

Computational approaches to control of industrial processes

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Abstract

This paper presents a numerical approach to assessment of process capability and product quality standards. The samples of this study were drawn from IBETO process for battery production. The statistical sampling method, the average range method and the single range method were used to establish the mean of distribution, the average standard deviation of samples and the average range of samples to establish the process limits. Process control was achieved using classical relations and analogies to establish process capability and Process capability index. The population mean was evaluated to be 3.4 years and the average standard deviation of the process as 0.75 years while the process capability was estimated to be 4.5 years. Also the action limits were found to be 4.4 years and 2.4 years for the mean. The upper and lower specification limits were evaluated as 5.53years and 1.27years while the process capability index was estimated as 1.2. The coefficient of variation was found to be 21%, indicating low variability. The IBETO process for battery production therefore produces within specification. Hence this paper has presented procedures for assessment and control of quality of manufactured products.

1. Introduction

To meet design and customers specifications for a product for quality assurance, Quality Engineers perform quality tests of a process before mass production. Vonderembse and white (1991) reported that the control of a process begins with the understanding of the variability of the process. The quality of a product depends upon the application of materials, men, machines and manufacturing conditions was reported by Hansen and Ghare(2006). It is a common misconception that automatic machines will produce identical components. Unfortunately, real life considerations interfere with this theoretical ideal, the properties of work piece material vary along the length of the bar, the machine tool slide way must have clearances to allow them to move, and lubrication conditions will constantly be changing, and such random variable will mean that the actual size of the parts produced will vary, distributed closely around the target sizes, Black et al. (1996). When the quantity involved is large the pattern of variation can be studied on a statistical basis. It then becomes possible to assess the quality achieved by the process without testing every piece produced. A statistical method which reveals the pattern of variation in a product provides a more certain basis for the assessment of the quality of a

large volume of work than would be provided by a detailed inspection of some parts made without reference to the pattern of variability present.

Quality failures occur due to various causes was reported by Sharma (2000). Studies indicated that more than 50% of quality failures are due to human errors at various levels, such as understanding of customer's requirements, manufacturing, inspection, testing, packaging, and design.

The objectives of quality control are, to decide about the standard of quality of a product that is easily acceptable to the customer and at the same time economical to maintain, to take necessary steps so that the products which are below standard should not reach to the customers, to take different measures to improve the standard of quality of product, to evaluate, maintain and improve quality standards during various stages of manufacture so as to build the desired quality in the finished product.

The process capability is commonly used to establish the relationship between the tolerance specified for the component and the standard deviation for the process that will make it. The capability of the process can be established once the specifications and the standard deviation of product parameter are measured and the process control is assured. This paper

presented procedures for assessment and control of quality of manufactured products of a process as well as procedures for the evaluation of product capability.

Classical numerical methods, the sampling method, the average range method and the single range method found in Hansen and Ghare(2006), Black et al. (1996), Sharma (2000), Koshal(1998), Dieter(2000),Walpole (1982),Stroud (1995)were used as mixed method to find the mean, range, and standard deviation of distribution of measurements from where the process capability and capability index were evaluated as in Dieter(2000),using the mean and average standard deviation of distribution evaluated through the three methods employed.

2. Methodology and analysis

Classical numerical methods, the sampling method, the average range method and the single range method found in Hansen and Ghare(2006), Black et al. (1996), Sharma (2000), Koshal(1998), Dieter(2000),Walpole (1982),Stroud (1995)were used as mixed method to find the mean, range, and standard deviation of distribution of measurements from where the process capability and capability index were evaluated as in Dieter(2000)using the mean and average standard deviation of distribution evaluated through the three methods employed. Forty batteries produced by battery production process of IBETO factory were used in order to access their service lives. Eight replicated samples were made and each of the samples has sample size of five as in Table 1.

