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Modelling of sand drying process in a sheet/hallow glass plant

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Abstract

The sand drying process in a sheet/hallow glass plant has been modelled under steady state and unsteady state conditions. A model equation was developed for a differential length of the cylindrical drum dryer based on reasonable theoretical assumptions and solved under steady state and unsteady state conditions subject to the applicable boundary conditions. The model predicted the actual length of the drum dryer to an accuracy of 98.7% (1.3% deviation) and the diameter of the drum dryer to an accuracy of 98.3% (1.7% deviation). This model can be used for design, simulation and control of drum dryers for sand drying in a sheet/hallow glass plant.

Keywords: Modelling; drying; sand; glass; steady state; unsteady state

1. Introduction

A mathematical model of a real chemical process is a mathematical description which combines experimental facts and establishing relationships among the process variables. For this purpose it uses both theory and experimentation (Babu and Ramakrishna, 2002; Babu, 2006).

Models are developed by application of the fundamental laws subject to sound theoretical assumptions, and solution must be found using the appropriate initial and boundary conditions inherent in the process. An adequate model may be used in plant design, synthesis of control systems and forecasting of future plant behaviour (Aris and Amundson, 1923; INGLEN, 1976; Bird, Stewart and Lightfoot, 2002)

Quartz sand is the chief particulate solid in glass making and constitutes 72% of the total raw materials mixed in batch cycles. The humid sand contains 4-5% moisture and has to be dried to about 0.01-0.1% moisture before being allowed into the furnace (Marshal and Pigford, 1947; Mickley et al, 1957; INGLEN, 1976).

If a high moisture sand quartz is fed to the furnace through the batch logs, all free waters will evaporate above $100^{O}C$ during firing and this will alter the desired composition of quartz sand in the glass and consequently the glass product will fail the packing test. These failed products will become cullet recycled back to the furnace, thus increasing the operational cost and subsequently the unit cost of each product (Marshal and Pigford, 1947; Bird, Stewart and Lightfoot, 2002).

This work models the concentration profile of sand moisture content under steady state and unsteady state conditions. The model was validated by comparing the results with values from a functional twenty- ton capacity sand drying plant.

2. Model development

The drum is cylindrical in shape and mounted horizontally through its longitudinal axis, hinged at both ends and (unnoticeably) slightly inclined to the output end for discharge of dry sand into a conveyor. It is equipped internally with metal blades that cut across humid sand masses as the drum rotates, thus increasing voidage and allowing for easy penetration of heat throughout the entire mass (Ogunnaike et al, 1983).

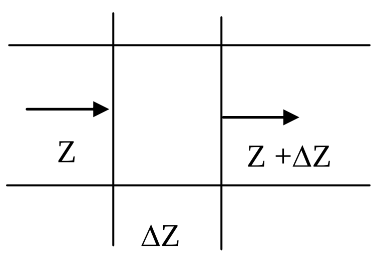

Fig. 1. Cross section of the rotary drum dryer.

The following assumptions were made:

- The rotary drum operates under plug flow conditions
- The rotary drum operates under isothermal conditions

- The sand velocity profile within the drum is flat
- The cross-sectional area of the drum (A_d) is constant

A mass balance will be taken on the differential section between z and z+Δz (Fig. 1). If the humid sand contains C_{AO} moles of water (A) per unit volume at a temperature T_O and it is fed into the drum at a velocity V over the time interval (t, t+Δt), a mass balance over the differential section gives:

$$\frac{dC_A}{dt} + V\frac{dC_A}{dz} = -K\,C_A \qquad (1)$$

Where, K is the vaporisation rate constant and C_A the concentration of the moisture in the sand(Szinfield and Lapidus, 1974;Ogunnaike, 1983). .

Equation (1) is the process model and a first order partial differential equation. For a solution to eqn. (1) we first make the model dimensionless by defining;

$$\phi = \frac{C_A - C_{AO}}{C_A^* - C_{AO}} \qquad (2)$$

Where, C_{AO} is humidity of sand at time, t = 0 and C_A* is the humidity of sand at the discharge end, this is set at 0.1%.

