as in aerospace, automobile and medical industries. One of the key factors which make composites attractive for engineering applications is the possibility of property enhancement through fibre reinforcement. Composites are widely used today in aerospace and automobile industries. Currently in the USA industries utilize over 100,000 tonnes of reinforced plastics out of a total consumption of over one million tones, Crawford (1998).

However Christensen and DeTeresa (1997) reported the compressive failure of fibre composite materials as the most limiting property, also reported is the fact that the compressive failure in the fibre direction is much less in magnitude than the corresponding tensile stress at failure. The objective of this work is to model the optimum compressive failure parameter as strength. Kyriakides etal (1998) consistently reported the compressive strength of typical fibre reinforced matrix composites to be only 50 to 60 percent of their tensile strength. The importance of complete knowledge of composites properties is reported by Shati etal (1991).

Black and Adams.(1981) reported need for ensuring that the design stress be less than yield stress for ductile material and less than the ultimate stress for brittle material. The design properties of composite elements are very important because of increasing demand for design of light weight structures. In the study of compression members, buckling which is the compressive failure of slender or thin section subjected to axial compression is important because it occurs before the elastic limit of the material. Koshal (1998) reported the tensile strength of GRP composites as 303 MPa while Benhan and Warknock (1981) reported 300 MPa. Budiansky (1994), Chung and Weitzsman (1994), Kyriakides(1994)and Hsu et al (1998) used idealized macro-buckling mechanical models of fibre reinforced composites to establish that the compressive strengths of fibre composites subjected to compressive loads are only about 50% to 60% of their ultimate strength intension, while this paper attempts to use finite difference method to optimize compressive failure response of GRP composites working under normal conditions using Ihueze (2005) multiple regression model which was transformed to two-dimensional Laplace equation to enable analysis by Finite difference methods.

The Finite Difference Method and Method of Separation of Variables were used to model compressive failure response of GRP composite material while the Leibman method, Visual Basic iterative programme and method of separation of variables product solution models were used to obtain solution for the compressive failure response model of GRP composite material. Elliptical and exponential functions are found to fairly represent the compressive failure behaviour of GRP.

2. Theoretical analysis and review

2.1. Some engineering phenomena.

Sundaram etal (2003) reported that engineering phenomena can be broadly be grouped into three kinds, namely wave phenomenon, diffusion phenomenon and potential phenomenon. However some complex engineering phenomenon may be a combination of these, so that a second order (Linear or quasi linear) partial differential equation in two independent variables may be classified as hyperbolic, parabolic and elliptic equations respectively. By following Zill and Cullen (1989) method in classifying partial differential equation using the relation of linear second order PDE as

$$A\, \partial^2 u/\partial x^2 + B\, \partial^2 u/\partial x \partial y + C\, \partial^2 u/\partial y^2 + D\, \partial u/\partial x + E\, \partial u/\partial y + Fu = G \quad (1)$$

Where A, B, C, D, E, F, G are functions of x and y
If G(x, y) = 0, the equation is Homogeneous .If G(x, y) ≠ 0, the equation is Non-homogeneous. The homogeneous equation can further be analyzed if

$B^2 - 4AC > 0$, as Hyperbolic:
$B^2 - 4AC = 0$, as Parabolic
$B^2 - 4AC < 0$, as Elliptic

This means that the multiple dimensional compression equation of Ihueze (2005) is an elliptic homogeneous equation since $B^2 - 4AC = -4$ (B = 0, A = 1, C = 0)

2.1.1. Possible functions for approximation

Laplace equation

Elliptical function of simple wave function classically has been expressed as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (2)$$

This is a laplace equation in two variables and homogeneous linear partial differential equation with constant coefficient which could be solved analytically or by Numerical methods.

Poisson's equation
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$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \quad (3)$$

This is a nonhomogeneous linear equation which may be a combination of a wave phenomenon and diffusion phenomenon.

2.2. IHUEZE (2005) Compressive model and transformations.

Ihueze (2005) multiple linear regression model was transformed to two-dimensional Laplace equation to enable analysis by Finite difference method and method of separation of variables. The model was expressed as:

$$S = 154.0432 - 2.6797\, x_{11} - 11.5726 x_{22} \quad (4)$$

Where, S = critical stress,x_{11} = slenderness ratio,x_{22} = height or thickness of section

Compressive failure as a homogeneous function

By expressing (4) as

$$u = 154.0432 - 2.6797x - 11.5726y \quad (5)$$

and by partial differentiation of function with respect to variables,

$$\partial u/\partial x = -2.6797 \quad (6)$$

$$\partial^2 u/\partial x^2 = 0 \quad (7)$$

$$\partial u/\partial y = -11.5726 \quad (8)$$

$$\partial^2 u/\partial x_{22}^2 = 0 \quad (9)$$

By adding (7) and (9)

$\partial^2 u/\partial x^2 + \partial^2 u /\partial y^2 = 0$ (10)

This represents a Laplacian function, u in two variables x_{11} and x_{22}.

Other compressive responses of GRP composites are found in the work of Argon (1972), Cains etal (1994), Ihueze and Enetanya (2004), Poulsen etal (1997),Volger and Kyriakides (1997) and Zang and Latour (1993).

3. Methodology and modelling

Ihueze (2005) compressive failure response model of GRP composites was approximated to two-dimensional Laplace equation by transformation. The Finite Difference Method and Method of Separation of Variables were used to model compressive failure response of GRP composite material, while the Leibman method, Visual Basic iterative programme and method of separation of variables product solution models were used to obtain solution for the compressive failure response model of GRP composite material. The Flowchart for finite difference modelling and solution is presented in Fig 1.

