JEAS

Journal of Engineering and Applied Sciences 4 (2008) 88 - 94

JOURNAL OF ENGINEERING AND

Investigation into the dynamic slenderness of reinforced concrete multi-cell box culverts for traffic uses

P.D. Onodagu, C.A. Chidolue, C.H. Aginam

Department of Civil Engineering, Nnamdi Azikiwe University, Awka

Abstract

This paper investigated the dynamic responses of most commonly cast-in-place reinforced concrete box culverts; and used the fundamental period of natural vibration as the adequate criterion to assess the dynamic slenderness of a given thickness of each of the simulated systems. In the course of this work, a given span and a given rise with variations in the thickness were numerically analysed until the fundamental period of vibration exceeded 0.7 seconds, (0.7s). And from the results obtained, it was found that almost all the cast-in-place reinforced concrete box culverts constructed for traffic uses are slender structures. This paper, consequently from the correlated results, recommended a range of ratios of thickness to rise to be adopted in the design and construction of reinforced concrete box culverts that would make the structures dynamically moderately rigid.

Keywords: Fundamental frequency and period; Single degree of freedom, (SDOF); Multi-cell box culvers; Dynamic slenderness

1. Introduction

A reinforced concrete (r. c.) box culvert is a structure specially constructed to serve dual purposes; the first is to act as a transverse drain or waterway under road; and the second is to carry the action (load) of moving traffic across the opposite sides of the opening channel. Consequent upon, two action cases must be analysed when designing a r. c. box culvert. The first action case is that due to static condition, and its analysis is not quite complicated since the analysis follows the principles of static equilibrium conditions. The second action case is that due to dynamic condition, and its analysis is much more complicated and it usually gives wholesome worrisomeness to the structural engineers, [Meyer, 1991]. The vibration induced to the r. c. box culvert by the transient laden traffic is essentially important for two reasons. The first is because of the registered increase in the stresses above those due to static action (load) application; and the second is because of the excessive vibration usually experienced by persons standing on the structure, [Biggs, 1964]. However, the first problem is empirically accounted for by the adoption of “impact factor”, which is simply a specified percentage of the axle actions above the actual evaluated load. The American Association of State Highway and Transportation Officials (AASHTO) recommended a 30% impact factor for a r. c. box culvert directly exposed to traffic; and any culvert which has a fill depth less than 600mm is considered to be directly exposed to traffic, [Portland Cement Association, 1975]. But the second problem which characteristically has psychological effect of impairing public confidence in the structure is not accounted for, even with the adoption of impact factor. Therefore, it is specially this second problem that this paper is sought to investigate on. Consequently, this paper is proposing investigation into the dynamic response of r. c. multi-cell box culverts with the view of evaluating the dynamic slenderness of most commonly cast-in-place r. c. box culverts. Thus, the simulated r. c. box culverts are the dual boxes and triple boxes with variations in spans and heights; and also with variations in thickness.

2. The fundamental frequency and period of vibration

The fundamental principle in the dynamic analysis of structures is the determination of the natural frequencies (eigen frequencies) associated with the natural vibration of the elastic system, [Biggs, 1964;

Rao, 2006]. However, only the fundamental frequency is paramount since resonance at the lowest frequency results in maximum dynamic effects, [Darkov, A. 1983]. The fundamental period, which is an inverse of the fundamental frequency, depends upon the mass and stiffness (rigidity) of the structure, and in the dynamic analysis these two quantities are of paramount importance. Flexible structures have longer natural periods, and are more susceptible to dynamic effects. And in the rigidity classification for dynamic structures, any structure having its fundamental period greater than 0.7 seconds is classified as flexible (slender) structure; and such a structure usually exhibits excessive vibrations during dynamic excitations, [Scarlat, A. S, 1996]. This paper, therefore, evaluates the fundamental periods of the simulated r. c. box culverts and thence determines the limiting thickness at which it might be considered slender structure.

