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Mathematical model for the prediction of the compressive strength characteristics of concrete made with granite chippings

I.E. Umeonyiagu, [1] I.O. Onyeyili

Department of Civil Engineering, Anambra State University, Uli
[1] Department of Civil Engineering, Nnamdi Azikiwe University, Awka

Abstract

The coarse aggregates used in this research work were granite chippings from Abakaliki in Ebonyi State and fine aggregates from Amansea River in Anambra State. These aggregates were tested for their physical and mechanical properties based on BS 812: Part 2:1975 and BS 812: Part 3: 1975. Concrete cubes were made, cured and tested according to BS 1881:1983. The research work made use of Scheffe's (4, 2) lattice polynomial with regression equations to develop mathematical models for the prediction of the compressive strength characteristics of concretes made with these coarse aggregates. The mathematical model developed was $\hat{Y} = 30x_1 + 32x_2 + 19x_3 + 12x_4 + 2.8x_1x_2 - 0.8x_1x_3 - 5.6x_1x_4 - 2.8x_2x_3 - 8x_2x_4 + 12x_3x_4$. The student's t-test and the Fisher test were used to test the adequacy of this model. The strengths predicted by the model were in complete agreement with the experimentally obtained values and the null hypothesis was satisfied.

1. Introduction

1.1. The scheffe's (4, 2) lattice polynomial

Simplex is the structural representation of the line or planes joining the assumed positions of the constituent materials (atoms) of a mixture (Jackson and Dhir, 1988). Scheffe(1958) considered experiments with mixtures of which the property studied depended on the proportions of the components present but not on the quantity of the mixture. If a mixture has a total of q components and x_i be the proportion of the ith component in the mixture such that $x_i \geq 0$ (i = 1, 2… q), then

$$x_1 + x_2 + x_3 + \ldots\ldots + x_q = 1 \text{ or} \quad (1)$$

H. Scheffe (1958) described mixture properties by reduced polynomials obtainable from Eq. 2:

$$\hat{Y} = b_0 + \Sigma b_i x_i + \Sigma b_{ij} x_i x_j + \Sigma b_{ijk} x_i x_j x_k + \Sigma b_{i1,i2\ldots i_n} x_{i1} x_{i2} x_{in} \quad (2)$$

Where ($1 \leq i \leq q$, $1 \leq i \leq j \leq q$, $1 \leq i \leq k \leq q$) respectively and b is constant coefficient. Multiplying eqn.1 by b_0 and multiplying the outcome by x_1, x_2, x_3 and x_4 in turn and substituting into equation 2, we have:

$$\hat{Y} = b_0x_1 + b_0x_2 + b_0x_3 + b_0x_3 + b_0x_4 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{34}x_3x_4 + b_{11}(x_1 - x_1x_2 - x_1x_3 - x_1x_4) + b_{22}(x_2 - x_1x_2 - x_2x_3 - x_2x_4) + b_{33}(x_3 - x_1x_3 - x_2x_3 - x_3x_4) + b_{44}(x_4 - x_1x_4 - x_2x_4 - x_3x_4) \quad (3)$$

Re-arranging Eq. 3, we have

$$\hat{Y} = \Sigma\propto_i x_i + \Sigma\propto_{ij} x_i x_j \quad (4)$$

Where $1 \leq i \leq q$, $1 \leq i \leq j \leq q$, $1 \leq i \leq j \leq q$ respectively and

$$\propto_i = b_0 + b_i + b_{ii} \text{ and } \propto_{ij} = b_{ij} + b_{ii} + b_{ii} \quad (5)$$

Let the response function to the pure components (x_i) be denoted by y_i and the response to a 1:1 binary mixture of components i and j be y_{ij}. From Eq. 4, it can be written that

$$\Sigma\propto_i x_i = \Sigma y_i x_i \quad (6)$$

Where (i = 1 to 4)
Evaluating y_i, for instance gives:

$$y_i = \propto_i \quad (7)$$

Also evaluating y_{ij}, gives in general the equations of the form

$$\propto_{ij} = 4y_{ij} - 2y_i - 2y_j \quad (8)$$

For the (4, 2) lattice polynomial, that is eqn. 4 becomes:

$$\hat{Y} = y_1 x_1 + y_2 x_2 + y_3 x_3 + y_4 x_4 + (4y_{12} - 2y_1 - 2y_2) x_1 x_2 + (4y_{13} - 2y_1 - 2y_3) x_1 x_3 + (4y_{14} - 2y_1 - 2y_4) x_1 x_4 + (4y_{23} - 2y_2 - 2y_3) x_2 x_3 + (4y_{24} - 2y_2 - 2y_4) x_2 x_4 + (4y_{34} - 2y_3 - 2y_4) x_3 x_4 \qquad (9)$$