3.1. Finite difference approximation of function

The approximation function for single valued function y(x) is established following Taylor's approximation method as follows:

$y(x + h) = y(x) + y^1(x)h + y^{11}(x)\, h^2/2 + y^{111}\, h^3/3! + \ldots$ (11)

also

$y(x - h) = y(x) - y^1(x)h + y^{11}(x)\, h^2/2 - y^{111}(x)\, h^3/3! + \ldots$ (12)

When h, step size is small, higher powers of h ie h^2, h^3, h^4 …. could be neglected so that

$y^1(x) = 1/h\, [y(x + h) - y(x)]$ (13)

$y^1 (x) = 1/h\, [y(x) - y (x - h)]$ (14)

By subtracting (11) and (12),

$y^1 (x) = 1/2h\, [y (x + h) - y (x - h)]$ (15)

By adding (11) and (12),

$y^{11} (x) = 1/h^2\, [y (x + h) - 2y(x) + y (x - h)]$ (16)

The right sides of (13), (14), (15), and (16) are called different quotients and the bracketed expressions in (13), (14), (15), and (16) are called finite differences

$$\left.\begin{array}{c} y(x+h) - y(x) \\ y(x) - y(x-h) \\ y(x+h) - y(x-h) \\ y(x+h) - 2y(x) + y(x-h) \end{array}\right] \text{Finite differences from (13), (14), (15) and (16)}$$

The first finite difference above is called the forward difference and the second is called the backward difference while both third and fourth finite differences are central difference approximations for the derivatives y^1 and y^{11} respectively. By considering a meshed rectangular region of mesh size h shown in Fig1 and employing Taylors expansion approximation, the difference quotients derived for the single valued function y(x) is employed for Laplacian equation of (2),

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

If u is the function of two variables, x and y the central differences can be expressed following Taylor's expansion approximation as

u(x+ h, y) –2u (x y) + u (x – h, y) and
u(x, y + h) –2u (x,y) + u (x, y - h)

So that following (16) we can write

$\partial^2 u/ \partial x^2 = 1/ h^2\, [u (x + h, y) - 2u (x, y) + u (x - h, y)]$ (17)

$\partial^2 u/ \partial y^2 = 1 / h^2\, [u (x, y + h) - 2u (x, y) + u (x, y - h)]$ (18)

By adding (17) and (18)

[u (x + h, y) + u (x, y + h) + u (x,y - h) – 4 u (x,y)]

$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 1/ h^2 = 0$

u(x + h,y) + u (x,y + h) + u (x – h, y) + u (x, y- h) – 4 (x, y) = 0 (19)

(19) can be simplified by using in (19)

$u(x, y) = u_{ij}$, $u(x + h,y) = u_{i+1,\, j}$, $u(x,y + h)\; u_{i,\, j+1}$, $u(x, y - h) = u_{i,\, j-1}$ to obtain

$u_{i+1,\, j} + u_{i,\, j+1} + u_{i-1,\, j} + u_{i,\, j-1} - 4u_{ij} = 0$ (20)

If the points of inter sections of mesh points are expressed as

Pij = P (ih, jh)

Where

i, j = integers of the lines designating horizontal and vertical mesh points. (20) could be rearranged to read

$u_{ij} = 1/4 (u_{i+1,\, j} + u_{i,\, j+1} + u_{i-1,\, j} + u_{i,\, j-1})$ (21)

(21), means that the value of four neighboring points must be evaluated to give the five-point approximation of Laplace equation.

3.1.1. Derivation of equations of function.

Problem Statement

Obtain the finite difference solution of the Laplacian function,

$\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 0$, subject to $0 < x < 2$, $0 < y < 2$

Procedures for solving systems of equations of function.

- Apply the finite difference function, uij at the nine interior mesh points.
- Apply boundary or Dirichlet condition to the nine nodes equations.
- Establish matrix equation of the nine equations above
- Solve the system of equations if diagonal dominance prevails by Gauss-seidel iteration
- Optimize solution by VB Language programme for Gauss –Seidel algorithms
- Compare result with any known classical report. This procedure is as presented in Fig1
- The boundary conditions specified a square region of length 2

By choosing the number of interval, n = 4
Mesh size, h = L/n = 2/4 = 1/2
Interior mesh points = $(n-1)^2 = (4-1)^2 = 9$
The problem is solved with Dirichlet conditions as shown in Fig.3

- Evaluation of u_{ij} at Interior Mesh Points.

The interior mesh points are by Fig 3, P_{11}, P_{12}, P_{13}, P_{21}, P_{22}, P_{23}, P_{31}, P_{32}, and P_{33},
So that the five points equation for function at 9 interior points are by (21) and considering the coordinates of P_{11}, P_{12}, P_{13}, P_{21}, P_{22}, P_{23}, P_{31}, P_{32}, and P_{33}

$u_{11} = \frac{1}{4} (u_{21} + u_{12} + u_{01} + u_{10}) = \frac{1}{4} (u_{21} + u_{12} + 308.0864)$ (22)

$u_{12} = \frac{1}{4} (u_{22} + u_{13} + u_{02} + u_{11}) = \frac{1}{4} (u_{22} + u_{13} + u_{11} + 154.0432)$ (23)

$u_{13} = \frac{1}{4} (u_{23} + u_{14} + u_{03} + u_{12}) = \frac{1}{4} (u_{23} + u_{12} + 308.0864)$ (24)

$u_{21} = \frac{1}{4} (u_{31} + u_{22} + u_{11} + u_{20}) = \frac{1}{4} (u_{31} + u_{22} + u_{11} + 154.0432)$ (25)

$u_{22} = \frac{1}{4} (u_{32} + u_{23} + u_{12} + u_{21})$ (26)

$u_{23} = \frac{1}{4} (u_{33} + u_{24} + u_{13} + u_{22}) = \frac{1}{4} (u_{33} + u_{13} + u_{22} + 154.0432)$ (27)

$u_{31} = \frac{1}{4} (u_{14} + u_{32} + u_{21} + u_{30}) = \frac{1}{4} (u_{32} + u_{21} + 308.0864)$ (28)

$u_{32} = \frac{1}{4} (u_{42} + u_{33} + u_{22} + u_{31}) = \frac{1}{4} (u_{33} + u_{22} + u_{31} + 154.0432)$ (29)

$u_{33} = \frac{1}{4} (u_{33} + u_{34} + u_{22} + u_{32}) = \frac{1}{4} (u_{22} + u_{32} + 308.0864)$ (30)

By setting $x_1 = u_{11}$, $x_2 = u_{12}$, $x_3 = u_{13}$, $x_4 = u_{21}$, $x_5 = u_{22}$, $x_6 = u_{23}$, $x_7 = u_{31}$, $x_8 = u_{32}$, $x_9 = u_{33}$.