3. Dynamic modelling of reinforced concrete multi-cell box culverts

Almost all engineering structures are continuous and possess infinite number of degrees of freedom, [Clough R. W. & Penzien, J. 1993; Polyakov, S. V. 1985; Osadebe, N. N., 1999]. However, in actual fact, the number of degrees of freedom is virtually being determined by the choice scheme of a design scheme, depending on the degrees of approximation to which the investigation of the real object is possible, [Feodosyev, V., 1973]. Therefore, in the course of this work, each of the simulated r. c. multi-cell box culverts is modelled as a structure with Single Degree of Freedom (SDOF) by assuming lumped mass element concentration at the right corner joint of roof slab and external wall, (see figures 2 & 3).

3.1. The governing differential equation for a structure with undamped sdof

The governing differential equation for undamped vibration is obtained by employing D'Alembert's principle for dynamic equilibrium equations.
Thus,

$$M\frac{d^2x}{dt^2} + Kx = P(t) \qquad (1)$$

Where
M = mass element
K = Stiffness Coefficient
x = displacement of the centre of mass
P (t) = exciting force.

In this work, we assume that the system is performing natural vibration, and therefore the exciting force is assumed to be zero. Then equation (1) becomes:

$$M\frac{d^2x}{dt^2} + Kx = 0 \qquad (2)$$

Or in compact form:

$$M\ddot{x} + Kx = 0 \qquad (2a)$$

Introducing the usual notation:

$$w^2 = \frac{K}{M} \qquad (3)$$

where
ω = natural frequency of vibrations.
Then the governing differential equation is:

$$\ddot{x} + w^2x = 0 \qquad (4)$$

In this work, what constitutes the mass element, M, are the weights of fill of 0.5m-lateritic soil and 50mm asphaltic surfacing. Our selection of the mass element was guided by the recommendation of minimum depth of fill material to be 20% of the internal width of the box culvert, [Standard Specification, 1991]. The evaluated mass element was thus 948.369kg/m. Since we are dealing with SDOF, equation (3) becomes much more important, hence:

$$w = \sqrt{\frac{K}{M}} \qquad (5)$$

And

$$T = \frac{2p}{w} \qquad (6)$$

Where
ω = fundamental cyclic frequency.
T = fundamental period.

3.2. Evaluation of stiffness coefficient

The analysis for Stiffness Coefficient, K, does not follow the Shear Building Model, (SBM) approach since it has been demonstrated that unless the flexural rigidity ratio of beam to column $\left[\frac{EI_b}{EI_c}\right]$ is greater than 15, the assumption of infinitely rigid horizontal member is inadequate, [Osadebe, N. N., 1998]. And in practice, r. c. box culverts are usually constructed with all the structural members having the same thickness. Therefore, the approach adopted follows a closed-analysis based on Classical Displacement method Model, (CDMM). By taking a unit width of the system, figures 4 and 5 show the basic systems for CDMM. Thus the compatibility equation generated in its matrix form is given by:

$$[r][x] + [R] = 0 \tag{7}$$

where
[r] = symmetrical matrix of Joint Unit Reaction due to unit joint rotations.
[x] = the column matrix of unknown rotation
[R] = the column matrix of moment reaction due to unit lateral displacement at the floor level.

The bending moment on each of the vertical structural member is given by:

$$M_{ij} = S_{ij} + \sum_{i=1}^{n} x_i \bar{M}_i \tag{8}$$

Where
S_{ij} = moment at point i due to unit lateral displacement at point j
x_i = actual rotation at point i
$\bar{M}_{ij}$ = moment at point i due to unit rotation at point j.

Therefore, the resulting Shearing force is given by:

$$Q_{ij} = \frac{\left[M_{ij} - M_{ji}\right]}{L_{ij}} \tag{9}$$

Subsequently

$$K = \sum_{i=1}^{n} Q_{ij} \tag{10}$$

Where
Q_{ij} = Shearing force at point i on the vertical structural member ij
n = number of vertical structural members.
K = Stiffness Coefficient.