2.1. Analysis of population data

Black et al (1996), Sharma(2000), Koshal(1998), Dieter(2000), Walpole (1982)and Stroud (1995) were used to establish statistical parameters of quality of the car battery lives as follows:

- *Grouping of data for analysis of table 1*
- Class intervals, c is chosen not less than 5 as recommended by Walpole (1982)
 $c = 7$

- Class width w , is estimated by dividing the range R , with the class interval

$$R = 4.7 - 1.6 = 3.1$$

$$W = R/C = 3.1/7 = 0.443$$

The approximate class width can not be less than 0.443, a class width of $w = 0.5$ is therefore chosen as recommended in Walpole (1982)

- *Establishment of class boundaries*

From data of table 1 the lowest measure is 1.6. The lowest class boundary is therefore set as $1.6 - 0.05 =$

1.55 the upper class boundary of 1.55 is obtained by adding class width, 0.5 to the lower class boundary 1.55 to obtain, $1.55 + 0.5 = 2.05$. The first class boundary is therefore $1.55 - 2.05$ The remaining six class boundaries of the seven intervals is obtained by adding the class width, 0.5 to the lower and upper class limits to obtain second class boundary as $1.55 + 0.5 - 2.05 + 0.5 = 2.05 - 2.55$ and third class as $2.05 + 0.5 - 2.55 + 0.5 = 2.55 - 3.05$, so that the seven class boundaries are established as presented in Table 2

- *Establishment of class interval*

Since the class interval is within the class boundaries and the first class boundary is $1.55 - 2.05$, adding 0.05 to the lower class boundary and removing 0.05 from the upper class boundary establishes the first class interval $1.6 - 2.0$. The remaining class interval are obtained by adding class width, 0.5 to the lower and upper class limits as $1.6 + 0.5 - 2.0 + 0.5 = 2.1 - 2.5$ for the second class interval. For the third class interval $2.1 + 0.5 - 2.5 + 0.5 = 2.6 - 3.05$, subsequently we have the following class intervals as in (Table 2)

- *Establish class mark and its frequency*

x = lower class boundary plus (+) upper class boundary
Divided by Two (2),Walpole (1982)

$$= \frac{LCB + UCB}{2}$$

The results of analysis are presented in Table 2.

2.1.1. Estimation of population parameters

Computation of Mean of population, \bar{x} is given by Walpole (1982), as

$$\bar{x} = \frac{\sum fx}{\sum f} \quad (1)$$

so that with Table 2 values in (1), $\bar{x} = 137.5/40 = 3.44$ years.

- *Computation of variance and standard deviation*

For grouped data, the computing relation for variance is expressed as

$$S^2 = \frac{(\sum f_i x_i^2 - (\sum x_i)^2)}{(N - 1)} \quad (2)$$

so that by substituting Table 3 values in (2)

$$S^2 = 0.50, \sigma = s = 0.71 \text{ years}$$

- Coefficient of Variation, $V = \sigma/\bar{x} = 0.71/3.44 = 0.21$

2.1.2. Computation of mean of 8 consecutive samples

The columns of Table1 represent measurements of 8 consecutive samples of battery lives, 5 samples taken at a time. It involves taking measurements of five batteries used at eight towns in Nigeria. The means and ranges are computed using Table 4

The mean of samples average, \bar{x}^{11} and Mean range, R^1 are computed with Table 4 as:

- *Mean of samples average (grand average of samples)*

This is estimated with the relation of Dieter (2000),as

$$\bar{X}^{11} = \frac{1}{m} \sum_{i=1,1,2,3,8}^m X_i^1 \tag{3}$$

Where, m = number of samples, \bar{x}^1 = sample average

By using the values of $\bar{x}_i^1 = 3.2, 3.4, 3.4, 3.7, 3.4, 3.4, 3.5, 3.3$ from Table 4 and $m = 8$ in (3), $\bar{x}^{11} = 3.41$ years

- *Mean range*

This is also estimated with the relation of [6], as

$$R^1 = \frac{1}{m} \sum_{i=1,1,2,3,8}^m R_i \tag{4}$$

By using the values of $R_i = 2.5, 2.7, 0.6, 1.9, 1, 1.1, 2.8, 1.1, m = 8$ from Table 4 in (4), $R^1 = 1.7$ years

2.2. Process control model and charts

Table 1 and Table 2 were used to compute the control limits for mean and range of samples following methods of Basterfield (1986), Mittag and Rinne(1993) and Breyfogles(1992) as warning and action limits as presented in Table5. The control charts are graphics for the mean and range of samples established after evaluation of average samples sizes and each sample range with Table2 as presented in Table6 and Figure1 and Figure2. Measurements are not expected to fall out of action limits and specifications.