The following 9 x 9 System of equations is obtained

$x_1 = 1/4\ (x_4 + x_2 + 308.0864)$ (31)

$x_2 = 1/4\ (x_5 + x_3 + x_1 + 154.0432)$ (32)

$x_3 = 1/4\ (x_6 + x_2 + 308.0864)$ (33)

$x_4 = 1/4\ (x_7 + x_5 + x_1 + 154.0432)$ (34)

$x_5 = 1/4\ (x_8 + x_6 + x_2 + x_4)$ (35)

$x_6 = 1/4\ (x_9 + x_3 + x_5 + 154.0432)$ (36)

$x_7 = 1/4\ (x_8 + x_4 + 308.0864)$ (37)

$x_8 = 1/4\ (x_9 + x_5 + x_7 + 154.0432)$ (38)

$x_9 = 1/4\ (x_6 + x_8 + 308.0432)$ (39)

To establish the diagonal dominance necessary for the application of Gauss-Seideliteration method the following matrix equation is established for (31) – (39) as

$$\begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} -308.0864 \\ -154.0432 \\ -308.0864 \\ -154.0432 \\ 0 \\ -154.0432 \\ -308.0864 \\ -154.0864 \\ -308.0864 \end{bmatrix} \quad (40)$$

This is an example of sparse and banded matrix. The absolute values of all the diagonal elements are respectively not less than the sum of the absolute values of the remaining elements of their rows; so that we have diagonal dominance of the coefficient matrix hence Gauss-Seidel iteration could be applied.

4. Computations and analysis

4.1. Numerical solution of FD model

Gauss-Seidel iteration and VB iterative programme were used to obtain solution for the system of linear equations of (31) – (39).

- Gauss-Seidel Iteration.

By taking the initial guesses as:

$x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = x_8 = x_9 = 0$

1st Iteration

By putting values in (31) – (39)

$x_1` = 1/4\ (0 + 0 + 308.0864) = 77.0216$
$x_2 = 1/4\ (0 + 0 + 77.0216 + 154.0432) = 57.7662$
$x_3 = 1/4\ (0 + 577662 + 308.0864) = 91.4632$
$x_4 = 1/4\ (0 + 0 + 77.0216 + 154.0432) = 57.7662$
$x_5 = 1/4\ (0 + 0 + 57.7662 + 57.7662) = 28.8831$
$x_6 = 1/4\ (0 + 91.4632 + 28.8831 + 154.0432) = 68.5974$
$x_7 = 1/4\ (0 + 57.7662 + 308.0864) = 91.4632$
$x_8 = 1/4\ (0 + 28.8831 + 91.4632 + 154.0432) = 68.5974$
$x_9 = 1/4\ (68.5974 + 68.5974 + 308.0864) = 111.3203$

2nd Iteration

$x_1` = 1/4\ (57.7662 + 57.7662 + 308.0864) = 105.9047$
$x_2 = 1/4\ (28.8831 + 91.4632 + 105.9047 + 154.0432) = 95.0730$
$x_3 = 1/4\ (68.5974 + 95.0736 + 308.0864) = 117.9394$
$x_4 = 1/4\ (91.4632 + 28.8831 + 105.9047 + 154.0432) = 72.5736$
$x_5 = 1/4\ (68.5974 + 68.5974 + 57.7662 + 57.7662) = 63.1818$
$x_6 = 1/4\ (111.3203 + 117.9394 + 63.1818 + 154.0432) = 111.6212$
$x_7 = 1/4\ (68.5974 + 72.5736 + 308.0864) = 112.3144$
$x_8 = 1/4\ (111.3203 + 63.1818 + 111.3149 + 154.0432) = 109.6950$
$x_9 = 1/4\ (111.6212 + 109.9650 + 308.0864) = 132.4182$

By similar procedures as above, 9 iterations that led to convergence are presented in Table1.

- VB Iterative Programme listing.

To still optimize FD results, VB Programme was developed as listed in Fig 6. The result of this programme is found in Table3. This programme solves the system (31)–(39) by iteration as listed in Fig.6.

4.2. Analytical solution of PDE model

- Separation of Variables Method.

Ihueze (2005) multiple regression model was transformed to PDE as in (10) as

$$\partial^2u/\partial x^2 + \partial^2u/\partial y^2 = 0$$

The method of separation of variables is well developed in Straud (2004) Sundaram etal (2003) and in Zill and Cullen (1989).

By considering the function of two variables u(x, y) and assuming the product solution of (1) in the form

$$u(x, y) = X(x)\ Y(y) \quad (41)$$

Where X is the function of x alone and Y is a function of y alone.

