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Mathematical model for the prediction of the compressive strength characteristics of concrete made with unwashed local gravel

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Abstract

This research work made use of Scheffe's (4, 2) lattice polynomial with regression equations to develop a mathematical model for the prediction of the compressive strength characteristics of concrete made with unwashed local gravel from Abagana, Anambra state. Concrete cubes were made, cured and tested according to BS 1881:1983.The mathematical model developed, i.e., $\hat{Y} = 20.67x_1 + 22.01\ x_2 + 13.04\ x_3 + 8.26\ x_4 + 1.72\ x_1\ x_2 - 0.46\ x_1\ x_3 - 3.78\ x_1\ x_4 - 1.7\ x_2\ x_3 - 5.5\ x_2\ x_4 + 8.6x_3\ x_4$, was subjected to both the student's t-test and the Fisher test to determine its adequacy. The strengths predicted by this model were in complete agreement with the experimentally-obtained values. The research also showed the effects of organic impurities on the strength of concrete with strengths of the unwashed local gravel concrete markedly less than those of washed local gravel concrete.

1. Introduction

1.1. The Scheffe's (4, 2) lattice polynomial

Simplex is the structural representation of the line or planes joining the assumed positions of the constituent materials (atoms) of a mixture (Jackson and Dhir, 1988). Scheffe, 1958 considered experiments with mixtures of which the property studied depended on the proportions of the components present but not on the quantity of the mixture. If a mixture has a total of q components and x_i be the proportion of the ith component in the mixture such that $x_i \geq 0$ (i = 1, 2… q), then

$$x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots\ldots\ldots + x_q = 1 \text{ or} \tag{1}$$

H. Scheffe [2] described mixture properties by reduced polynomials obtainable from Eq. 2:

$$\hat{Y} = b_0 + \Sigma b_i x_i + \Sigma b_{ij}\, x_i\, x_j + \Sigma b_{i\,jk}\, x_i\, x_j\, x_k + \Sigma b_{i1, i\,2} \ldots i_n\, x_{i1}\, x_{i2}\, x_{i\,n} \tag{2}$$

Where ($1\leq i \leq q$, $1\leq i \leq j \leq q$, $1 \leq i \leq k \leq q$) respectively and b is constant coefficient.
Multiplying Eq.1 by b_0 and multiplying the outcome by x_1, x_2, x_3 and x_4 in turn and substituting into equation 2, we have:

$$\hat{Y} = b_0\ x_1 + b_0\ x_2 + b_0\ x_3 + b_0\ x_3 + b_0\ x_4 + b_1\ x_1 + b_2\ x_2 + b_3x_3 + b_4x_4 + b_{12}\ x_1\ x_2 + b_{13}\ x_1\ x_3 + b_{14}\ x_1\ x_4 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{34}x_3x_4 + b_{11}(x_1 - x_1x_2 - x_1x_3 - x_1x_4) + b_{22}(x_2 - x_1x_2 - x_2x_3 - x_2x_4) + b_{33}(x_3 - x_1x_3 - x_2x_3 - x_3x_4) + b_{44}(x_4 - x_1x_4 - x_2x_4 - x_3x_4) \tag{3}$$

Re-arranging Eq. 3, we have

$$\hat{Y} = \Sigma\propto_i x_i + \Sigma\propto_{ij}\, x_i\, x_j \tag{4}$$

Where $1 \leq i \leq q$, $1\leq i \leq j \leq q$, $1\leq i \leq j \leq q$ respectively and

$$\propto_i = b_0 + b_i + b_{ii} \text{ and } \propto_{ij} = b_{ij} + b_{i\,i} + b_{ii} \tag{5}$$

