JEAS

Journal of Engineering and Applied Sciences 7 (2011) 45-49

JOURNAL OF ENGINEERING
AND
APPLIED SCIENCES

Mathematical modelling and simulation of solar drying of cassava

J.T. Nwabanne, V. N. Okafor

Department of Chemical Engineering, Nnamdi Azikiwe University, PMB 5025, Awka, Nigeria

Abstract

A mathematical model for the solar drying of cassava, was developed to predict the moisture content and temperature of the samples, using a finite difference numerical solution procedure to generate a computer simulation in Visual Basic 6.The model predicts the moisture content and temperature of the cassava using input data such as initial moisture content, solar radiation intensity, relative humidity, ambient temperature, and change in time. To validate the model, predicted moisture content and temperature were compared with experimental data. The model was in good agreement with experimental results. The model showed that drying was insensitive to ambient relative humidity but sensitive to factors affecting the temperature of the sample.

Keywords: Solar drying; modeling; simulation; cassava

1. Introduction

With the rising demand for food by ever increasing human population, research has continually been carried out to find out ways to maximize the level of food production to satisfy man's insatiable hunger. Sometime farmers sell at very low prices during the harvest season because they cannot store or preserve their products. Drying is the most widespread heat and mass transport process and one of the most energy consuming operations (Oliveira and Haghighi, 1992). During drying, heat and mass (air, vapour, free water, bound water) transfers occur. The basic laws of conservation of each quantity (momentum, energy and mass) are at the origin of the formulation of heat and mass transfer in the porous media in which the divergence of different fluxes is required (Perré and Turner, 1997). Drying of agricultural crops is essentially a means of increasing food availability and reducing losses. Proper drying inhibits germination of seeds and growth of fungi and bacteria; it also prevents attacks on grains by insects and mites. The basic essence of drying is to reduce the moisture content of the product to a level that prevents deterioration within a certain period of time normally regarded as the "safe storage period" (Ekechukwu, 1987). Drying is a dual process of heat transfer to the products from the heating source and mass transfer of moisture from the interior of the product to its surface and from the surface to the surrounding air. The quality of an agricultural material is influenced by its temperature and moisture content (Ismail et al, 2008). Solar drying technology involves the use of various solar energy dryers to improve the effectiveness of sun drying technique (Ekechukwu, 1987). Solar drying offers the advantages of faster drying than regular sun drying. The use of solar dryer systems to conserve fruits, crops and other kinds of food is practical, economical and environmentally responsible (Salom et al, 1996). Solar drying has helped to increase the position of farmers. At present the increasing demand for healthy low-cost natural foods and the need for sustainable income, are bringing solar drying to the fore as a useful alternative for surplus products.

The knowledge of the physical properties of agricultural products is essential when investigating the heat and mass transfer phenomena in product drying (Morita, 1979). A knowledge of the thermal properties of agricultural products such as specific heat, thermal conductivity, thermal diffusivity, surface conductance as well as some physical properties are essential for design of drying equipment and predicting the processes. (Okoli, 1991; Nwabanne, 2009).

Cassava is a common staple food item known the world over, especially in sub-Saharan Africa. They are of the carbohydrate class of food (Ihekoronye and Ngoddy, 1985). Nigeria is the world's largest producer of cassava with an estimated production of 34, 410, 000 out of 214, 515, 149 tonnes representing 16% of global production in 2007 (FAO). Cassava (marihot esculenta) represents the main source of energy for 200 to 300 million people all over the world (Ademiliyu et al, 2006). Modelling of the drying process ultimately has the potential to quickly identify the consequences of schedule alterations and provide focus to the solution of drying problems (Pang and Haslett, 1997). Mathematical modeling and numerical simulation not only allows some rather expensive and repetitive experimentation to be avoided but perhaps most importantly, can be use to elucidate on the underlying physics associated with the convoluted heat and mass transport phenomena which are generated with the porous media during the drying process (Turner and Perré, 1997)

2. Materials and methods

2.1. Sourcing and preparation of food samples

Cassava tubers were obtained from a farm at the University of Nigeria, Nsukka. The cassava tubers were washed with water, peeled and sliced ready for loading into the solar dryer. Thereafter the sliced samples of equal length, width and thickness were loaded into the trays of the solar dryer. During drying, air was allowed to enter the dryer through the tiny holes at the bottom of the dryer. The physical properties of the samples (temperature, mass, moisture content) were determined. Oven drying method was used to determine the moisture content of the samples. The environmental conditions such as temperature of the surrounding air, relative humidity and solar radiation intensity were determined using digital thermocouple, hygrometer and digital solarimeter respectively.

Drying was carried out for 8 hours (8 am to 4 pm) each day. Every hour the moisture loss was determined. After each day's operation, the samples were kept in polythene bags for drying the next day.