By substituting (41)in (10) and separating variables

$$X^{11}Y + XY^{11} = 0 \quad (42)$$

$$X^{11}/X = Y^{11}/Y \quad (43)$$

L.H.S is a function of x and R.H.S is a function of y and for the equality to hold each must be a constant say p, so that (43)can be expressed as

$$X^{11}/X = Y^{11}/Y = p \quad (44)$$

Three possibilities of p: positive, zero or negative

Case 1: $p = \wedge^2$, being real

From (5)

$X^{11} - X\wedge^2 = 0,\ Y^{11} + Y\wedge^2 = 0$

By solving the homogeneous equations

$X = A_1 e^{\wedge x} + A_2 e^{-\wedge x},\ Y = A_3 \cos \wedge y + A_4 \sin \wedge y$

A_1- A_4 are arbitrary constant

$$u(x, y) = X(x)\ Y(y) = (A_1e^{\wedge x} + A_2e^{-\wedge x})(A_3 \cos \wedge y + A_4 \sin \wedge y) \quad (45)$$

Case 2: p = 0

From (45)

$X^{11} = 0$ and $Y^{11} = 0$

By solving for x and y

$x = B_1x + B_2,\ y = B_3y + B_4$

Where B_1, B_2, B_3 and B_4 are arbitrary constants.

$$u(x, y) = X(x)\ Y(y) = (B_1x + B_2)(B_3y + B_4) \quad (46)$$

Case 3: $p = -\wedge^2$, being real

From (44)

$X^{11} + \wedge^2 X = 0$, $Y^{11} - \wedge^2 Y = 0$
By solving the equations for x and y

$X = c_1 \cos \wedge x + c_2 \sin \wedge x$, $Y = c_3 e^{\wedge y} + c_4 e^{-\wedge y}$
Where c_1, c_2, c_3, and c_4 are arbitrary constants.

$$u(x, y) = (c_1 \cos \wedge x + c_2 \sin \wedge x)(c_3 e^{\wedge y} + c_4 e^{-\wedge y}) \quad (47)$$

As the equation $u_{xx} + u_{yy} = 0$ describes a potential phenomenon, any of the product solutions of (45)–(47)may suit the physical phenomenon. So one has to understand the physical problem and select the solution accordingly.

- Simulation with Equations (45), (46) and (47)

This requires that appropriate boundary conditions of the physical problem must be chosen to evaluate the constants of (45)– (47).

Boundary conditions: By considering a square region of composite structure subjected to buckling, the following boundary conditions are used to solve the boundary value problem
$\leq x \leq 2, 0 \leq y \leq 2$
u(0, 0) = 154 MPa, u(0, 2) = 154 MPa,
u(2, 0) = 154 MPa, u(2, 2) = 154 MPa

- Evaluation with Boundary Conditions.
- Case 1

$u(x, y) = (A_1 e^{\wedge x} + A_2 e^{-\wedge x}) (A_3 \cos \wedge y + A_4 \sin \wedge y)$

- By expanding the R.H.S of (45)and assuming $\wedge = 1$

$$u(x, y) = A_1 A_3 e^x \cos y + A_1 A_4 e^x \sin y + A_2 A_3 e^{-x} \cos y + A_2 A_4 e^{-x} \sin y \quad (48)$$

By putting the boundary conditions u(0,0) = 154, u(0,2) = 154, u(2,0) = 154, u(2,2) = 154 in (45)respectively to solve for the constants, systems of equations are obtained from equations as

$$A_1A_3 + 0A_1A_4 + A_2A_3 + 0A_2A_4 = 154 \quad (49)$$

$$0.9994A_1A_3 + 0.0349A_1A_4 + 0.9994A_2A_3 + 0.0349A_2A_4 = 154 \quad (50)$$

$$7.3890A_1A_3 + 0A_1A_4 + 0.1353A_2A_3 + 0.0047A_2A_4 = 154 \quad (51)$$

$$7.385A_1A_3 + 0.2579A_1A_4 + A_2A_3 + 0.0047A_2A_4 = 154 \quad (52)$$

$$\begin{bmatrix} 1.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.9994 & 0.0349 & 0.9994 & 0.0349 \\ 7.3891 & 0.0000 & 0.1353 & 0.0000 \\ 7.385 & 0.2579 & 1.0000 & 0.0047 \end{bmatrix} \begin{bmatrix} A_1A_3 \\ A_1A_4 \\ A_2A_3 \\ A_2A_4 \end{bmatrix} = \begin{bmatrix} 154 \\ 154 \\ 154 \\ 154 \end{bmatrix} \quad (53)$$

(53)is solved by L U – decomposition to obtain

$A_1A_3 = 18.3578$, $A_1A_4 = -462.9818$, $A_2A_3 = 135.6422$, $A_2A_4 = 465.6294$
So that (45) or (47)can be expressed as

$$u(x, y) = 18.3578\ e^x \cos y - 462.9818\ e^x \sin y + 135.6422\ e^{-x} \cos y + 465.6294\ e^{-x} \sin y \quad (54)$$

- Case 2

By expanding (46)

$$u(x, y) = B_1B_3xy + B_1B_4x + B_2B_3y + B_2B_4 \quad (55)$$

By putting the boundary conditions u(0, 0) = 154, u(0, 2) = 154, u(2, 0) = 154, u(2, 2) = 154 in (55) respectively to solve for the constants, systems of equations are obtained as

$$B_2B_4 = 154 \quad (56)$$

$$2B_2B_3 + B_2B_4 = 154 \quad (57)$$

$$2B_1B_4 + B_2B_4 = 154 \quad (58)$$

$$4B_1B_3 + 2B_1B_4 + 2B_2B_3 + B_2B_4 = 154 \quad (59)$$

$$\begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 2.0000 & 1.0000 \\ 0.0000 & 2.0000 & 0.0000 & 1.0000 \\ 2.0000 & 2.0000 & 1.0000 & 4.0000 \end{bmatrix} \begin{bmatrix} B_1B_3 \\ B_1B_4 \\ B_2B_3 \\ B_2B_4 \end{bmatrix} = \begin{bmatrix} 154 \\ 154 \\ 154 \\ 154 \end{bmatrix} \quad (60)$$