Let the response function to the pure components (x_i) be denoted by y_i and the response to a 1:1 binary mixture of components i and j be y_{ij}. From Eq. 4, it can be written that

$$\Sigma\propto_i x_i = \Sigma y_i\, x_i \tag{6}$$

Where (i = 1 to 4)
Evaluating y_i, for instance gives:

$$y_i = \propto_i \tag{7}$$

Also evaluating y_{ij}, gives in general the equations of the form

$$\propto_{ij} = 4y_{ij} - 2\, y_i - 2y_j \tag{8}$$

For the (4, 2) lattice polynomial, that is Eq. 4 becomes:

$\hat{Y} = y_1 x_1 + y_2 x_2 + y_3 x_3 + y_4 x_4 + (4y_{12} - 2y_1 - 2y_2) x_1 x_2 + (4y_{13} - 2y_1 - 2y_3) x_1 x_3 + (4y_{14} - 2y_1 - 2y_4) x_1 x_4 + (4y_{23} - 2y_2 - 2y_3) x_2 x_3 + (4y_{24} - 2y_2 - 2y_4) x_2 x_4 + (4y_{34} - 2y_3 - 2y_4) x_3 x_4$ (9)

1.2. The student's t-test

The unbiased estimate of the unknown variance S^2 is given by Biyi, 1975,

$$S_Y^2 = \frac{\sum\left(y_i - \breve{y}\right)^2}{n-1} \quad (10)$$

If $a_i = x_i (2x_i - 1)$, $a_{ij} = 4 x_i x_j$; for $(1 \le i \le q)$ and $(1 \le i \le j \le q)$ respectively.

Then, $\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$ (11)

where ε is the error of the predicted values of the response. The t-test statistic is given by Biyi, 1975

$t = (\Delta y \sqrt{n}/\boldsymbol{s}_y)\sqrt{(1+\varepsilon)}\}$ (12)

where $\Delta y = y_0 - y_t$; y_0 = observed value, y_t = theoretical value; n = number of replicate observations at every point; ε = as defined in Eq.11.

1.3. The Fisher's test

The Fishers-test statistic is given by

$F = \mathbf{S}_1^2/\boldsymbol{S}_2^2$ (13)

The values of $\mathbf{S_1}$(lower value) and $\mathbf{S_2}$ (upper value) are calculated from Eq. 10.

2. Materials and methods

2.1. Preparations, curing and testing of cube samples

The aggregates were sampled in accordance with the methods prescribed in BS 812: Part 1:1975. The test sieves were selected according to BS 410:1986. The water absorption, the apparent specific gravity and the bulk density of the coarse aggregates were determined following the procedures prescribed in BS 812: Part 2: 1975. The Los Angeles abrasion test was carried out in accordance with ASTM. Standard C131: 1976. The sieve analyses of the fine and coarse aggregate samples were done in accordance with BS 812: Part 1: 1975 and satisfied BS 882:1992. The sieving was performed by a sieve shaker. The water used in preparing the experimental samples satisfied the conditions prescribed in BS 3148:1980. The required concrete specimens were made in threes in accordance with the method specified in BS 1881: 108:1983. These specimens were cured for 28 days in accordance with BS 1881: Part 111: 1983. The testing was done in accordance with BS 1881: Part 116:1983 using compressive testing machine.

Table 1
Responses of the actual components

S/NO	Z_1	Z_2	Z_3	Z_4	Y	Average compressive strength[N/mm^2]
1	0.6	1	1.5	2	Y_1	20.67
2	0.5	1	1	2	Y_2	22.01
3	0.55	1	2	5	Y_3	13.04
4	0.65	1	3	6	Y_4	8.26
5	0.55	1	1.25	2	Y_{12}	21.77
6	0.575	1	1.75	3.5	Y_{13}	16.74
7	0.625	1	2.25	4	Y_{14}	13.52
8	0.525	1	1.5	3.5	Y_{23}	17.1
9	0.575	1	2	4	Y_{24}	13.76
10	0.6	1	2.5	5.5	Y_{34}	12.8

Legend: Z_1= water/cement ratio; Z_2=Cement; Z_3=Fine aggregate; Z_4=Coarse aggregate; Y = responses.