2.2. Model development

Basic assumptions

The following assumptions are made in the formulation of the mathematical model

1. The process is a batch process. It is an unsteady state process
2. Heat losses by conduction are negligible
3. Wind velocity is negligible
4. Moisture loss is limited to internal diffusion

2.3. Mass balance

The mass balance takes into account the rate of moisture loss. That is a measure of the drying rate.

$$\frac{dX}{dt} = -K(X - X_e) \tag{1}$$

The value of K was given by the Arrhenius' equation [Phoungchandang and Woods, 2000].

$$K = 0.6854\exp\left[\frac{-3316.1}{T_s + 273.15}\right] \tag{2}$$

The equilibrium moisture content, X_e was obtained using modified Oswin equation given below:

$$X_e = \frac{(C_1 + C_2 T_s)}{\left(\frac{1}{RH_s} - 1\right)^{\frac{1}{C_2}}} \tag{3}$$

The values of C_1, C_2, and C_3 are given as 16.68, -0.1212 and 0.9020 respectively. [Phoungchandang and Woods, 2000].

The relative humidity at the surface of the sample, RH_s was obtained from the correlation [Phoungchandang and Woods, 2000].

$$RH_s = \frac{1}{x}\left[\frac{H}{0.622 + H}\right] \tag{4}$$

The value of x is a ratio of the saturation pressure of the sample surface to the atmospheric pressure.

$$x = \frac{P_{ws}}{P_{atm}} \tag{5}$$

2.4. Energy balance

The rate of sensible energy gain is equal to the solar radiation absorbed less energy losses due to convection, radiation and evaporation. [Woods, 1991]

Algebraically,

$$M_s C_{ps}\frac{dT_s}{dt} = Q_{abs} - (Q_c + Q_r + Q_e) \tag{6}$$

The specific heat of the sample could be obtained from the relation, (Anon, 1989),

$$C_s = 1.675 + 0.025 X_{wb} \tag{7}$$

I. Solar absorption: The solar energy absorbed for unshaded food samples can be written as

$$Q_{abs} = \alpha(ld)q$$

Where *ld* is the projected area of the sample.

II. Convection: The convective heat loss is predicted using dimensionless correlations from the literature. These are for natural or forced convection processes, but for this model, the heat loss due to forced convection is assumed negligible. The correlation for natural convection is given by

$$Nu = (h_c d)/k_A = a(Gr\,\mathrm{Pr})^n$$

For *GrPr* < 10^9, the flow is laminar, and a = 0.53, and n = 0.25 (Simonson, 1988). In this model development, the value of *GrPr* is always well into the laminar range.

III. Energy loss due to radiation**:** For a surface with a long-wave emissivity, ε at a temperature, T_s in a black body enclosure at a temperature, $T_{a,}$ the equation can be written as [Poungchandang and Woods, 2000].

$$Q_r = \varepsilon\sigma(\pi dl)(T_{ks}^4 - T_{ka}^4)$$

Linearization gives:

$$Q_r = hr(\pi dl)(T_{kb} - T_{ka})$$ [Duffie and Beckman, 1991]

Where $h_r = \varepsilon\sigma(T_{kb} - T_{ka})(T_{kb}^2 - T_{ka}^2)$

IV. Energy Loss by Convection**:** The energy loss by convection is given by (Anon, 1987)

$$Q_e = (L - fT_s)m_{H_2O}$$

Where L = 2448kj/kg and f = 0.2386kJ/kg-K

2.5. Model solving procedure

The change in moisture content defined by equation (1) was written in differential form:

$$\Delta X = \Delta t\,.\,k(X - X_e) \quad (8)$$

The change in temperature defined in equation (6) was also written in finite difference form as:

$$\Delta T_s = \frac{\Delta t}{M_s C_{ps}}[\alpha l d q_i - \pi dl(h_c + h_r)(T_s - T_a)] - (L - fT_s)\frac{M_{ds}\left(\frac{\Delta X}{100}\right)}{M_s C_s} \quad (9)$$

The solution to the above two equations were programmed in Visual Basic 6.0 language, with the procedure repeated for the updated temperature and moisture content. The time step used in the model throughout this work was Δt = 900s.

3. Results and discussion

The determined ambient conditions for the drying of cassava samples are shown in Table 1. The average values of relative humidity, ambient temperature, radiation intensity obtained were 0.60, 34^0C and 661 W/m^2 respectively. These values were similar to the values obtained by Phoungchandang, and Woods, 2000.

Table 1:
Values of the average ambient conditions for cassava

Day(s)	Relative humidity	Ambient Temp. (^{0}C)	Radiation intensity (W/m^2)
1	0.59	33.92	658
2	0.62	33.88	663
3	0.60	34.20	660
4	0.58	34.08	664
5	0.64	34.12	659
6	0.57	33.80	662
Average	0.60	34.00	661

The effect of ambient temperature, while holding all other variables constant at specified values is shown in Figure 1. Increasing the ambient temperature increased the temperature of the samples and also the drying rate.