Similarly L U – Decomposition or triangulation method is used to obtain the value of the constants as:

$B_1B_3 = 0$, $B_1B_4 = 0$, $B_2B_3 = 0$, $B_2B_4 = 154$

When values of these constants are replaced in (55)
u(x, y) = 154

- Case 3

By expanding (47)
and using $\wedge = 1$

$$u(x, y) = c_1c_3 \cos xe^{-y} + c_1c_4 \cos xe^{y} + c_2c_3 \sin xe^{-y} + c_2c_4 \sin xe^{y} \quad (61)$$

By substituting the boundary conditions,
u(0, 0) = 154, u(0, 2) = 154, u(2, 0) = 154 and u(2, 2) = 154 in turn into (61) ,the following system of linear algebraic equation is obtained.

$$c_1c_3 + c_1c_4 = 154 \quad (62)$$

$$0.1353c_1c_3 + 7.3891c_1c_4 = 154 \quad (63)$$

$$0.9994c_1c_3 + 0.9994c_1c_4 = 154 \quad (64)$$

$0.1352c_1c_3 + 7.385c_1c_4 + 0.0047c_2c_3 + 0.2579c_2c_4 = 154$ (65)

$$\begin{bmatrix} 1.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.1353 & 7.3891 & 0.0000 & 0.0000 \\ 0.9994 & 0.9994 & 0.0349 & 0.0349 \\ 0.1353 & 7.3850 & 0.0047 & 0.2579 \end{bmatrix} \begin{bmatrix} C_1C_3 \\ C_1C_4 \\ C_2C_3 \\ C_2C_4 \end{bmatrix} = \begin{bmatrix} 154 \\ 154 \\ 154 \\ 154 \end{bmatrix} \quad (66)$$

By employing LU – decomposition
$c_1c_3 = 135.6422$, $c_1c_4 = 18.3578$, $c_2c_3 = 2.3994$, $c_2c_4 = 0.2481$

so that (61)becomes
$u(x, y) = 135.6422 \cos xe^{-y} + 18.3578 \cos x\, e^{y} + 2.3994 \sin x\, e^{-y} + 0.2481 \sin xe^{y}$ (67)

4.2.1. Numerical analysis with the 3 cases analytical models

The computations using the three cases equations of (54), (55) and (67) are presented in Table3.

4.3. 3-D graphics

The 3-D Graphics were produced with excel spread sheet package using Table 4 ,Table 5 and Table 6 and presented in figures 7 , 8 and 9 for analysis. 3-D Graphics of case1 using Table 6 is presented in Fig.9.Fig.7and 8 represents elliptical function while Fig. 9 represents exponential function. Tables 4, 5 and 6 are predictions of three product solution models at interior and regional points.

5. Discussion of results

Both finite difference method and method of separation of variables showed that the compressive failure response may be approximated by potential phenomenon or elliptical function SUCH as the two dimensional Laplace function.

Though solution of PDE may lead to infinite number of solutions but physical problems need unique solutions however, the method of separation of variables yields three product solutions whose unique solution is established by understanding the physical problem. Inclusion of geometric terms may indicate potential problem solution so that the product solution model containing geometric terms may be selected as a solution of u(x,y).

The optimum compressive and buckling strengths of GRP composites have been evaluated as 154MPa and 100MPa respectively. While Table 1 and Table 2 of Guass-seidel iterations for finite difference solution gave the approximate compressive strength of GRP as 154MPa, Table 4 of analytical solution gave 154MPa and 100MPa representing ultimate and buckling strengths of GRP.

The 3 -D plots of Fig.7 also showed that stress distribution in composites is described by mixed phenomena which could be approximated by an elliptical function, the centre of the GRP composite having the lowest strength. The 3-D plots Fig.8 of analytical solution also while showing the composite response function as elliptical portrayed the fact that the composite under compression is weaker at the centre. The strength of GRP composite at the centre or along its neutral axis is lower than its compressive strength of 154MPa. This means that composites under compressive loading could fail within loading less than its elastic limit stress, estimated as 100MPa.This study also indicates that the elastic limit of GRP is more than 100MPa.

The 3-D graphics of Fig.7-9 also show the maximum and minimum compressive strengths as 154MPa. and 100MPa.respectively.

6. Conclusions

The finite difference method and method of separation of variables with approximating function as Laplace function representing compressive failure response of GRP composite strength has been successfully applied with the following deductions:

- The compressive failure response of GRP composites can be represented by a Laplacian homogeneous function.
- The finite difference grid or meshing method enables evaluation of function at interior nodes when the values of at least four neighbouring nodes are known.
- Gauss-Seidel iteration method provides a converging algorithm for evaluation of a Laplacian function
- For a homogeneous GRP composite the value of the function at any interior point is the same
- Given the coordinate of an interior point of FDM (mesh), (i,j) as in Fig. 7 ,of Laplacian function, $u_{i,j}$ of two dimension and other four surrounding boundary points coordinates, (i+1,j), (i-1,j), (i,j+1) and (i,j-1) the value of the function, $u_{i,j}$ at the interior node is estimated as

$$u_{ij} = 1/4\,(u_{i+1,\,j} + u_{i,\,j+1} + u_{i-1,\,j} + u_{i,\,j-1})$$

- Elliptical and exponential functions are found to fairly represent the compressive failure behaviour of GRP
- The method of separation of variables showed that given the coordinates of points,(x,y) the compressive strength could be estimated with the models,

ELLIPTICAL

$u(x, y) = 135.6422 \cos x e^{-y} + 18.3578 \cos x\, e^{y} + 2.3994 \sin x\, e^{-y} + 0.2481 \sin x e^{y}$ of (54)

EXPONENTIAL

- $u(x, y) = 18.3578\, e^{x} \cos y - 462.9818\, e^{x} \sin y + 135.6422\, e^{-x} \cos y + 465.6294\, e^{-x} \sin y$ of (67)

The compressive strengths of GRP composites are about 154 MPa and 100 MPa.for maxima and minima (buckling strength) respectively.