3.3. Dynamic response of simulated reinforced concrete multi-cell box culverts

The dynamic responses of each of the simulated r. c. multi-cell box culvert with a constant lumped mass but with variations in the thickness of the structural members are numerically investigated. With the Stiffness Coefficient, K, evaluated from equation (10), and by applying equation (5) and subsequently equation (6), the respective fundamental periods are evaluated. The dynamic responses were investigated for various variations in effective height, Z as well as in effective span, X for dual boxes and triple boxes.
Where

$$Z = B + h$$
$$X = A + h$$

4. Results and discussions

Tables 1-19 show the dynamic responses of the simulated structures at various sizes and thicknesses. From the results obtained, it was found that at certain thicknesses for a given span and varied rises (heights), the structure is flexible (slender). Similarly, at certain thicknesses for a given rise and varied spans, the structure is slender (flexible). Also, from tables 3 and 5, it was found that rise (height) of box culvert influences the dynamic response more than the span.
Figures 6 - 11 are the graphical representations of the fundamental periods against thicknesses. By using a limiting fundamental period of 0.7s, it was found that the required ratio of thickness to rise, $\left(h/Z\right)$, for most commonly cast-in-place multi-cell box culverts to be dynamically moderately rigid is in the range of 17.5% to 21%; and less than 17.5% will render the structure to be dynamically slender.

5. Conclusion and recommendation

From the above results, it is quite obvious that most of the r. c. multi-cell box culverts constructed for traffic uses are dynamically slender structures. It is essential that adequate measures be taken by the Highway and Transportation Officials to reduce excessive vibrations due to traffic on such important structures by ensuring that adequate thicknesses for the structural members are maintained, both during designs and constructions. It is equally recommended that the designers of r. c. box culverts should use the thickness to rise ratio $\left(h/Z\right)$

range found in this work as a guide in the selection of thickness of the structural members.

References

Adrian, S. S., 1996. Approximate Methods in Structural Seismic Designs. E and FN Spon, Chapman and Hall, London.

Biggs, J. M., 1964. Introduction to Structural Dynamics. McGraw – Hill Book Company, New York.

Clough, R. W., Penzien, J., 1993. Dynamics of Structures. McGraw – Hill, New York.

Darkov, A., 1983. Structural Mechanics. Mir Publishers. Moscow, 4th Edition.

Feodosyev, V., 1973. Strength of Materials. Mir Publishers, Moscow, 2nd Edition.

Meyer, C., 1991. Reinforced – Concrete Frames Subjected to Cyclic Load. Structures Subjected to Repeated Loading, Stability and Strength; Edited by Narayanan, R and T. M. Roberts., Elsevier Science Publishers ltd, London.

Polyakov, S. V., 1985. Design of Earthquake Resistant Structures. Mir Publishers, Moscow.

Portland Cement Association, 1975. Concrete Culvers and Conduits; Printed in U. S. A.

Osadebe, N. N., 1998. Free vibrations of multi-storey frames with flexible horizontal members. NSE Technical Transactions, Vol. 33, No. 4, 30 – 36.

Osadebe, N. N., 1999. An Improved MODF model simulating some system with distributed mass. A Journal of the University of Science and Technology, Kumasi, Ghana. Vol. 19, No. 1, 2 and 3.

Rao, S. S., 2006. Mechanical Vibrations. Pearson Education, Inc. and Dorling Kindersley Publishing Inc., Low Price Edition..

Standard Specifications, 1991; www.boxculverts.org.uk/publications/standard-specification-1991 PDF.