2.2.1. Measurement and evaluation with population data

- *Control limits for average.*

Black et al. (1996) expressed Action Limits and Warning limits for sample average respectively as

$$x = \bar{x}^{11} \pm (3.09\sigma)/\sqrt{n} \tag{5}$$

and

$$x = \bar{x}^{11} \pm (1.96\sigma)/\sqrt{n} \tag{6}$$

where

σ = bulk standard deviation,

n = sample size,

\bar{x}^{11} = bulk mean

By putting $\sigma = 0.71, \bar{x}^{11} = 3.44, n = 5$ in (5) and (6)

The values for action limit and warning limits were obtained as:

Upper action limit, UAL = 4.4 years, Lower action limit, LAL = 2.5 years

Upper Warning Limit, UWL = 4.1 years, Lower Warning Limit, LWL = 2.8 years

Alternatively the control limits for mean could be established using the relations of Sharma(2000),and Table 6 in (7) as

$$x = \bar{x}^1 \pm A_2 R^1 \tag{7}$$

$$UAL = \bar{x}^1 + A_2 R^1, \quad LAL = \bar{x}^1 - A_2 R^1$$

$$UAL = 3.4 + 0.58 * 1.7 = 4.4 \text{ years}, \quad LAL = 3.4 - 0.58 * 1.7 = 2.4$$

- *Control limits for range*

Following Sharma(2000),and Table 9, the upper and lower limits for the range are estimated as

$$UCL_R = D_4 * R^1 = 2.11 * 1.7 = 3.6 \text{ years}, \quad LCL_R = D_3 * R^1 = 0 * 1.7 = 0$$

Koshal (1998) approach could be used with Table 10 as:

- Action limits for mean = $\bar{x}^1 \pm A^1 0.001 R^1, UAL = 3.4 + 0.594 * 1.7 = 4.4, LAL = 2.4$

- Warning limits for mean = $\bar{x}^1 \pm A^1 0.025 R^1, UWL = 3.4 + 0.377 * 1.7 = 4.4, LWL = 2.4$

- Action limit for Range = $R^1 * D_{0.999} = 1.7 * 2.34 = 4.4$ years,

- Warning limit for Range = $R^1 * D_{0.975} = 1.7 * 1.81 = 3.1$ years

The values of various factors, $A^1, D_4, D_{0.999}, A_2, D_3, D_{0.975}$ used in estimating the control charts action and warning limits for mean and range are found in Koshal (1998), Table 9 and Table 10.

2.2.2. Measurement and evaluation with samples data

- *Evaluation with mean of samples*

This involves finding the average of all samples measurements, finding the grand average of samples and Computing the upper and lower control limits ,

By using Table 6 values in (3) $\bar{x}^{11} = 3.41$ years, $R^1 = 1.7$ years

- *Control limit for mean*

Following Sharma(2000),using Table 9 the control limits were established as follows. For a sample size of 5

$$A_2 = 0.58, D_3 = 0, D_4 = 2.11, d_2 = 2.33$$

- *Action limit*

Upper control limit

$$x^1 = \bar{x}^{11} + A_2 R^1 = 3.41 + 0.58 * 1.7 = 4.4 \text{ years} = \text{UCL}$$

Lower control limit

$$x^1 = \bar{x}^{11} - A_2 R^1 = 3.41 - 0.58 * 1.7 = 2.4 \text{ years} = \text{LCL}$$