The increase in the temperature of the cassava samples increases the rate of drying in two ways (Phoungchandang, and Woods, 2000). First, from the model equation described by Arrhenius' equation (2), it can be seen that the rate of internal diffusion varies proportionally with the temperature of the sample. Second, an inverse relationship between the sample temperature and equilibrium moisture content can be inferred by fixing the value of the relative humidity at the surface of the sample. Thus increasing sample temperature, T_s reduces equilibrium moisture content, X_e. The overall effect of this can then be seen from the equation for the rate of drying. Since K increases and X_e decreases with increasing temperature of the cassava sample, the change in moisture content gradually increases with the drying time and the higher the change in moisture content, the faster the drying rate under the conditions under study.

In like manner, fixing arbitrary values for other variables and increasing only the solar radiation gave faster rates for drying and higher temperature for the samples. This means if the thermal energy from the sun is high then this favours the drying of food materials, though excessive rise in temperature of the food to be dried should be avoided to avoid product damage.

A look at the shapes of Figs. 1 to 2 for the temperature and moisture content (wet basis) of the samples shows that the results of the model are slightly greater than their corresponding experimental values. This is because at the end of every day, conditions of overnight storage in polythene bags caused moisture to return to the surface, and thus evaporation is greater than predicted by the model making the experimental temperature rise each day to be slower than that predicted by the computer simulation. This also explains the reason why the experimental drying rate is slightly greater than that of the model.

Each day there was a rapid transient heating of the samples, which flattens off when the heat losses equal the energy absorbed by the radiant heat source. As drying rate declined with time, the evaporative heat loss was reduced, and in general, higher temperatures were achieved towards the end of drying as shown in Figure 2.

Generally, the model was in good agreement with experimental results. The model showed that drying was insensitive to ambient relative humidity but sensitive to factors affecting the temperature of the sample.

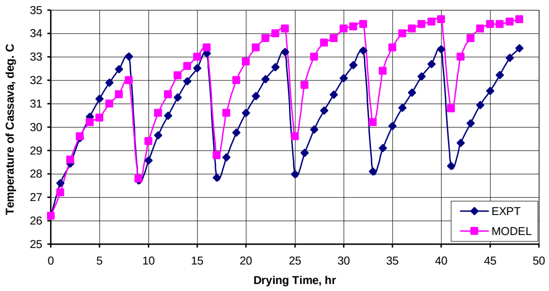

Fig. 1. Variation in cassava temperature readings with time.

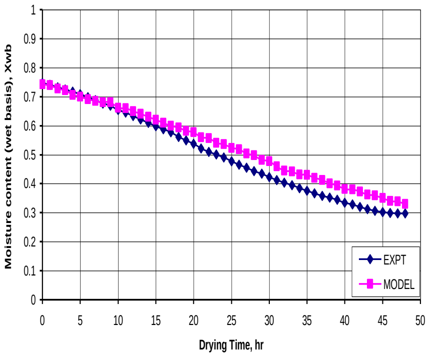

Fig. 2. Variation in cassava moisture content readings with time

4. Conclusion

Mathematical model of the solar drying of cassava was developed and shown to be in good agreement with experimental results. The average model error was found out to be ±0.33 for sample temperature and ±0.007 for moisture content (wet basis); while the Standard Error of Estimate was calculated to be 1.09%, for sample temperature and 1.35%, for moisture content (wet basis), all for cassava. The model predicts the drying process to be sensitive to factors that increase sample temperature but not to relative humidity and also helps determine the effect of any variable on the drying. This can be done by varying only the test variable and fixing arbitrary values for every other variable with subsequent results compared.

Nomenclature

a:	constant
b:	constant
C_1-C_3:	constant
C_p:	specific heat capacity
d:	diameter (or thickness) of sample (m)
f:	constant (kJ/kg-K)
Gr:	Grash of number
h_c:	convective heat transfer coefficient (W/m^{2}K)
h_r:	radiative heat transfer coefficient (W/m^{2}K)
H:	absolute humidity (kgH$_2$O/kg dry air)
k_A:	thermal conductivity of air (W/m-K)
K:	drying constant (s^{-1})
l:	length of sample (m)
L:	latent heat of vaporization (kJ/kg)
m_{H2O}:	rate of moisture evaporation (kg/s)
M_{ds}:	mass of sample dry matter (kg)
M_s:	mass of sample (kg)
n:	constant
Nu:	Nusselt number
P_a:	atmospheric pressure (kPa)
P_r:	Prandtl number
P_{ws}:	saturation vapour pressure of water (kPa)
q_i:	incident radiant energy (W/m^2)
Q_{abs}:	absorbed radiant energy (W)
Q_c:	surface convection heat loss (W)
Q_e:	evaporative heat loss (W)
Q_r:	radiation heat loss (W)
Re:	Reynolds number
RH:	relative humidity (decimal)
t:	time (s)
T:	temperature (^{0}C)
T_K:	absolute Temperature (K)
x:	pressure ratio (P_{ws}/P_a)
X:	moisture content (%d.b.)
X_{wb}:	moisture content (%w.b.)
X_e:	equilibrium moisture content