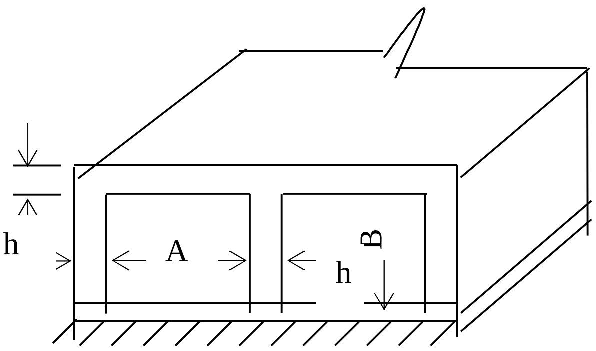

Fig.1. Dual-cell box culvert showing clear dimensions and thickness.

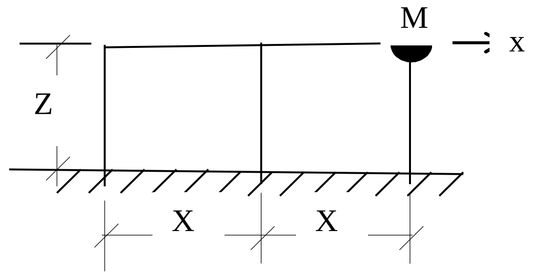

Fig. 2. Dynamic model for a dual-cell box culverts.

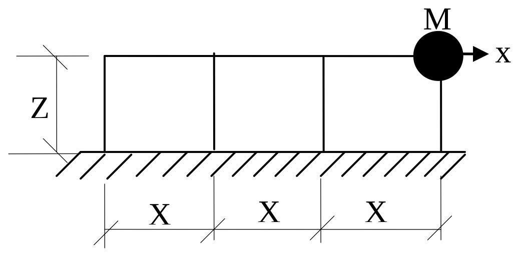

Fig.3. Dynamic model for a triple-cell box culvert.

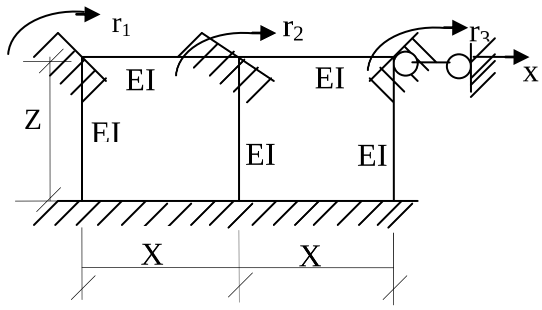

Fig. 4. Basic system for dual-cell box culvert.

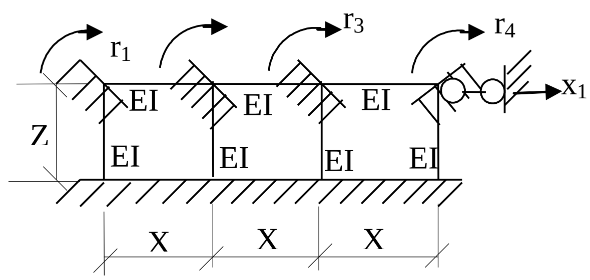

Fig. 5. Basic system for triple-cell box culvert.

Table 1
Dynamic response of 2m x 2m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.15*	*0.20*	*0.25*	*0.30*	*0.35*	*0.40*
T(sec.)	2.50	1.63	1.16	0.88	0.70	0.57

Table 2
Dynamic response of 2m x 1.75m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.15*	*0.20*	*0.25*	*0.30*	*0.35*
T(sec.)	2.08	1.35	0.83	0.74	0.58

Table 3
Dynamic response of 2m x 1.5m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.15*	*0.20*	*0.25*	*0.30*
T(sec.)	1.68	1.09	0.78	0.59

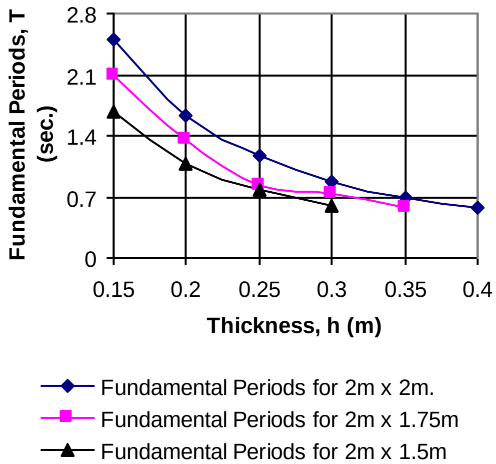

Fig. 6. Graphical representations of tables 1, 2 and 3.