2.2. Testing the fit of the quadratic polynomials

The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. The null hypothesis was denoted by H_0 and the alternative by H_1.

3. Results and discussion

3.1. The regression equation for the compressive strength tests results

Applying the responses (average compressive strengths) of table 1 in determining the coefficients of the (4, 2) lattice polynomial to Eqs. 7 and 8, we had α_1=20.67, α_2=22.01, α_3=13.04, α_4=8.26.

Similarly, α_{12}=1.72, α_{13}=-0.46, α_{14}=-3.78, α_{23}=-1.7, α_{24}=-5.5, α_{34}=8.6. Thus, from Eq. 9:

$\hat{Y} = 20.67x_1 + 22.01\,x_2 + 13.04\,x_3 + 8.26\,x_4 + 1.72\,x_1\,x_2 - 0.46\,x_1\,x_3 - 3.78\,x_1\,x_4 - 1.7\,x_2\,x_3 - 5.5\,x_2\,x_4 + 8.6x_3\,x_4.$

This is the mathematical model for the prediction of the compressive strength characteristics of the unwashed local gravel concrete, based on Scheffe's (4, 2) polynomial.

3.2. Fit of the polynomial

The scope of the work was represented as the design matrix for Scheffe's (4, 2) lattice polynomial (table 2). The polynomial regression equation developed i.e., $\hat{Y} = 20.67x_1 + 22.01\,x_2 + 13.04\,x_3 + 8.26\,x_4 + 1.72\,x_1\,x_2 - 0.46\,x_1\,x_3 - 3.78\,x_1\,x_4 - 1.7\,x_2\,x_3 - 5.5\,x_2\,x_4 + 8.6x_3\,x_4$, was tested to see if the model agreed with the actual experimental results. There was no significant difference between the experimental and the theoretically expected results. The null hypothesis, H_0 was satisfied.

Table 2
Design matrix for scheffe's (4, 2) lattice polynomial

Pseudo-components					Response Components	Actual components			
S/N	X_1	X_2	X_3	X_4	Y	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.6	1	1.5	2
2	0	1	0	0	Y_2	0.5	1	1	2
3	0	0	1	0	Y_3	0.55	1	2	5
4	0	0	0	1	Y_4	0.65	1	3	6
5	½	½	0	0	Y_{12}	0.55	1	1.25	2
6	½	0	½	0	Y_{13}	0.575	1	1.75	3.5
7	½	0	0	½	Y_{14}	0.625	1	2.25	4
8	0	½	½	0	Y_{23}	0.525	1	1.5	3.5
9	0	½	0	½	Y_{24}	0.575	1	2	4
10	0	0	½	½	Y_{34}	0.6	1	2.5	5.5
					CONTROL				
11	½	¼	¼	0	C_1	0.5625	1	1.5	2.75
12	½	0	¼	¼	C_2	0.6	1	2.0	3.75
13	0	½	¼	¼	C_3	0.55	1	1.75	3.75
14	¼	¼	¼	¼	C_4	0.575	1	1.875	3.75
15	¾	¼	0	0	C_5	0.575	1	1.375	2
16	¾	0	¼	0	C_6	0.5875	1	1.625	2.75
17	¾	0	0	¼	C_7	0.6125	1	1.875	3.0
18	0	¾	¼	0	C_8	0.5125	1	1.25	2.75
19	0	¾	0	¼	C_9	0.5375	1	1.5	3.0
20	0	0	¾	¼	C_{10}	0.5850	1	2.25	5.25

3.4. t -value from table

The t-student's test had a significance level, α = 0.05 and $t_{\alpha/l(ve)} = t_{0.005(9)}$=3.69. This was greater than any of the t values calculated in table 3. Therefore, the regression equation for the crushed granite chippings concrete was adequate.

3.5. F-statistic analysis

The sample variances $\boldsymbol{S_1}^2$ and $\boldsymbol{S_2}^2$ for the two sets of data were not significantly different (table 4).