1.2. The student's t-test

The unbiased estimate of the unknown variance $\boldsymbol{S}^2$ is given by Biyi (1975).

$$S_Y^2 = \frac{\sum \left(y_i - \breve{y} \right)^2}{n-1} \qquad (10)$$

If $a_i = x_i (2x_i - 1)$, $a_{ij} = 4 x_i x_j$; for ($1 \le i \le q$) and ($1 \le i \le j \le q$) respectively.

Then, $\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$ (11)

where ε is the error of the predicted values of the response. The t-test statistic is given by Biyi(1975).

$$t = (\Delta y \sqrt{n}/\boldsymbol{s}_y)\sqrt{(1+\varepsilon)}\} \qquad (12)$$

where $\Delta y = y_0 - y_t$; y_0 = observed value, y_t = theoretical value; n = number of replicate observations at every point; ε = as defined in Eq.11.

1.3. The Fisher's test

The Fishers-test statistic is given by

$$F = S_1^2/S_2^2 \qquad (13)$$

The values of S_1(lower value) and S_2 (upper value) are calculated from Eq. 10.

2. Materials and method

2.1. Preparations, curing and testing of cube samples

The aggregates were sampled in accordance with the methods prescribed in BS 812: Part 1:1975. The test sieves were selected according to BS 410:1986. The water absorption, the apparent specific gravity and the bulk density of the coarse aggregates were determined following the procedures prescribed in BS 812: Part 2: 1975. The Los Angeles abrasion test was carried out in accordance with ASTM. Standard C131: 1976. The sieve analyses of the fine and coarse aggregate samples were done in accordance with BS 812: Part 1: 1975 and satisfied BS 882:1992. The sieving was performed by a sieve shaker. The water used in preparing the experimental samples satisfied the conditions prescribed in BS 3148:1980. The required concrete specimens were made in threes in accordance with the method specified in BS 1881: 108:1983. These specimens were cured for 28 days in accordance with BS 1881: Part 111: 1983. The testing was done in accordance with BS 1881: Part 116:1983 using compressive testing machine.

Table 1
Responses of the actual components

S/NO	Z_1	Z_2	Z_3	Z_4	Y	Average compressive strength[N/mm^2]
1	0.6	1	1.5	2	Y_1	30
2	0.5	1	1	2	Y_2	32
3	0.55	1	2	5	Y_3	19
4	0.65	1	3	6	Y_4	12
5	0.55	1	1.25	2	Y_{12}	31.7
6	0.575	1	1.75	3.5	Y_{13}	24.3
7	0.625	1	2.25	4	Y_{14}	19.6
8	0.525	1	1.5	3.5	Y_{23}	24.8
9	0.575	1	2	4	Y_{24}	20
10	0.6	1	2.5	5.5	Y_{34}	18.5

Legend**:** Z_1= water/cement ratio; Z_2=Cement; Z_3=Fine aggregate; Z_4=Coarse aggregate; Y=responses.

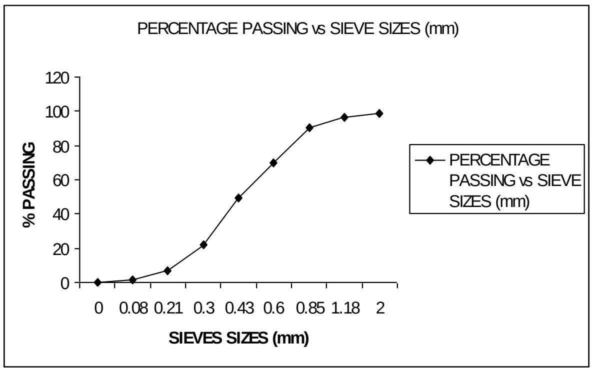

Fig. 1. Grading curve for the fine aggregate.

