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Mathematical model for evaluating internal volume shrinkage of fired clays

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Abstract

A model was derived showing a mathematical relationship between volume shrinkage and fired linear shrinkage. The derivation indicates that fired linear shrinkage plays a vital role in controlling the volume shrinkage of ceramic materials. The model was found to be a third order polynomial equation which calculates volume shrinkage from known values of fired shrinkage hence maintaining same experimental data as calculated using the already-in-use conventional formular for volume shrinkage. It was also discovered that volume shrinkage increases with increase in fired shrinkage; a direct relationship. It was observed that maximum volume shrinkage occurs at maximum fired shrinkage.

Keywords: Mathematical model; Evaluation; Volume shrinkage; Fired shrinkage; Ceramic materials

1. Introduction

Recent analysis shows that fine particles shrink more, denser and exhibits excellent mechanical properties (Viewey and Larrly, 1978). They also came out with the following relationship when they investigated the relationship between particle size and size distribution with linear drying shrinkage, firing shrinkage and apparent porosity. No visible relationship was found to exist between particles size and linear drying shrinkage. Finer particles tend to shrink more. They also stated that the finer the particle size, the lesser the apparent porosity and greater the bulk density.

It was discovered that on heating dried clays, water is given off. With time, a hard but porous piece forms. A swollen appearance might occur during the release of some gases, but overall shrinkage must occur when vitrification sets in leading to a strong dense piece (Singer and Singer, 1963).

The present work is an attempt to derive a model which relates volume shrinkage directly to fired shrinkage as a law enroute a clear understanding of the behaviour of clay materials under processing at the firing temperature. The model will create room for innovations which will enhance the performance of ceramic products where they are needed and applied.

2. Materials and methods

In this work the raw materials used were Olokoro, Ukpor and Otamiri clays. Each of these clay samples were crushed to fine particle size less than 100μm and homogenized separately. Samples were thoroughly mixed with water of known quantity. The plastic clays were kneaded using hand to expel any trapped air from the clays. A binder (starch) of known amount was also added. The samples were then extruded and the extrusion dates were cold pressed at 10Mpa. Marks for identification were made on each test sample after extrusion**.** Also these samples were marked, immediately after air drying following pressing, with two long parallel lines 70mm apart.

Electric kiln (furnace) was used for firing the clay samples. The samples were charged at a lower temperature (125^0C) after which the temperature was increased until the samples were fired at 1200°C. This was done for 48 hours before allowing the samples to cool in the furnace for another 48 hours.

3. Derivation

The green samples marked immediately after extrusion were examined after firing, and the distance between the two parallel lines before and after firing (fired length) were determined for each sample and recorded.

In accordance with common knowledge, shrinkage can be calculated by the formular

$$F_s = \frac{L - L_2}{L} \quad \text{dry basis} \quad (1)$$

$$VS_L = 1 - \left\{1 - \left(\frac{L - L_2}{L}\right)^3\right\} \quad (2)$$

(Conventional Equation)
Where F_s = Fractional fried linear shrinkage
VS_L = Fractional volume shrinkage in terms of length
L = Original length (mm)
L_2 = Fired Length (mm)
For convenience let $F_s = \alpha$ and $V_s = \beta$, also fired linear shrinkage is taken to be fired shrinkage. Substituting equation (1) in (2)

$$V_s = 1 - \left\{(1 - F_s)^3\right\} \quad (3)$$

Substituting α and β in equation (3)

$$\beta = 1 - \left\{(1 - \alpha)^3\right\} \quad (4)$$

Expanding equation (4) using Pascal's Triangle value for third power (1331)

$$\beta = 1 - \left\{1 - 3\alpha + 3\alpha^2 - \alpha^3\right\} \quad (5)$$

Opening the brackets to collect like terms

$$\beta = 1 - 1 + 3\alpha - 3\alpha^2 + \alpha^3 \quad (6)$$

$$\beta = 3\alpha - 3\alpha^2 + \alpha^3 \quad (7)$$

Rearranging equation (7) in descending order of power of the fired shrinkage α

$$\beta = \alpha^3 - 3\alpha^2 + 3\alpha \quad (8)$$

Volume shrinkage β is related to fire shrinkage by equation (8) and the relation is polynomial.