From eqn. (2) we obtain the following:

$$\frac{dC_A}{dt} = \frac{d\phi}{dt}[C_A^* - C_{AO}] \qquad (3)$$

$$\frac{dC_A}{dz} = \frac{d\phi}{dz}[C_A^* - C_{AO}] \qquad (4)$$

Rearranging eqn. (2) gives;

$$C_A - C_A^* = (\phi - 1)(C_A^* - C_{AO}) \qquad (5)$$

For $C_A >> C_A$*

$$C_A = (\phi - 1)(C_A^* - C_{AO}) \qquad (6)$$

Substituting eqn. (3), (4) and (6) into eqn. (1) gives a first order PDE;

$$\frac{\partial\phi}{\partial t} + V\frac{\partial\phi}{\partial z} = K(1-\phi) \qquad (7)$$

At steady state, $\frac{\partial\phi}{\partial t} = 0$, thus;

$$V\frac{\partial\phi}{\partial z} - K(1-\phi) = 0 \qquad (8)$$

Equation (8) is a first order ODE, which when solved for the initial conditions: z = 0, $C_A = C_{AO}$ and Φ = 0, gives the solution to the steady state drying process as;

$$\phi = 1 - \exp\left(-\left(\frac{K}{V}\right)z\right) \qquad (9)$$

Substituting eqn. (2) into eqn. (9) we have variation in sand humidity along the drum as;

$$C_A = (C_A^* - C_{AO})\left(1 - \exp\left(-\left(\frac{K}{V}\right)z\right)\right) + C_{AO} \qquad (10)$$

For unsteady state operations, the PDE in eqn. (7) can be solved using the laplace transformation technique. On transformation, with zero initial conditions, we obtain;

$$\frac{d\bar{\phi}}{dz} + \frac{(s+K)}{V}\bar{\phi} = \frac{K}{Vs} \qquad (11)$$

Equation (11) is a linear first order ODE, which when solved subject the initial conditions above (z = 0, Φ = 0) gives;

$$\bar{\phi} = \frac{K}{s(s+K)}\left[1 - \exp\left(-\left(\frac{s+K}{V}\right)z\right)\right] \qquad (12)$$

Inversion of eqn. (12) gives;

$$\phi = \frac{C_A - C_{AO}}{C_A^* - C_{AO}} = \begin{cases} 1-\exp(-Kt);\ t<\frac{z}{V} \\ 1-\exp(-K\frac{z}{V});\ t\geq\frac{z}{V} \end{cases} \qquad (13)$$

Thus

$$C_A(z,t) = (C_A^* - C_{AO})\left(\begin{cases} 1-\exp(-Kt);\ t<\frac{z}{V} \\ 1-\exp(-K\frac{z}{V});\ t\geq\frac{z}{V} \end{cases}\right) + C_{AO} \qquad (14)$$

Equation (14) predicts sand humidity along the drum under unsteady state conditions.

3. Analysis of model

At the outlet of the rotary sand dryer, z = L, the solution in equation (9) becomes;

$$\phi = 1 - \exp\left(-\left(\frac{K}{V}\right)L\right) \qquad (15)$$

If the angular speed of the drum is very high, the velocity, V, of the sand in the drum will be very high as well, $(-(\frac{K}{V})L)$ will be small and the exponential term will be close to unity. Thus from eqn. (2), Φ ≅ 0 and $C_A \cong C_{AO}$. If the angular speed of the drum is low the

velocity of the sand in the drum will be low, $(-(\frac{K}{V})L)$ will be very large and the exponential term will almost varnish. Thus $\Phi \cong 1$ and $C_A \cong C_A^*$.
From eqn. (15) we have that L/V= τ (residence time). Thus for longer length (L) or residence time (τ), $(-(\frac{K}{V})L)$ is large and the exponential term nears zero. Thus $\Phi \cong 1$ and $C_A \cong C_A^*$, while for shorter length and residence time $(-(\frac{K}{V})L)$ tends to zero and the exponential term to unity, thus $\Phi \cong 0$ and $C_A \cong C_{AO}$
The meaning of the model is limited unless (K/V) is evaluated. From eqn. (9) we observe that; at z = 0, Φ = 0, and at z = ∞ , Φ = 1.

The two boundary conditions are adequately modelled by the equation;

$$z = \frac{\phi}{1-\phi} \qquad (16)$$

Which gives; z = 0 at Φ = 0, and z = ∞ at Φ = 1.
Fig. 2 shows a fit for values generated from eqn. (16) to the model of equation (9). Fitting the model, $\Phi = 1 - \exp(-(K/V)*z)$, to points generated from eqn. (16) gave (K/V) = 0.623 with R^2 of 0.9509, which shows a good correlation thus validating the use of eqn. (16) and value of (K/V) estimated. Thus the particular solution is;

$$\phi = 1 - \exp(-0.623z) \qquad (17)$$

Equation (17) can be used to predict length of the rotary dryer. Drying is complete when the exponential term has approximately disappeared and $\Phi \cong 1$.
For Φ = 0.99, equation (17) gives z = In (0.01)/-0.623 = 7.3919m.
Foster (1973) reports a drying efficiency of 78% for this type of dryer. The actual length can be obtained as: 7.3919/0.78 = 9.4768m.
For the 20-ton rotary drum; (π D^2/4) x L x density of sand = 20,000kg
Density of sand is 1804kg/m3, for L = 9.4768m, we have D = 1.2204m.