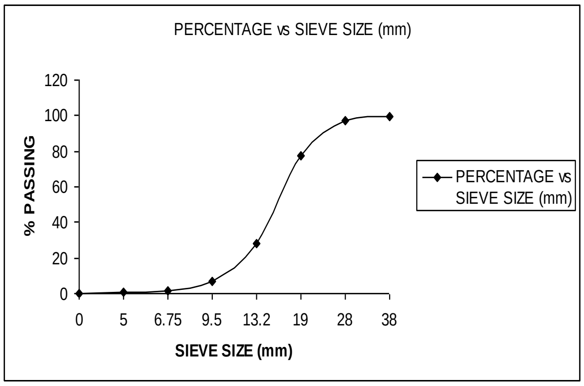

Fig. 2. Grading curve for the granite chippings.

2.2. Testing the fit of the quadratic polynomials

The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. The null hypothesis was denoted by H_0 and the alternative by H_1.

3. Results and discussion

3.1. Physical and mechanical properties of aggregates

Sieve analyses of both the fine and coarse aggregates were performed and the grading curves shown in figures 1 and 2. These grading curves showed the particle size distribution of the aggregates. The maximum aggregate size for the granite chipping was 20 mm and 2mm for the fine sand. The granite chippings had water absorption of 2.7%, moisture content of 44.2%, apparent specific gravity of 2.26, Los Angeles abrasion value of 22% and bulk density of 2072.4 kg/m^3.

3.2. The regression equation for the compressive strength tests results

Applying the responses (average compressive strengths) of table 1 in determining the coefficients of the (4, 2) lattice polynomial to eqns. 7 and 8, we had $\alpha_1 = 30$, α_2=32, α_3=19, α_4=12, α_{12}= 4 x 31.7 – 2 x 30 – 2 x 32 = 2.8. Similarly, α_{13}= - 0.8, α_{14}=- 5.6, α_{23}= -2.8, α_{24}= -8, α_{34}= 12. Thus, from eqn.9:

Table 2
Design matrix for scheffe's (4, 2) lattice polynomial

Pseudo-components					Response Component	Actual components			
S/N	X_1	X_2	X_3	X_4	Y	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.6	1	1.5	2
2	0	1	0	0	Y_2	0.5	1	1	2
3	0	0	1	0	Y_3	0.55	1	2	5
4	0	0	0	1	Y_4	0.65	1	3	6
5	½	½	0	0	Y_{12}	0.55	1	1.25	2
6	½	0	½	0	Y_{13}	0.575	1	1.75	3.5
7	½	0	0	½	Y_{14}	0.625	1	2.25	4
8	0	½	½	0	Y_{23}	0.525	1	1.5	3.5
9	0	½	0	½	Y_{24}	0.575	1	2	4
10	0	0	½	½	Y_{34}	0.6	1	2.5	5.5
					CONTROL				
11	½	¼	¼	0	C_1	0.5625	1	1.5	2.75
12	½	0	¼	¼	C_2	0.6	1	2.0	3.75
13	0	½	¼	¼	C_3	0.55	1	1.75	3.75
14	¼	¼	¼	¼	C_4	0.575	1	1.875	3.75
15	¾	¼	0	0	C_5	0.575	1	1.375	2
16	¾	0	¼	0	C_6	0.5875	1	1.625	2.75
17	¾	0	0	¼	C_7	0.6125	1	1.875	3.0
18	0	¾	¼	0	C_8	0.5125	1	1.25	2.75
19	0	¾	0	¼	C_9	0.5375	1	1.5	3.0
20	0	0	¾	¼	C_{10}	0.5850	1	2.25	5.25

$\hat{Y} = 30 x_1 + 32 x_2 + 19 x_3 + 12 x_4 + 2.8 x_1 x_2 - 0.8 x_1 x_3 - 5.6 x_1 x_4 - 2.8 x_2 x_3 - 8 x_2 x_4 + 12 x_3 x_4.$

This is the mathematical model for the prediction of the compressive strength characteristics of granite chippings concrete, based on Scheffe's (4, 2) polynomial.

3.3. Fit of the polynomial

The scope of the work was represented as the design matrix for Scheffe's (4, 2) lattice polynomial (table 2). The polynomial regression equation developed i.e., $\hat{Y} = 30 x_1 + 32 x_2 + 19 x_3 + 12 x_4 + 2.8 x_1 x_2 - 0.8 x_1 x_3 - 5.6 x_1 x_4 - 2.8 x_2 x_3 - 8 x_2 x_4 + 12 x_3 x_4$, was tested to see if the model agreed with the actual experimental results. There was no significant difference between the experimental and the theoretically expected results. The null hypothesis, H_0 was satisfied.