Table 4
Dynamic response of 1.75m x 2m
(X by Z) dual-cell boxes with varied thicknesses

h (m)	*0.15*	*0.20*	*0.25*	*0.30*	*0.35*
T(sec.)	2.31	1.50	1.07	0.93	0.65

Table 5
Dynamic response of 1.50m x 2m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.15*	*0.20*	*0.25*	*0.30*	*0.35*
T(sec.)	2.10	1.37	0.98	0.74	0.59

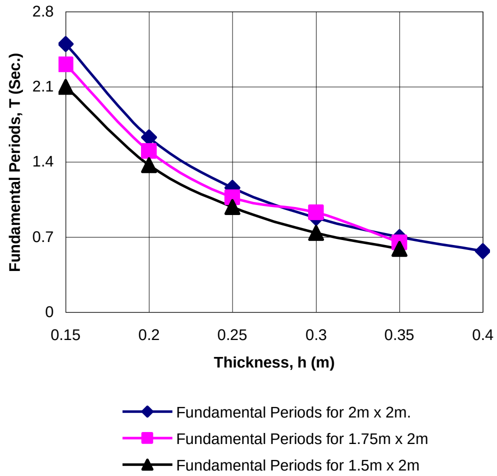

Fig. 7: Graphical representations of tables 1, 4, and 5.

Table 6
Dynamic response of 3m x 3m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
T(sec.)	3.66	1.99	1.29	0.93	0.70

Table 7
Dynamic response of 3m x 2.75m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
T(sec.)	3.24	1.76	1.15	0.82	0.62

Table 8
Dynamic response of 3m x 2.50m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
T(sec.)	2.84	1.55	1.00	0.72	0.55

Table 9
Dynamic response of 3m x 2m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*
T(sec.)	2.09	1.14	0.74	0.53

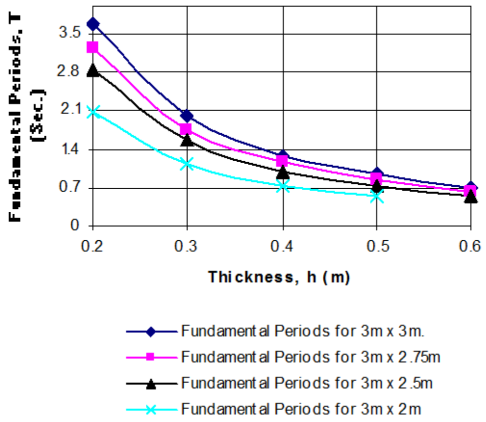

Fig. 8. Graphical representations of tables 6, 7, 8 and 9

Table 10
Dynamic response of 2.75m x3m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
T(sec.)	3.47	1.89	1.23	0.88	0.67

Table 11
Dynamic response 2.5m x 3m
(X by Z) dual-cell box culvert with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
T(sec.)	3.27	1.78	1.16	0.83	0.63

Table 12
Dynamic response of 2m x 3m
(X by Z) dual-cell boxes with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
T(sec.)	2.87	1.56	1.01	0.73	0.55

Table 13
Dynamic response of 2m x 2m
(X by Z) triple-cell boxes with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*
T(sec.)	1.70	0.92	0.60	0.43

Table 14
Dynamic response of 2m x 1.75m
(X by Z) triple-cell boxes with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*
T(sec.)	1.41	0.77	0.50	0.36