- *Control limit for range*

The Control Limits for range are established as:

$$\text{UCL}_R = D_4 * R^1 = 2.11 * 1.7 = 3.6 \text{ years,}$$

$$\text{LCL}_R = D_3 * R^1 = 0 * 1.7 = 0 [2, 4, 5]$$

- *Average range method*

This is an alternative method for finding process capability from samples of product being produced. By using results of (4) in

$$\sigma^1 = R^1/d_2 [2] \quad (8)$$

σ^1 = range relative standard deviation.

and also by employing Table A7 of [2] under sample size 5

$$d_2 = 2.326, \sigma^1 = 1.7/2.326 = 0.73 \text{ years}$$

- *Single range method*

This method depends on the average of samples standard deviations; it involves the initial computations of each sample standard deviation, s^1 with the classical relation for variance used as,

$$S^2 = \frac{\sum (x_i - \bar{x}^1)^2}{n-1} \quad (9)$$

Computations for standard deviation of each sample were made as follows using Table 2 and results presented in Table 7. From Table 7 a, b, c, d, e, f, g, h. the standard deviation of samples were estimated with (9) and presented in Table 8 as follows:

$$s^2 = 2.18/4 = 0.545, s = 0.74, s^2 = 4.79/4 = 1.1975, s = 1.1$$

$$s^2 = 0.23/4 = 0.0575, s = 0.24, s^2 = 2.51/4 = 0.6275, s = 0.79$$

$$s^2 = 0.74/4 = 0.185, s = 0.43, s^2 = 0.84/4 = 0.21, s = 0.46$$

$$s^2 = 4.9/4 = 1.225, s = 1.11, s^2 = 0.74/4 = 0.185, s = 0.4$$

The average sample standard deviation is related in [2] as

$$s^1 = c_2 \sigma \quad (10)$$

where

σ = Bulk standard deviation of distribution or population standard deviation

c_2 = Factor, estimated in Table A7 of Hansen and Ghare (2006) for sample size 5

$$c_2 = 0.8407$$

From (10) and Table 7, $s^1 = 5.3/8 = 0.66 \text{ years}$

$$\sigma = s^1/c_2 = 0.66/0.8407 = 0.785 \text{ years}$$

2.2.3. Process capability index and tolerance specification

Breyfogles,(1992) gave relations for predicting the following process specification estimates as follows:

$$\text{USL} = \bar{x}^{11} + 3 \sigma \quad (11)$$

$$\text{LSL} = \bar{x}^{11} - 3 \sigma \quad (12)$$

$$C_p = (\text{USL} - \text{LSL})/6 \sigma \quad (13)$$

Where, USL= upper specification limit, LSL=lower specification limit,

C_p = process capability index

The process capability index expressed by Dieter (2000) is the ideal or theoretical capability index, because the individual observations may not be centered on the mean, Dieter (200) gave two relations for predicting the actual process capability index as:

$$C_{pk1} = (\text{USL} - \bar{x}^{11})/3\sigma \quad (14)$$

$$C_{pk2} = (\bar{x}^{11} - \text{LSL})/3\sigma \quad (15)$$

By using $\bar{x}^{11} = 3.4 \text{ years}$ and the average standard deviation of distribution, C_p is obtained as follows:

$$\sigma = (0.71 + 0.73 + 0.785) / 3 = 0.75 \text{ years, USL} = 5.65 \text{ years,}$$

$$\text{LSL} = 1.15 \text{ years,}$$

$$C_{p1} = 1, C_{p2} = 1 \text{ by (14) and (15), } \sigma = 0.71 \text{ and using (11) and (12)}$$

$$\text{USL} = 5.53, \text{LSL} = 1.27, 3 \sigma = 2.13 \text{ and by employing (14, 15),}$$

$$C_p = 1.2$$

2.2.4. Design specification and process capability

The three major steps in the production of any item are, design, production and inspection so that the essence of process control is to ensure that the process produces within design specifications. Process capability is the best quality product of a process which is evaluated in [6, 9, 10, and 11] as

$$PC = 6 \sigma \tag{16}$$

So that by using $\sigma = 0.75$ in (16)
 $PC = 4.5$ years

2.2.5. Process validation

The upper and lower production limits were evaluated as 4years and 2.4years respectively while upper and lower specification limits were evaluated as 5.53years and 1.27years respectively showing that the process is under control. The excel graphics of Table 6 are presented in Figure 1 and Figure 2 as mean and range control charts for process control.