It implied that the error(s) from experimental procedure were similar and that the sample variances being tested are estimates of the same population variance. Based on eqn.10, we had that S_K^2 = 52.14626/9 = 5.794, S_E^2 = 52.17572/9 = 5.797 & **F** = 5.797 /5.794= 1.001. . From Fisher's table, $F_{0.95(9,9)}$ = 3.3, hence the regression equation for the compressive strength of the unwashed local gravel concrete was adequate.

Table 3
t –statistic for the controlled points, unwashed local gravel concrete compressive test, based on scheffe's (4, 2) polynomial

RESPONSE SYMBOL	i	J	ai	aij	ai2	aij2	ε	ў	Ŷ	t
C_1	1	2	0	0.5	0	0.25	0.6093	19.13	19.15	-1.95283
	1	3	0	0.5	0	0.25				
	1	4	0	0	0	0				
	2	3	-0.125	0.25	0.0156	0.0625				
	2	4	-0.125	0	0.0156	0				
	3	4	-0.125	0	0.0156	0				
	4	—	0	—	0	—				
				∑	0.0468	0.5625				
	Similarly									
C_2	—	—	—	—	—	—	0.4842	14.59	14.59	0
C_3	—	—	—	—	—	—	0.7343	15.94	15.97	-2.71813
C_4	—	—	—	—	—	—	0.5939	15.9	15.925	-2.46463
C_5	—	—	—	—	—	—	0.2893	21.31	21.328	-2.19377
C_6	—	—	—	—	—	—	0.8593	18.7	18.676	2.02831
C_7	—	—	—	—	—	—	0.5937	16.86	16.859	0.098598
C_8	—	—	—	—	—	—	0.4833	19.45	19.449	0.105936
C_9	—	—	—	—	—	—	0.6405	17.51	17.541	-2.96933
C_{10}	—	—	—	—	—	—	0.4697	13.46	13.458	0.213833

Legend: c_i =response; $a_i = x_i (2x_i - 1)$; $a_{ij} = 4 x_i x_j$; $\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$; ў = experimentally-observed value; Ŷ= theoretical value; t = t-test statistic.

Table 4
F –statistic for the controlled points, granite concrete compressive strength, based on scheffe's (4, 2) polynomial.

Response symbol	YK	YE	YK- ỸK	YE-ỸE	(YK- ỸK)2	(YE-ỸE)2
C_1	19.13	19.15	1.845	1.8554	3.404025	3.442509
C_2	14.59	14.59	-2.695	-2.7046	7.263025	7.314861
C_3	15.94	15.97	-1.345	-1.3246	1.809025	1.754565
C_4	15.9	15.925	-1.385	-1.3696	1.918225	1.875804
C_5	21.31	21.328	4.025	4.0334	16.20063	16.26832
C_6	18.7	18.676	1.415	1.3814	2.002225	1.908266
C_7	16.86	16.859	-0.425	-0.4356	0.180625	0.189747
C_8	19.45	19.449	2.165	2.1544	4.687225	4.641439
C_9	17.51	17.541	0.225	0.2464	0.050625	0.060713
C_{10}	13.46	13.458	-3.825	-3.8366	14.63063	14.7195
∑	172.85	172.946			52.14626	52.17572

Legend: $\check{Y} = \Sigma y/n$ where y is the response and n, the number of observed data (responses)
y_k is the experimental value (response)
y_E is the expected or theoretically calculated value(response)

Conclusion

The strengths (responses) of concrete were a function of the proportions of its ingredients: water, cement, fine aggregate and coarse aggregates. Since the predicted strengths by the model were in total agreement with the corresponding experimentally -observed values, the null hypothesis was satisfied. This meant that the model equation was valid. Also, the responses of the cubes made with the unwashed local gravel were markedly less than those of washed local gravel concrete suggesting the effect of organic impurities on the compressive strengths of concrete.

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