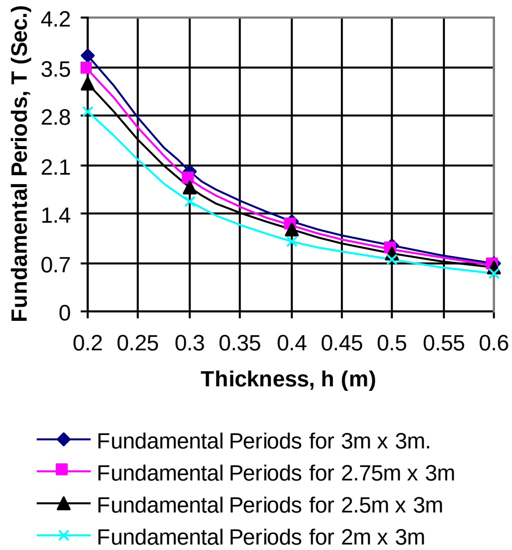

Fig. 9. Graphical representations of tables 6, 10, 11 and 12

Table 15
Dynamic response of 2m x 1.5m
(X by Z) triple-cell boxes with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*
T(sec.)	1.14	0.62	0.40	0.29

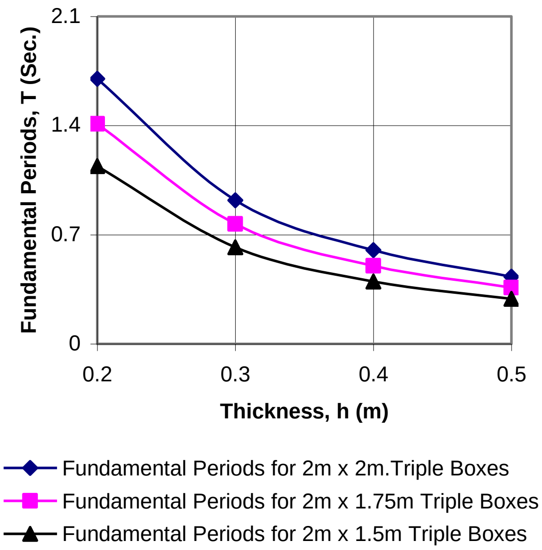

Fig. 10. Graphical representations of Tables 13, 14 and 15.

Table 16
Dynamic response of 3m x 3m
(X by Z) triple-cell boxes with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*	*0.70*
T(sec.)	3.82	2.08	1.35	0.97	0.74	0.58

Table 17
Dynamic response of 3m x 2.75m
(X by Z) triple-cell boxes with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*	*0.70*
T(sec.)	3.38	1.84	1.20	0.86	0.65	0.52

Table 18:
Dynamic response of 3m x 2.5m
(X by Z) triple-cell boxes with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*	*0.70*
T(sec.)	2.96	1.61	1.05	0.75	0.57	0.43

Table 19
Dynamic response of 3m x 2m
(X by Z) triple-cell boxes with varied thicknesses

h (m)	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*	*0.70*
T(sec.)	2.18	1.19	0.76	0.55	0.42	0.33

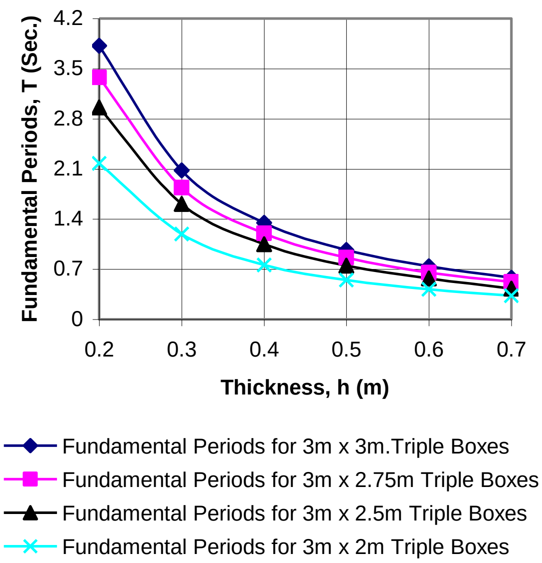

Fig. 11. Graphical representations of tables 16, 17, 18 and 19.