3. Discussions

The parameters for the assurance of process capability were numerically evaluated. The means of eight samples were evaluated as presented in Table 4, while the action limits (upper and lower) were evaluated as 4years and 2.4years for upper action and lower action limits respectively considering the method of [4] as presented in Tables 5. Since the estimated means of Table 4 are within the action limits the process is within control.

The variability of the process was also estimated by measuring the standard deviation of distribution as 0.71years showing that variability of the process is low

with the coefficient of variation estimated as 21%. Figure3 shows the variability of samples standard deviation within the distribution confirming the population or distribution standard deviation to be within the range 0.66 – 0.75.

The process capability index and tolerance specification evaluation show that the process variability is low and the $C_p = 1.2$, shows that the process is producing within the upper and lower specification limit evaluated as 5.53years and 1.27years, above all, the mean control chart of Figure1 and range control chart of Figure 2 show that the process under study is producing within the mean and is under control.

4. Conclusions

It is Important that before producing in large quantities to ensure that the implementing process and its supervisors via control charts is actually capable of meeting the required specifications with respect to the expectation. The process capability index of 1.2 and tolerance specification evaluation that showed process variability as being low showed that the process is producing within the upper and lower specification limits evaluated as 5.53 years and 1.27 years. The IBETO Process for battery production is hence appropriate battery production process.

Table 1
 IBETO car battery lives

Sample	Car Battery lives, years							
	1	2	3	4	5	6	7	8
Measurements, years	2.2	4.1	3.5	4.5	3.2	3.7	3.0	2.6
	3.4	1.6	3.1	3.3	3.8	3.1	4.7	3.7
	2.5	4.3	3.4	3.6	2.9	3.3	3.9	3.7
	3.3	3.1	3.7	4.4	3.2	4.1	1.9	3.4
	4.7	3.8	3.2	2.6	3.9	3.0	4.2	3.5

Table 2
 Group class data distribution

Class Interval	Class Boundaries	Class Mark (x) = (LCB + UCB)/2	Frequency (f)	f(x)
1.6 – 2.0	1.55 – 2.05	1.8	2	3.6
2.1 – 2.5	2.05 – 2.55	2.3	2	4.6
2.6 – 3.0	2.55 – 3.05	2.8	5	14
3.1 – 3.5	3.05 – 3.55	3.3	15	49.5
3.6 – 4.0	3.55 – 4.05	3.8	8	30.4
4.1 – 4.5	4.05 – 4.55	4.3	6	25.8
4.6 – 5.0	4.55 – 0.05	4.8	2	9.6
SUM		23.1	40	137.5

Table 3
Computation of variance data

	<i>Class Mark x_i</i>	<i>Frequency f_i</i>	x_i^2	$f_i x_i$	$f_i x_i^2$
	1.8	2	3.24	3.6	6.48
	2.3	2	5.29	4.6	10.58
	2.8	5	7.84	14	39.20
	3.3	15	10.89	49.5	163.35
	3.8	8	14.44	30.4	115.52
	4.3	8	18.49	25.8	110.94
	4.8	2	23.04	9.6	46.08
SUM	23.1	N = 40		137.5	492.15

Table 4
Computation of means and range of consecutive samples of battery

	<i>Car Battery lives, years</i>							
Sample	1	2	3	4	5	6	7	8
Measurements, years	2.2	4.1	3.5	4.5	3.2	3.7	3.0	2.6
	3.4	1.6	3.1	3.3	3.8	3.1	4.7	3.7
	2.5	4.3	3.4	3.6	2.9	3.3	3.9	3.7
	3.3	3.1	3.7	4.4	3.2	4.1	1.9	3.4
	4.7	3.8	3.2	2.6	3.9	3.0	4.2	3.5
Sum, years	16.1	16.9	16.9	18.4	17	17.2	17.7	16.3
Mean, years	3.2	3.4	3.4	3.7	3.4	3.4	3.5	3.3
Range, years	2.5	2.7	0.6	1.9	1	1.1	2.8	1.1

