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# Free vibrations of multi-storey, multi-bay reinforced concrete frames with strong column – weak beam model.

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**Abstract**

This paper investigated the dynamic responses of multistorey, multibay reinforced concrete framed buildings, using the concept of strong column – weak beam model. In the course of this work, three simulated frames were dynamically investigated; the natural frequencies of each frame at various ratios of column flexural strengths to beam flexural strengths were numerically evaluated at a given concrete grade and unit weight; using classical method of displacement; and subsequently, the fundamental tone of vibration at a given $EI_c/EI_b$ ratio was explicitly compared with the fundamental tone of vibration of the same system, evaluated using the Shear Building model approach. And from the results obtained, it was found that the column flexural strength demand for dynamic stability varied in accordance with the number of floor levels of the frames. This paper, therefore, establishes no fixed empirical value for the ratio of column flexural strength to beam flexural strength; and consequently recommends that the column flexural strength participation factor(s) in the analysis of dynamic stability of frames with strong column – weak beam concept be adequately evaluated, using Classical method of analysis.
*Keywords: Capacity design concept; Strong column – weak beam model; multistorey – multibay frames; Finite degree of freedom; Classical displacement method; Fundamental frequency.*

## Introduction

Multi-storey, multi-bay reinforced concrete (r. c.) frames are increasingly becoming important engineering structures of our modern time, in the sense that, most of the high-rise buildings are designed and constructed as multi-storey, multi-bay frames. However, the ultimate objective of structural design of such important structures is to devise structures that are capable of fulfilling their intended purposes with a minimum risk of failure, given all statistical uncertainties of loads and resistances, and economic constraints, [Meyer, 1991]. In practice, two load cases are usually considered when analyzing multi-storey frames; the static conditions and the dynamic conditions. However, the static conditions impose lesser difficulties when compared with the dynamic conditions. Predominant among the dynamic load conditions on multi-storey frames are those associated with lateral load applications, probably because of their catastrophic effects, especially those that are accompanied with reversal of stress inputs. Different design approaches and analytical models have been formulated by different stakeholders in structural engineering, for use in the evaluation of dynamic stability of multi-storey framed structures. Among them is the capacity design concept. In Capacity Design Model, (CDM), a special hierarchy of members' strengths is devised that assures the formation of plastic hinges in pre-specified zones. The simplest and most current in use is the "strong column – weak beam" concept, [Park, 1986]. Also, in the design guidelines given in the report 352 of ACI Committee, it is recommended that since it is preferable to have plastic hinges formed in the beams rather than in columns, the columns should have flexural strengths 1.4 times those of the beams framing into the same joints if these joints are part of the primary system that resists seismic lateral load, [ACI – ASCE, 1985]. Fundamentally, this design concept assumes that all or most of the structural elements participate equally in the task of energy dissipation, with the result that any damage occurring is uniformly distributed over the entire frame. Consequent upon, it is on the basis of this design concept of strong columns – weak beams and the assumption of equal participation in the task of energy dissipation by all the structural members of the frame that this paper is poised to investigate the free vibrations of multi-storey, multi-bay r. c. frames with strong column – weak beam model. The main objective of this paper is to evaluate the dynamic responses of multi-storey, multi-bay r. c. framed structures under self-excitation, using strong column – weak beam model, analyzed using Classical Displacement Method Approach (CDMA) and compare the results with the results obtained by using shear Building Model (SBM) method of analysis for the same framed structural buildings.

## STRUCTURAL MODEL AND DYNAMIC FLOOR MASS:

The building configuration adopted in this study is simple and regular; a typical office block building with a plan view as shown in Figure 1. In this study, a reinforced concrete frame of a three-floor simulated model, a five-floor simulated model and a ten-floor simulated model are considered to represent low, medium and tall buildings respectively, as shown in Figures 2, 3 and 4 respectively; using the same plan view of Figure 1. The floor thickness for each floor level is 150mm except the topmost floor level, which is 125mm thick. A concrete grade of 25 with Elastic modulus of 26 x $10^6$kN/m$^2$ and a unit weight of 24kN/m$^3$ is adopted. A provision for finishes at each floor is given to be 0.75kN/m$^2$ and is constant for all the floors; thereby giving the numerical value of dynamic floor mass, $M_1 = M_2 = \ldots = M_{n-1}$ = 14047.706kg, and $M_n$ = 12110.092kg; where n = number of floor levels.