Greek Letters

α:	absorptivity of solar radiation
ε:	long-wave emissivity
σ:	Stefan-Boltzmann constant (W/m^{2}K^4)
°:	angle of incidence of solar radiation to the vertical (degrees)

Subscripts

a:	ambient
s:	sample
d. b.:	dry basis
e:	equilibrium
w. b.:	wet basis

References

Ademiliyu, T., Abowei, M. F., Achinewhu, S. C., Fabara, T. E., 2006. Effect of variety in the drying and engineering properties of fermented ground cassava. NEAM 1(1), 80 – 96.

Anon, 1987. ASAE Standards Psychometric Data. D271.2. New York: American Society of Agricultural Engineers, 4 – 11.

Anon, 1989. ASHRAE Fundamentals Handbook: Psychometric: Thermal Properties of Foods. Atlanta: American Society of Heating, Refrigerating and Air Conditioning Engineers, 30.1 - 30.28.

Duffie, J. A., Beckman W.A., 1991. Solar Engineering of Thermal Processes, 2nd ed. Wiley, New York, 919.

Ekechukwu, O.V., 1987. Elemental Study of Integral-type Natural-Circulation Solar Energy: Tropical Crop Dryers. Ph.D. Thesis, Cranfield Institute of Technology, United kingdom, 23-52.

FAO Statistics webpage: retrieved on June 2010 from http://www.faostat.fao.

Ihekoronye, A. I., Ngoddy, P.O., 1985. Technology of Food Preservation. In: Integrated Food Science and Technology for the Tropics. Macmillan Publishers Ltd., London, 115-148.

Ismail, M. A., Douglass M. P., Stenning, B. C., 2008. Modelling and simulation moisture transfer in wheat stored in a simulated sealed pit. International Journal of Food Engineering, 4(3), Article 9.

Morita, N.J., 1979. Physical Properties of Plant and Animal Materials. Gordon and Breach publisher Inc., New York, 20-35.

Nwabanne, J.T., 2009. Drying characteristics and engineering properties of fermented ground cassava. African J. Biotechnology, 8(5), 873 – 876.

Okoli, E.C., 1991. Design of a Deep Layer Dryer Equipment. B.Eng. Thesis, Department of Agricultural Engineering, University of Nigeria, Nsukka, 35-54.

Oliveira, L. S., Haghighi, K., 1997. Finite Element Modeling of Grain Drying. In: Mathematical Modelling and Numerical Techniques in Drying Technology. Ed. Turner, I. and Mujumdar, A. S. Marcel Dekker Inc., New York, 2, 309 – 310.

Pang, S., Haslett, A. N., 1997. High-temperature Kiln Drying of Softwood Timber: the Role of Mathematical Modeling. In: Mathematical Modelling and Numerical Techniques in Drying technology. Ed. Turner, I. and Mujumdar, A. S. Marcel Dekker Inc., New York, 2, 181.

Perré, P., Turner, I., 1997. The use of Macroscopic Equations to Simulate Heat and Mass Transfer in Porous Media: some possibilities illustrated by a wide range of configurations that emphasize the role of internal pressure. In: Mathematical Modelling and Numerical Ttechniques in Drying Technology. Ed. Turner, I. and Mujumdar, A. S. Marcel Dekker Inc., New York, 2, 85.

Phoungchandang, S., 1999. The Solar Drying of Bananas. Ph.D. thesis. University of Newcastle, Newcastle, 291.

Phoungchandang, S., Woods, J. L., 2000. Moisture diffusion and desorption isotherms for banana. J. Food Sci. 65(4), 651 – 657.

Salom, J., Ortega O., Felipe, J. J., 1996. A mathematical model for indirect solar driers. In EUROSUN 96, 346 – 52.

Simonson, J. R., 1988. Engineering Heat Transfer. 2nd ed. Basingstoke, Macmillan, 266.

Turner, I., Perré, P., 1997. A Synopsis of the Strategies and Efficient Resolution Techniques used for Modeling and Numerically Simulating the Drying Process. In: Mathematical Modelling and Numerical Techniques in Drying Technology. Ed. Turner, I. and Mujumdar, A. S. Marcel Dekker Inc., New York, 2.

Woods, J. I., 1991. Heat and Mass Transfer in the Drying and Cooling of Crops. In: Progress in Agricultural Engineering and Physics. Mathews J. editor. London: CAB International, 203 – 229.