Table 5
Control limits estimated with different models

	<i>Mean</i>				<i>Range</i>	
	UAL	LAL	UWL	LWL	UCL _R	LCL _R
Black (1996)	4.4 yrs	2.5 yrs	4.1	2.8	4.8	3.7
Sharma 2000	4yrs	2.4yrs			3.6 yrs	0
Koshal (1993)	4.4	2.4	4.4	2.4	4.4	3.1

Table 6
Means and ranges of samples, extracted from table 4

<i>Sample Number</i>	\bar{x}^j	<i>R</i>
1	3.2	2.5
2	3.4	2.7
3	3.4	0.6
4	3.7	1.9
5	3.4	1.0
6	3.4	1.1
7	3.5	2.8
8	3.3	1.1

Table 7
Computation of data for standard deviation of samples

a) Sample 1, $\bar{x}^1 = 3.2$, $n = 5$, $n-1 = 4$

<i>x</i>	$x - \bar{x}^1$	$(x - \bar{x}^1)^2$
2.2	-1	1
3.4	0.2	0.04
2.5	-0.7	0.49
3.3	0.1	0.01

4.7	0.8	0.64
Sum		2.18

b) Sample 2, $\bar{x}^1 = 3.4$, $n = 5$, $n-1 = 4$

x	$x-\bar{x}^1$	$(x-\bar{x}^1)^2$
4.1	0.7	0.49
1.6	-1.8	3.24
4.3	0.9	0.81
3.1	-0.3	0.09
3.8	0.4	0.16
Sum		4.79

c) Sample 3, $\bar{x}^1 = 3.4$, $n=5$, $n-1 = 4$

x	$x-\bar{x}^1$	$(x-\bar{x}^1)^2$
4.5	0.8	0.64
3.3	-0.4	0.16
3.6	-0.1	0.01
4.4	0.7	0.49
2.6	-1.1	1.21
Sum		2.51

d) Sample 4, $\bar{x}^1 = 3.7$, $n = 5$, $n-1 = 4$

x	$x-\bar{x}^1$	$(x-\bar{x}^1)^2$
3.5	0.1	0.01
3.1	-0.3	0.09
3.4	0	0
3.7	0.3	0.09
3.2	-0.2	0.04
Sum		0.23

e) Sample 5, $\bar{x}^1 = 3.4$, $n=5$, $n-1 =4$

x	$x-\bar{x}^1$	$(x-\bar{x}^1)^2$
3.2	-0.2	0.04
3.8	0.4	0.16
2.9	-0.5	0.25
3.2	-0.2	0.04
3.9	0.5	0.25
Sum		0.74

f) Sample 6, $\bar{x}^1 = 3.4$, $n=5$, $n-1 = 4$

x	$x-\bar{x}^1$	$(x-\bar{x}^1)^2$
3.7	0.3	0.09
3.1	-0.3	0.09
3.3	-0.1	0.01
4.1	0.7	0.49
3.0	-0.4	0.16
Sum		0.84

g) Sample 7, $\bar{x}^1 = 3.5$, $n = 5$, $n-1 = 4$

x	$x-\bar{x}^1$	$(x-\bar{x}^1)^2$
3	-0.5	0.25
4.7	1.2	1.44
3.9	0.4	0.16
1.9	-1.6	2.56

4.2	0.7	0.49
Sum		4.9

h) Sample 8, $\bar{x} = 3.3$, $n = 5$, $n-1 = 4$

x	$x-\bar{x}$	$(x-\bar{x})^2$
2.6	-0.7	0.49
3.7	0.4	0.16
3.1	-0.2	0.04
3.4	0.1	0.01
3.5	0.2	0.04
Sum		0.74

Table 8

Variation of standard deviation of product samples.