## DYNAMIC MODEL:

In spite of the fact that almost all engineering structures are continuous and possess infinite number of degrees of freedom, [Clough, & Penzien, 1993; Polyakov, 1985], the adoption of lumped mass system in the dynamic analysis provides satisfactorily approximate results, [Feodosyev, 1973]. Thus, in the course of this work, each of the simulated r. c. multi-storey, multi-bay frames is modeled as a structure with finite number of degrees of freedom, by assuming lumped mass element concentration at the right corner of each floor level, as shown in Figure 5.

## DYNAMIC ANALYSIS:

The fundamental principle in the dynamic analysis of structural system is the determination of the natural frequencies associated with the natural vibration of the system, [Biggs, 1964; Rao, 2006]. For regular building structures, it is only the fundamental tone of vibration that is usually considered to be paramount, as resonance at the lowest frequency will result in maximum dynamic effects, [Darkov, 1983]. However, in a recent publication, [Hemant et al 2006], it was pointed out that with increasing number of floors, the flexibility of building structures increases, thereby bringing higher mode effects into the picture. Therefore, it is recommended that in dynamic analysis of tall buildings (ten or more floors), even if the buildings are regular, the higher modes should be considered in addition to the first mode.

## EQUATION OF MOTION:

Most often, the governing differential equation employed is the D'Alembert's principle for dynamic equilibrium equations; thus:

$$\mathbf{M}\ddot{x}_{(t)} + \mathbf{C}\dot{x}_{(t)} + \mathbf{K}x_{(t)} = \mathbf{P}_{(t)} \qquad (1)$$

where **M**, **C**, **K** are matrices of mass elements, damping elements and stiffness coefficients respectively; $\mathbf{P}_{(t)}$ is the column vector representing the external excitations; $x_{(t)}, \dot{x}_{(t)}, \ddot{x}_{(t)}$ are vectors of displacements, velocities and accelerations of a system with finite number of degrees of freedom. In this work, we assume that the system is performing natural vibration, and therefore the exciting force vector is assumed to be zero. Thus equation (1) becomes:

$$\mathbf{M}\ddot{x}_{(t)} + \mathbf{C}\dot{x}_{(t)} + \mathbf{K}x_{(t)} = 0 \qquad (2)$$

Generally, all vibrating systems are, to certain degree, subject to damping effect due to the fact that energy is dissipated by friction and other resistances. However, in some cases, damping is very small or the dynamic disturbances on the vibrating system (buildings) act for relatively short duration, so that the effect of damping becomes unimportant and very negligible, [Humar, 2002; Meirovvitch, 1986; Coates, Coutie &Kong, 1980]. Therefore, in this work, we completely neglect the effect of damping; and equation (2) reduces to:

$$\mathbf{M}\ddot{x}_{(t)} + \mathbf{K}x_{(t)} = 0 \qquad (3)$$

By adopting the stiffness matrix and displacement approach, the natural frequencies of the simulated frames can be evaluated by solving the non-trivial equation:

$$\left| M^{-1}K_S - \omega^2 I \right| = 0 \qquad (4)$$

where

$M^{-1}K_s$ = dynamic matrix

and

$\mathbf{M}^{-1}$ = inverse matrix of mass system
$\mathbf{K}_s$ = Stiffness matrix
$\boldsymbol{\omega}$ = eigenvalues
**I** = identity matrix

## METHODOLOGY

For a given concrete grade and unit weight, the flexural strength of the structural members is a function of the sectional moment of inertia. Consequently, by adopting a constant concrete grade and unit weight in this work, both the columns and beams are made to have equal initial flexural strength, (i.e. $EI_c = EI_b$ ).

### Procedure of Methodology

(i) The ratios of column flexural strength, $EI_c$ to beam flexural strength, $EI_b$, $\left(\alpha = EI_c / EI_b\right)$ are established, in which α = 1, 5, 10, 15, 20.

(ii) The natural frequencies of the simulated r. c. frame, for a given value of α, are evaluated using the Classical method of Flexible Horizontal Members Model, (FHMM).

(iii) The natural frequencies of the simulated r. c. frames are evaluated using the Shear Building Model approach.