Table 1
Finite difference solution results by traditional gauss -seidel iteration

variable / *iteration*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
1	77.0216	57.7662	91.4632	57.7662	28.8831	68.5974	91.4632	68.5974	111.3203
2	105.9047	95.0736	117.9394	72.5736	63.1818	111.6212	112.3144	109.9650	132.4182
3	118.9334	113.5245	133.3080	112.1182	111.8072	132.8942	132.5424	132.7028	143.4209
4	108.4323	126.8977	141.9696	126.7063	129.8002	142.3085	141.8739	142.2845	148.1699
5	140.4226	141.5589	147.9884	141.5350	141.9217	148.0308	147.9765	148.0278	151.0363
6	147.7951	147.9371	151.0136	147.9341	147.9825	151.0188	151.0121	151.0185	152.5309
7	150.9894	151.0072	152.5281	151.0068	151.0128	152.5288	152.5279	152.5287	153.2860
8	152.2251	152.5273	152.5273	151.0068	152.5280	153.2857	153.2856	153.2857	153.6645
9	153.2853	153.2855	153.6644	153.6644	153.2856	153.5697	153.6644	153.6644	153.8301

Table2
Visual basic programme output

iteration / *Variable*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
x_1	77.0216	105.9047	124.5584	136.8186	145.1207	149.5431	151.7883	152.9152
x_2	154.0410	154.0421	154.0426	154.0429	154.0431	154.0431	154.0432	154.0432
x_3	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_4	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_5	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_6	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_7	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_8	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_9	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
Iteration / Variable	9	10	11	12	13	14	15	16
x_1	153.4791	153.7611	153.9022	153.9727	154.0079	154.0256	154.0344	154.0388
x_2	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_3	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_4	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_5	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_6	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_7	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_8	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432
x_9	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432	154.0432

Table 3
Analysis of separation of variables models

Coordinates		*Case 1(uij)*	*Case 2 (uij)*	*Case 3 (uij)*
x	y			
0.5	0.5	108.3370	154.0000	112.5510
0.5	1.0	104.1330	154.0000	99.8114
0.5	1.5	99.9107	154.0000	112.5510
1.0	0.5	90.3102	154.0000	112.5510
1.0	1.0	80.8119	154.0000	99.8114
1.0	1.5	71.3074	154.0000	112.5510
1.5	0.5	95.3391	154.0000	112.5510
1.5	1.0	78.1219	154.0000	99.8114
1.5	1.5	60.90528	154.0000	112.5510
0.0	0.0	154.0000	154.0000	153.9853
0.0	2.0	153.9985	154.0000	153.9853
2.0	0.0	153.8001	154.0000	153.9853
2.0	2.0	36.7384	154.0000	153.9853

Table 4
Gauss-Seidel iteration result and 3-D analysis

154.0432	*154.0432*	*154.0432*	*154.0432*	*154.0432*
154.0432	153.6644	153.5697	153.8301	154.0432
154.0432	153.2855	153.2856	153.6644	154.0432
154.0432	153.2853	153.2855	153.6644	154.0432
154.0432	154.0432	154.0432	154.0432	154.0432

Table 5
Separation of variable case 3 and 3-D analysis

153.9853	*153.9853*	*153.9853*	*153.9853*	*153.9853*
153.9853	112.2694	112.5513	112.5443	153.9853
153.9853	99.8114	99.8136	99.8008	153.9853
153.9853	112.5510	112.5533	112.5513	153.9853
153.9853	153.9853	153.9853	153.9853	153.9853

Table 6
Separation of variable case 1 and 3-D analysis

153.9985	*95.7833*	*61.9128*	*43.7654*	*36.7384*
154.3944	99.9107	90.3102	95.3391	148.7999
154.4226	104.1330	80.8119	78.1219	95.1145
154.3954	108.3370	71.3074	60.90528	127.0669
154.0000	112.5983	99.8021	112.3599	153.8001

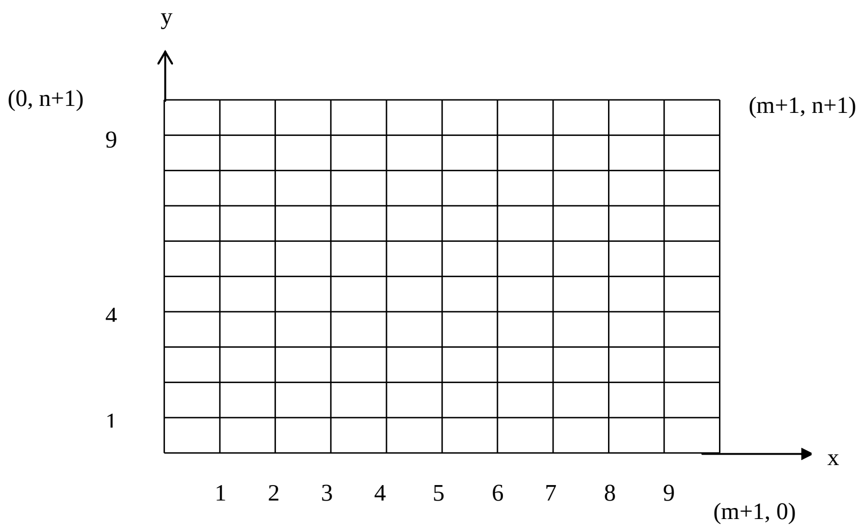


Fig. 1. Flowchart for FD modeling and solution.