Sample Number, m	Standard deviation, s
1	0.74
2	1.1
3	0.24
4	0.79
5	0.43
6	0.46
7	1.11
8	0.43
Sum	5.3

Table 9

Factors A_2 , D_3 , D_4 , d_2 , Sharma (2000)

No of units in sample	A_2	D_3	D_4	d_2
2	1.88	0	3.27	1.13
3	1.02	0	2.57	1.69
4	0.73	0	2.28	2.06
5	0.58	0	2.11	2.33
6	0.48	0	2.00	2.53
7	0.42	0.08	1.92	2.70
8	0.37	0.14	1.86	2.85
9	0.33	0.18	1.82	2.97
10	0.31	0.22	1.78	3.08
11	0.27	0.26	1.72	3.17
12	0.27	0.28	1.72	3.26
13	0.25	0.31	1.69	3.34
14	0.24	0.33	1.67	3.41
15	0.22	0.35	1.65	3.47

Table 10

Constants for limits on mean and Range control charts (Koshal, 1998)

Sample size, n	Normal mean Chart		Range Chart		Modified Limits	
	Warning limits $A^{10.025}$	Action limits $A^{10.001}$	Warning limits $D_{0.975}$	Action limits $D_{0.999}$	Warning limits $A^{110.025}$	Action limits $A^{110.01}$
2	1.128	1.937	2.81	4.12	1.51	0.80
3	0.668	1.054	2.17	2.98	1.16	0.77
4	0.476	0.750	1.93	2.57	1.02	0.75
5	0.377	0.594	1.81	2.34	0.95	0.73

6	0.3116	0.498	1.72	2.21	0.90	0.71
7	0.274	0.432	1.66	2.11		
8	0.274	0.384	1.62	2.04		
9	0.244	0.347	1.58	1.99		
10	0.202	0.317	1.56	1.93		
11	0.186	0.294	1.53	1.91		
12	0.174	0.274	1.51	1.87		

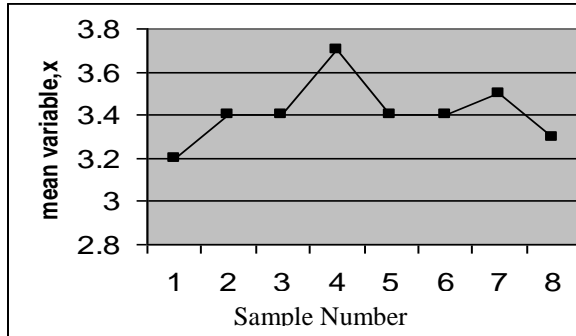


Fig. 1. Mean control chart for measurements.

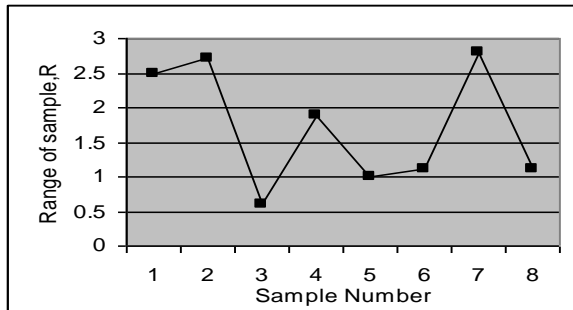


Fig. 2. Range control chart for measurements.

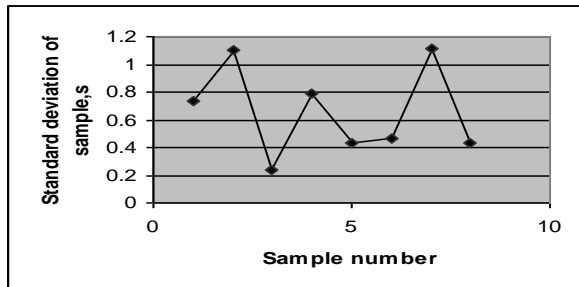


Fig. 3. Variation of standard deviation of distribution.

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