(iv) The fundamental tone of vibration of each simulated system, for a given value of α, is normalized with the fundamental tone of vibration of that obtained by using the corresponding SBM approach; and the normalized fundamental frequencies are represented graphically against α, with the α as the abscissas.

**RESULTS AND DISCUSSIONS**:

Tables 1, 2, and 3 show the dynamic responses of the simulated r. c. framed structures at various $\alpha = EI_c / EI_b$, evaluated using the Classical Method, (CM) of Flexible Horizontal Members Model (FHMM) approach; and Table 4 shows the dynamic responses of the same reinforced concrete (r. c.) multi-storey framed buildings of ten, five and three floors respectively, evaluated using the Shear Building Model, (SBM) approach. In the analysis of the results, the fundamental tone of vibrations of the simulated frames for a given α – value is compared with the corresponding fundamental tone of vibrations obtained using SBM. In the process, the fundamental frequencies of each simulated r. c. frame at various α – values are normalized with the fundamental frequency of the corresponding SBM, (i. e. $\omega_1 / \theta_1$); and the results are graphically represented in Figure 6. The abscissa coordinate value at which $\omega_1 / \theta_1$ equals unity is read off from each graph as required column flexural strength multiplication factor. From the graphical interpretations, it is deduced that in capacity design of strong column – weak beam model, the column dynamically flexural strength demand is of the order of 4.65; 8.75; and 10 times that of the beam dynamically flexural strength demand for a 3- floor storey, 5-floor storey and 10-floor storey building respectively, using the SBM dynamical response as a standard.

**CONCLUSION AND RECOMMENDATION**:

From the results obtained, it is quite obvious that in the Capacity Design Concept of strong column – weak beam model, the ratio of column flexural strength to beam flexural strength is not constant, but varies accordingly with the increase in the number of floors of the buildings. Therefore, the idea of adopting a fixed empirical value for the ratio of column flexural strengths to beam flexural strengths is not adequately justified in the dynamic analysis of multi-storey, multi-bay r. c. framed buildings. It is essentially recommended, therefore, that the column flexural strength participation factor(s) in the analysis of dynamic stability of multi-storey, multi-bay r. c. framed buildings be adequately evaluated using Classical method of analysis. Also, for economic implications, it is recommended that this design concept should be limited in its application to low and medium rise buildings, as the dimensional requirements of the columns are quite enormously so demanding with the increase in the number of floor levels.

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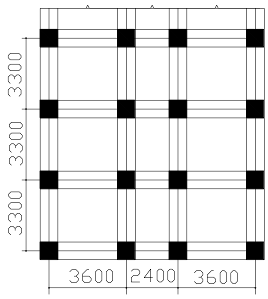


**Figure** 1: The Plan view of the Simulated Multi-storey, Multi-bay Reinforced Concrete Building.

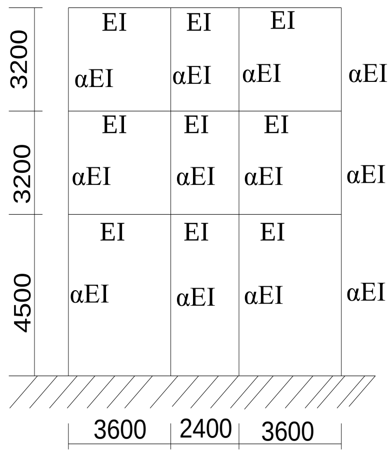


**Figure 2**: A Three-Floor, Multi-bay Simulated Reinforced Concrete Frame showing the Flexural Rigidity Arrangement for the Structural Members.

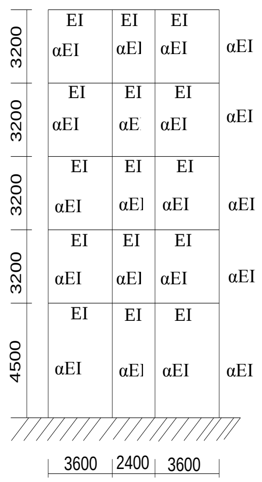


**Figure 3**: A Five-Floor, Multi-bay Simulated Reinforced Concrete Frame showing the Flexural Rigidity Arrangement for the Structural Members.