3.4. t -value from table

The t-student's test had a significance level, $\alpha = 0.05$ and $t_{\alpha/l(ve)} = t_{0.005(9)} = 3.69$. This was greater than any of the t values calculated in table 3. Therefore, the regression equation for the crushed granite chippings concrete was adequate.

3.5. F-statistic analysis

The sample variances S_1^2 and S_2^2 for the two sets of data were not significantly different (table 4). It implied that the error(s) from experimental procedure were similar and that the sample variances being tested are estimates of the same population variance. Based on Eq.10, we had that $S_K^2 = 102.9929/9 = 11.444$, $S_E^2 = 101.5538/9 = 1.284$ & $\mathbf{F} = 11.444/11.284 = 1.014$. From Fisher's table, $F_{0.95(9,9)} = 3.3$, hence the regression equation for the compressive strength of the crushed-granite concrete was adequate.

Table 3
T –statistic for the controlled points, granite-concrete compressive test, based on scheffe's (4, 2) polynomial

Response Symbol	i	j	a_i	a_{ij}	a_i^2	a_{ij}^2	ε	y̌	Ŷ	t
C_1	1	2	0	0.5	0	0.25	0.6093	28	27.825	1.689953
	1	3	0	0.5	0	0.25				
	1	4	0	0	0	0				
	2	3	-0.125	0.25	0.0156	0.0625				
	2	4	-0.125	0	0.0156	0				
	3	4	-0.125	0	0.0156	0				
	4	—	0	—	0	—				
				∑	0.0468	0.5625				
						Similarly				
C_2	—	—	—	—	—	—	0.4842	22.5	22.7	-1.56204
C_3	—	—	—	—	—	—	0.7343	23.3	23.15	1.002587
C_4	—	—	—	—	—	—	0.5939	22.9	23.1	-1.45453
C_5	—	—	—	—	—	—	0.2893	30.8	31.03	-2.02294
C_6	—	—	—	—	—	—	0.8593	27.6	27.1	3.117279
C_7	—	—	—	—	—	—	0.5937	24.5	24.45	0.363679
C_8	—	—	—	—	—	—	0.4833	28	28.225	-1.75836
C_9	—	—	—	—	—	—	0.6405	25.3	25.5	-1.89464
C_{10}	—	—	—	—	—	—	0.4697	19.4	19.5	-0.78873

Legend: c_i = response; $a_i = x_i (2x_i - 1)$; $a_{ij} = 4 x_i x_j$; $\varepsilon = \Sigma a_i^2 + \Sigma a_{ij}^2$; y̌ = experimentally-observed value; Ŷ= theoretical value; t = t-test statistic.

Table 4
F –statistic for the controlled points, granite concrete compressive strength, based on scheffe's (4, 2) polynomial

Response symbol	Y_K	Y_E	Y_K- $\breve{Y}_K$	Y_E-$\breve{Y}_E$	$(Y_K$- $\breve{Y}_K)^2$	$(Y_E$-$\breve{Y}_E)^2$
C_1	28	27.825	2.77	2.5675	7.6729	6.592056
C_2	22.5	22.7	-1.4	-1.1585	7.29	6.540806
C_3	23.3	23.15	-0.6	-0.7085	3.61	4.441556
C_4	22.9	23.1	-1	-0.7585	5.29	4.654806
C_5	30.8	31.025	6.9	7.1715	31.36	33.26406
C_6	27.6	27.1	3.7	3.2415	5.76	3.394806
C_7	24.5	24.45	0.6	0.5915	0.49	0.652056
C_8	28	28.225	4.1	4.3665	7.84	8.806056
C_9	25.3	25.5	-0.2	0.2425	0.04	0.058806
C_{10}	19.4	19.5	-4.5	-4.3585	33.64	33.14881
$\sum$	252.3	252.575			102.9929	101.5538

Legend: $\breve{Y}=\Sigma y/n$ where y is the response and n, the number of observed data (responses)
y_k is the experimental value (response)
y_E is the expected or theoretically calculated value(response).

Conclusion

The strengths (responses) of concrete were a function of the proportions of its ingredients: water, cement, fine aggregate and coarse aggregates. Since the predicted strengths by the model were in total agreement with the corresponding experimentally - observed values, the null hypothesis was satisfied. This meant that the model equation was valid.

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