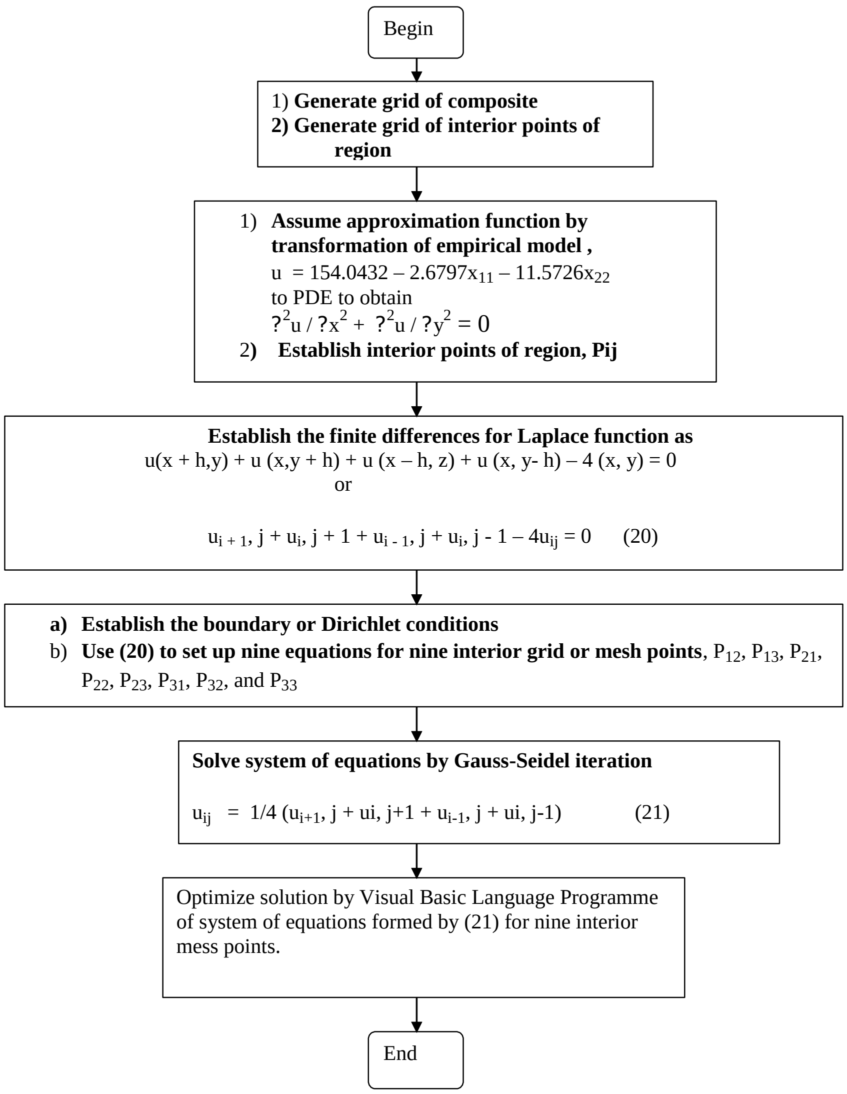
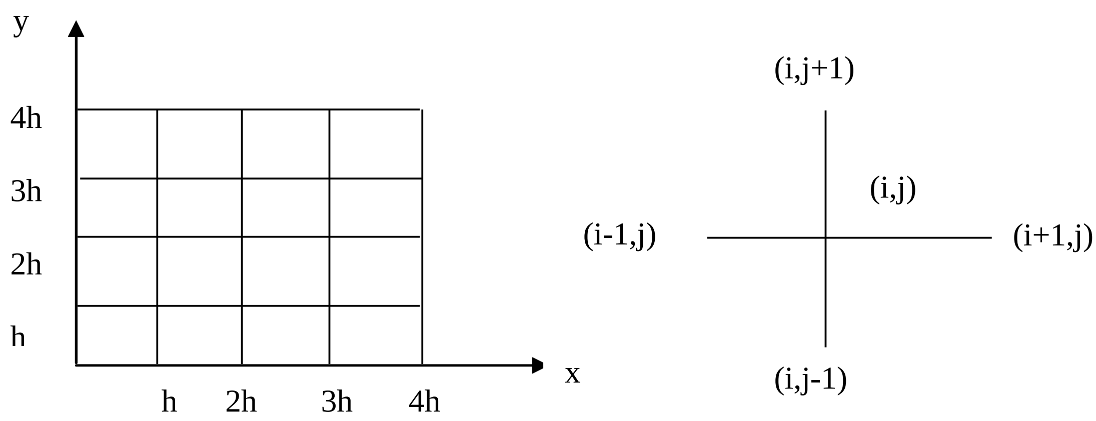


Fig. 2. Typical grid system for FD solution of functions of two variables.



a) Regional mesh, b) Interior and boundary points.

Fig. 3. Finite Difference Model (FDM):
m = number of intervals for coordinating x variable,
n = number of intervals for coordinating y variable.