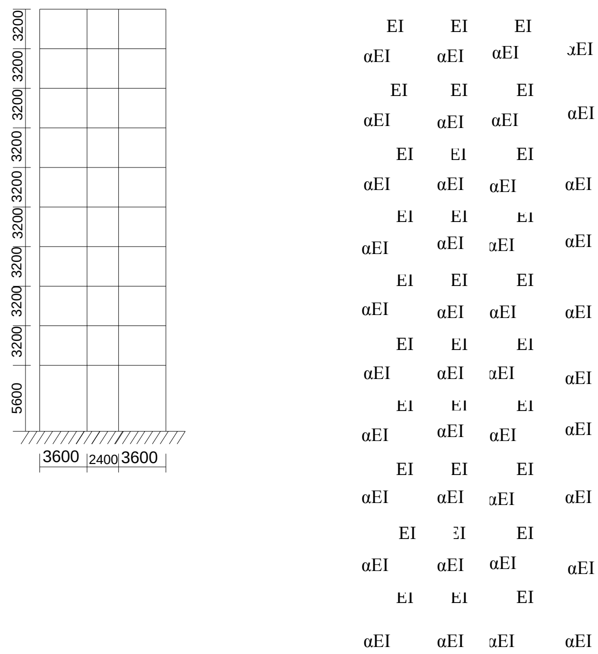


**Figure 4**: A Ten-Floor, Multi-bay Simulated Reinforced Concrete Frame showing the Flexural Rigidity Arrangement for the Structural Members.

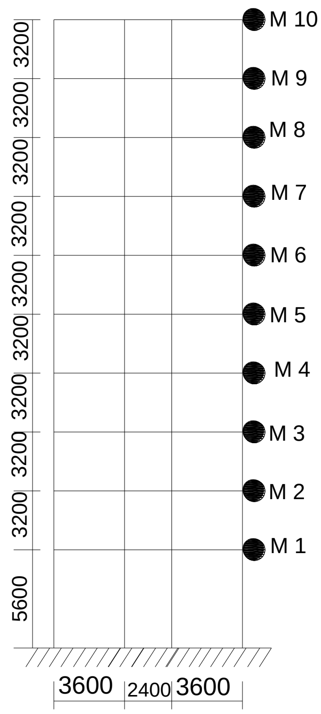


**Figure 5:** Dynamic Model for the Ten-Floor, Multi-bay Simulated Reinforced Concrete Frame.

**Table 1**: The Natural Frequencies of the Three-Floor Storey Reinforced Concrete Model evaluated using the Classical Method of Flexible Horizontal Members Model approach at various α-values.

| Frequency | $\alpha = 1$ | $\alpha = 5$ | $\alpha = 10$ |
|---|---|---|---|
| $\omega_1$ | 0.1699rad/sec | 0.2522rad/sec | 0.2899rad/sec |
| $\omega_2$ | 0.6482rad/sec | 1.0667rad/sec | 1.3260rad/sec |
| $\omega_3$ | 1.2109rad/sec | 2.4097rad/sec | 3.2688rad/sec |

**Table 2**: The Natural Frequencies of the Five-Floor Storey Reinforced Concrete Model evaluated using the Classical Method of Flexible Horizontal Members Model approach at various α-values.

| Frequency | $\alpha = 1$ | $\alpha = 5$ | $\alpha = 10$ |
|---|---|---|---|
| $\omega_1$ | 0.3261rad/sec | 0.4597rad/sec | 0.4985rad/sec |
| $\omega_2$ | 1.1062rad/sec | 1.6588rad/sec | 1.9283rad/sec |
| $\omega_3$ | 2.0476rad/sec | 3.4304rad/sec | 4.3135rad/sec |
| $\omega_4$ | 3.0573rad/sec | 5.8258rad/sec | 7.7790rad/sec |
| $\omega_5$ | 3.9333rad/sec | 8.3801rad/sec | 11.6530rad/sec |

**Table 3**: The Natural Frequencies of the Ten-Floor Storey Reinforced Concrete Model evaluated using the Classical Method of Flexible Horizontal Members Model approach at various α-values.