4. Discussion

The result from the analysis of the model show that increase in angular speed of the drum will result reduction of rate of drying of sand due to increase in the velocity of the sand in the drum In a similar way reducing length of the drum or residence time will also negatively impart drying.

Reduction in the angular speed of the drum, increase in drum length or residence time of sand in the drum will increase the extent of drying achievable.

The humidity of the sand in the dryer decreases exponentially along the length of the drum, initially at a fast rate and later slowly (Fig. 3), and the dimensionless parameter increases exponentially along the length of the drum (Fig. 4). The model was able to predict the length and diameter for a drum dryer designed for 20 tons of sand with deviations of 1.2833% and 1.7% respectively.

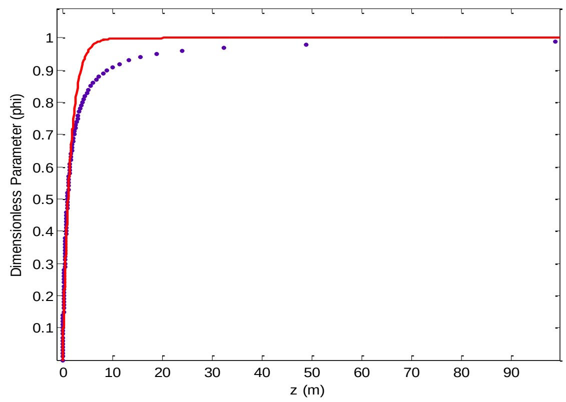

Fig. 2. Plot of Φ versus z for data from equation (16) against model solution

Table 1
Comparison of Model Predictions with Actual Plant values

Design Parameter	Model Prediction	Actual Plant Value	Percentage Deviation
Drum Length (m)	9.4768	9.6*	1.2833 %
Drum Diameter (m)	1.2204	1.2*	1.7 %

*source (INGLEN,1976)

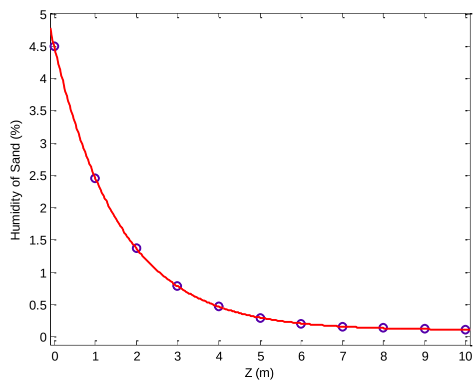

Fig. 3. Plot of % humidity versus length, z (m) for the drum dryer model.

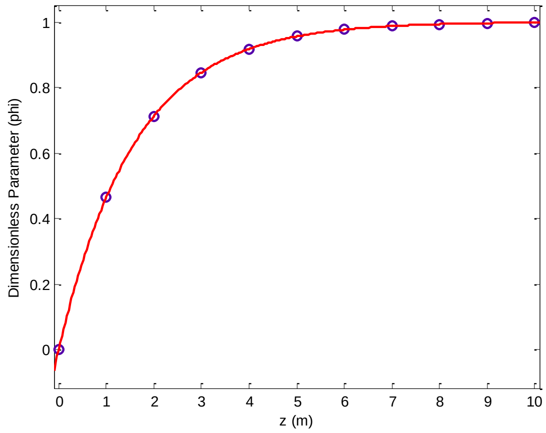

Fig. 4. Plot of Dimensionless parameter,Φ versus z (m) for the model.

5. Conclusion

The humidity profile during sand drying in a sheet/hallow glass plant can be modelled under steady state and unsteady state conditions. The model predicted the actual length of the drum dryer to an accuracy of 98.7% (1.3% deviation) and the diameter of the drum dryer to an accuracy of 98.3% (1.7% deviation). This model can be used for design, simulation and control of drum dryers for sand drying in a sheet/hallow glass plant.

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