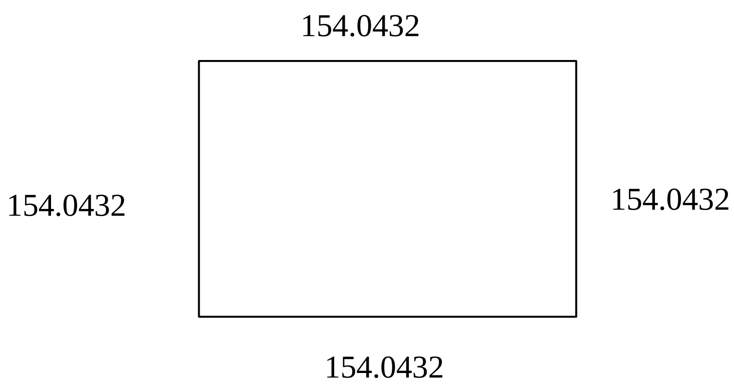
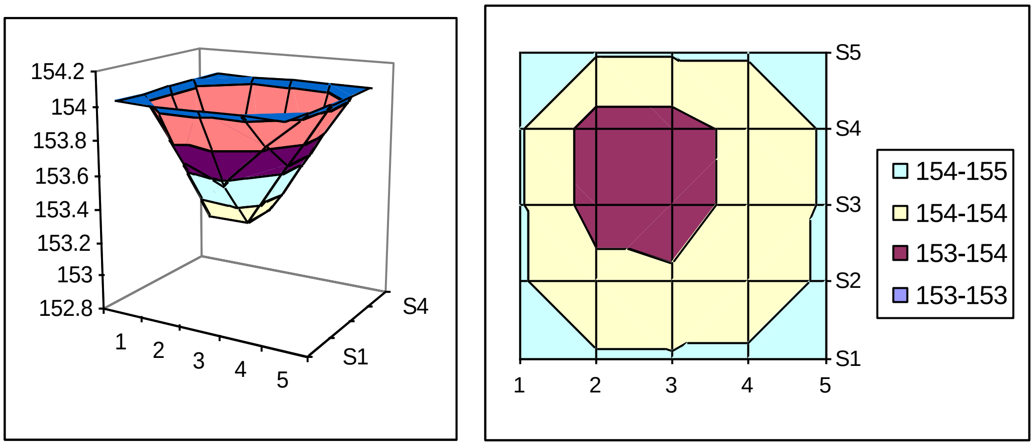


Fig. 4. Dirichlet boundary conditions.

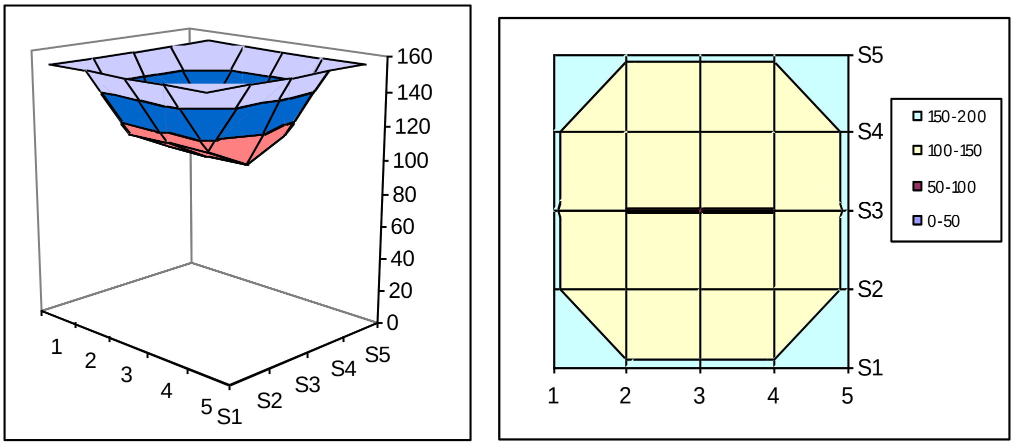
153.6644	153.5697	153.8301
153.2855	153.2856	153.6644
153.2853	153.2855	153.6644

Fig..5. Compressive stress distribution in 9 interior meshes predicted by Gauss – Seidel algorithm.



a) 3-D Plot b) surface plot

Fig.7. 3-D Graphics of Gauss-Seidel iteration function results.



a) 3-D plot b) surface plot

Fig. 8. 3-D Graphics of separation of variables case3, showing elliptical model.

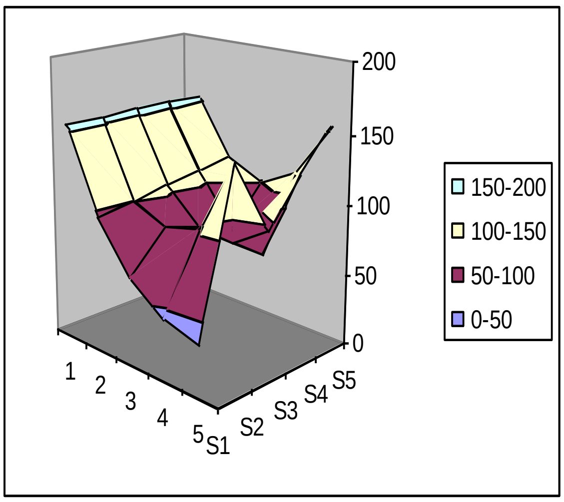


Fig. 9. 3-D Graphics of separation of variables case1, showing exponential model.

Visual Basic Programme Listing

```
Private Sub Command1_Click ()
Rem ITERATION
Dim a(20, 20) 'declaring a table to hold iteration values
Dim x(1 To 9)
Rem initializing 9 variables as zero
For i = 1 To 9
 x(i) = 0
Next i
Form1.Print
"===================================="
Form1.Print “ITERATION"
          Form1.Print
For t = 1 To 9    'controls number of variables
For u = 1 To 16     'controls number of iterations
x(1) = 0.25 * (x(4) + x(2) + 308.0864)
x(2) = 0.25 * (x(5) + x(3) + x(1) + 154.0432)
x(3) = 0.25 * (x(6) + x(2) + 308.0864)
x(4) = 0.25 * (x(7) + x(5) + x(1) + 154.0432)
x(5) = 0.25 * (x(8) + x(6) + x(2) + x(4))
x(6) = 0.25 * (x(9) + x(3) + x(5) + 154.0432)
x(7) = 0.25 * (x(8) + x(4) + 308.0864)
x(8) = 0.25 * (x(9) + x(5) + x(7) + 154.0432)
x(9) = 0.25 * (x(6) + x(8) + 308.0864)
 a(t, u) = x(t)
  Form1.Print Format$(a(t, u), "#000.0000"); "    ";
          Next u
Form1.Print
          Next t
End Sub
```

Fig. 6. Visual basic programme listing.

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