| Frequency | $\alpha = 1$ | $\alpha = 10$ | $\alpha = 20$ |
|---|---|---|---|
| $\omega_1$ | 0.1618rad/sec | 0.2360rad/sec | 0.2358rad/sec |
| $\omega_2$ | 0.5322rad/sec | 0.8065rad/sec | 0.7908rad/sec |
| $\omega_3$ | 0.9633rad/sec | 1.5933rad/sec | 1.4071rad/sec |
| $\omega_4$ | 1.4325rad/sec | 2.6347rad/sec | 2.0140rad/sec |
| $\omega_5$ | 1.9345rad/sec | 3.9567rad/sec | 2.5791rad/sec |
| $\omega_6$ | 2.4579rad/sec | 5.5800rad/sec | 3.0830rad/sec |
| $\omega_7$ | 2.9864rad/sec | 7.4780rad/sec | 3.5114rad/sec |
| $\omega_8$ | 3.4854rad/sec | 9.5409rad/sec | 3.8539rad/sec |
| $\omega_9$ | 3.9101rad/sec | 11.5282rad/sec | 4.1030rad/sec |
| $\omega_{10}$ | 4.1998rad/sec | 13.0374rad/sec | 4.2540rad/sec |

**Table 4**: The Natural Frequencies of the Simulated Frames evaluated using Shear Building Model approach.

| Frequency | Ten-Floor Storey Frame | Five-Floor Storey Frame | Three-Floor Storey Frame |
|---|---|---|---|
| $\theta_1$ | 0.2358rad/sec | 0.4835rad/sec | 0.2459rad/sec |
| $\theta_2$ | 0.7908rad/sec | 1.5756rad/sec | 0.8577rad/sec |
| $\theta_3$ | 1.4071rad/sec | 2.6738rad/sec | 1.3494rad/sec |
| $\theta_4$ | 2.0140rad/sec | 3.5580rad/sec | |
| $\theta_5$ | 2.5791rad/sec | 4.1159rad/sec | |
| $\theta_6$ | 3.0830rad/sec | | |
| $\theta_7$ | 3.5114rad/sec | | |
| $\theta_8$ | 3.8539rad/sec | | |
| $\theta_9$ | 4.1030rad/sec | | |
| $\theta_{10}$ | 4.2540rad/sec | | |

**Table 5**: The Normalized Fundamental Frequencies of the Three-Floor Storey and Five-Floor Storey Frame Models with the Associated Fundamental Frequency of the Shear Building Model

| $\frac{EI_c}{EI_b} = \alpha_i$ | 3-Floor Storey Frame: | 5-Floor Storey frame: |
|---|---|---|
| | $\frac{\omega_1}{\theta_1}$ | $\frac{\omega_1}{\theta_1}$ |
| $\alpha = 1$ | 0.69 | 0.67 |
| $\alpha = 5$ | 1.03 | 0.95 |
| $\alpha = 10$ | 1.18 | 1.03 |

**Table 6**: The Normalized Fundamental Frequencies of the Ten-Floor Storey Frame Model with the Associated Fundamental Frequency of the Shear Building Model.

| | Normalized Frequency | | |
|---|---|---|---|
| | $\alpha = 1$ | $\alpha = 10$ | $\alpha = 20$ |
| $\frac{\omega_1}{\theta_1}$ | 0.69 | 1.00 | 1.04 |

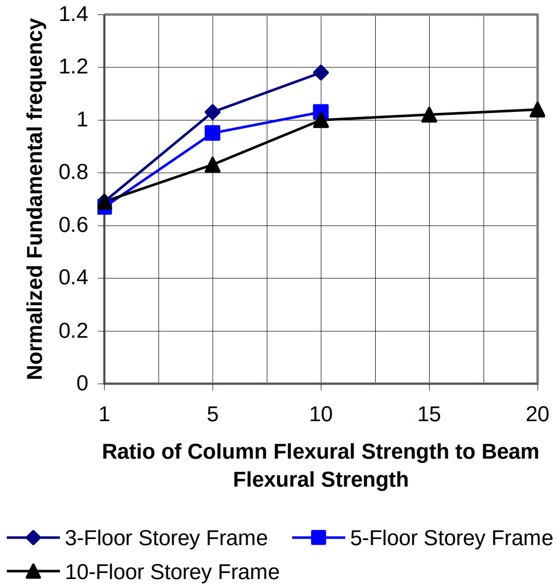


**Figure 6**: The Graphical Representations of various Normalized Fundamental Frequencies.