Differentiating equation (8) wrt α and equating to zero in order to determine the value of α at which volume shrinkage β is maximum.

$$\frac{d\beta}{d\alpha} = \frac{d(\alpha^3)}{d\alpha} - \frac{3d(\alpha^2)}{d\alpha} + \frac{3d(\alpha d)}{d\alpha} \quad (9)$$

$$\frac{d\beta}{d\alpha} = 3\alpha^3 - 6\alpha + 3 \quad (10)$$

At maximum β

$$\frac{d\beta}{d\alpha} = 0 \quad (11)$$

$$3\alpha^2 - 6\alpha + 3 = 0 \quad (12)$$

Factorizing equation (12)

$$(3\alpha - 3)(\alpha - 1) = 0 \quad (13)$$

$$3\alpha - 3 = 0 \text{ and } \alpha - 1 = 0 \quad (14)$$

$\alpha = 1$ and $\alpha = 1$

Considering the fact that both β and α are fractional values, the unit value of α obtained indicates that at maximum volume shrinkage β, fired shrinkage α is also at maximum which is unity.

4. Testing of the model and discussion

The chemical composition of the raw material used is given in Table 1. In order to confirm the validity of the model for calculating volume shrinkage β, values of fired shrinkage α calculated based on experimental data using equation (1) were slotted into the derived model in equation (8) for each of the clay type. On comparing, the values of the volume shrinkage β were exactly equal and the same for all the clay types. This is a very clear indication that volume shrinkage β can be calculated in terms of fired shrinkage α using the model equation derived. The results of the experimental work showing the calculated values of the fired shrinkage α and volume shrinkage β for each clay type are shown in Tables 2-4.

It was also found that the volume shrinkage increases with increase in the fired shrinkage. Furthermore the model equation indicates that there is a direct relationship between the β and α culminating in a third order polynomial equation. It was discovered that Olokoro clay has the highest shrinkage followed by Ukpor Clay while Otamiri has the lowest shrinkage. This is confirmed in Tables 2-4.

5. Conclusion

This model equation derived calculates the value of the volume shrinkage β when the value of fired shrinkage α is known. The equation is a third order polynomial. Volume shrinkage β increases with increase in fired shrinkage this is a direct relationship.
Olokoro clay has the highest shrinkage, followed by Ukpor Clay while Otamiri has the lowest.

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Viewey, F., Larry, P., 1978. Ceramic Processing before Firing. John Wiley and Sons, New York, P. 3-8.

Table 1.
Chemical composition of raw materials used

Source	Al_2O_3	Fe_2O_3	TiO_2	MgO	CaO	SiO_2	Na_2O	K_2O	Loss of ignition
Ukpor	31.34	0.63	2.43	0.14	0.06	51.43	0.04	0.10	12.04
Olokoro	29.10	7.95	-	0.75	1.26	45.31	0.05	0.09	11.90
Otamiri	15.56	0.05	1.09	-	0.29	69.45	0.01	0.21	13.01

Table 2
Fired and volume shrinkage of olokoro clay fired at 1200^0C

Initial length (mm)	Fired Length (mm)	Fired shrinkage α $\frac{L - L_2}{L}$	Volume shrinkage VS_L (from conventional equation) $1 - \left\{1 - \left(\frac{L - L_2}{L}\right)^3\right\}$	Volume shrinkage β (from Derived model) $\alpha^3 - 3\alpha^2 + 3\alpha$
70	63.40	0.0943	0.2571	0.2571
70	63.38	0.0946	02578	0.2578
70	63.40	0.0943	0.2571	0.2571
70	63.42	0.0940	0.2563	0.2563
70	63.52	0.0926	0.2529	0.2529

Table 3
Fired and volume shrinkage of Ukpor clay fired at 1200^0C

Initial length (mm)	Fired Length (mm)	Fired shrinkage α $\frac{L - L_2}{L}$	Volume shrinkage VS_L (from conventional equation) $1 - \left\{1 - \left(\frac{L - L_2}{L}\right)^3\right\}$	Volume shrinkage β (from Derived model) $\alpha^3 - 3\alpha^2 + 3\alpha$
70	63.20	0.0829	0.2287	0.2287
70	63.99	0.0859	0.2362	0.2362
70	64.12	0.0840	0.2314	0.2314
70	64.00	0.0857	0.2357	0.2357
70	63.70	0.0900	0.2464	0.2464

Table 4
Fired and volume shrinkage of Otamiri clay fired at 1200^0C

Initial length (mm)	Fired Length (mm)	Fired shrinkage α $\frac{L - L_2}{L}$	Volume shrinkage VS_L (from conventional equation) $1 - \left\{1 - \left(\frac{L - L_2}{L}\right)^3\right\}$	Volume shrinkage β (from Derived model) $\alpha^3 - 3\alpha^2 + 3\alpha$
70	64.80	0.0743	0.2067	0.2067
70	64.98	0.0717	0.2000	0.2000
70	64.75	0.0750	0.2085	0.2085
70	64.99	0.0716	0.1998	0.1988
70	64.67	0.0761	0.2114	0.2114