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Stability characterization of a turning process

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Abstract

It will always be the desire of the machinist to simultaneously meet the two opposed demands: achieving high productivity and high surface quality. It is seen from this work that the extent to which these two demands are considered opposed depends on the machines stability information at the disposal of the machinist. The parameters adopted for the turning process considered are as follows; mass, $\mathrm{m} = 0.0431\mathrm{kg}$; Natural frequency, $\omega_n = 5700\ \mathrm{rad/sec}$; damping factor, $\xi = 0.02$; feed speed, $\mathrm{v} = 0.0025\mathrm{m/sec}$ and material cutting coefficient, $\mathrm{C} = 3.5 \times 10^7 \mathrm{Nm}^{\frac{-7}{4}}$. Stability chart was constructed from which a number of comments and recommendations were made. It was seen from the stability chart that the machine would have very low productivity at low spindle speed requiring very small dept of cut for stable operation. For example at a spindle speed of 2000rpm the maximum depth of cut for stable operation would be about 0.25mm. This result points to the need of chatter stability enhancement for low speed range turning machine through any means of chatter suppression.

Keywords: Chatter vibration, turning process, delay differential equation, machine tools, chatter suppression

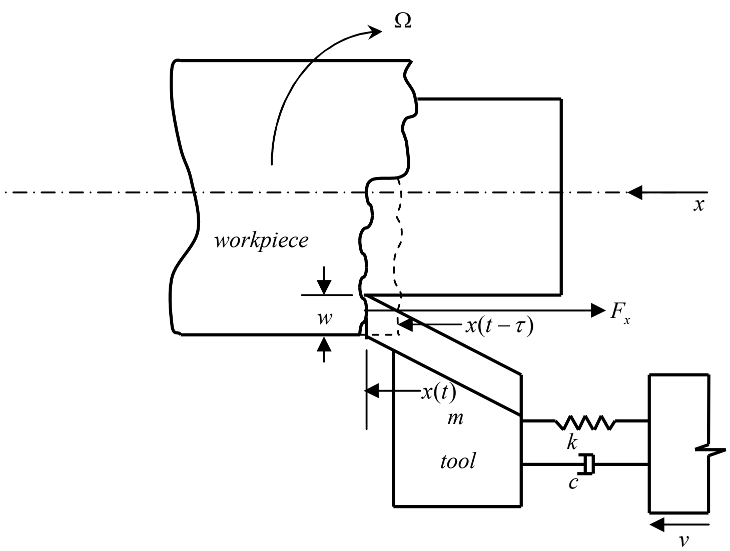
1. Introduction

Turning like most machining processes, is described by delay differential equation which is an equation in which the present rate of change is dependent on both the present and past solutions of the system. Full-immersion or low speed turning being a continuous process is described by autonomous delay differential equation. Chatter in such a turning process could build up progressively when there is no destructive interference stemming from any disturbance. For this reason it is logical to suppose that turning chatter could be suppressed by some form of periodic disturbance. The result that parametric excitation effects suppress chatter derives from the knowledge of stabilizing effects of parametric excitation on inverted pendulum (Insperger, 2002). Though not proven formally, it is qualitatively deduced from time domain analysis that low frequency periodic disturbance would be effective at suppressing turning chatter (Ozoegwu, 2011). Suppressive effect of spindle speed modulation on turning chatter is given a good attention by Insperger (2002). Bifurcation analysis of turning process has been conducted by Stepan et al (2003). Analytical proof has been given by Stepan and Kalmar-Nagy (1997) that turning chatter at loss of stability is subcritical in nature, which is confirmation of an earlier experimental finding by Shi and Tobias as reported by Insperger (2002).

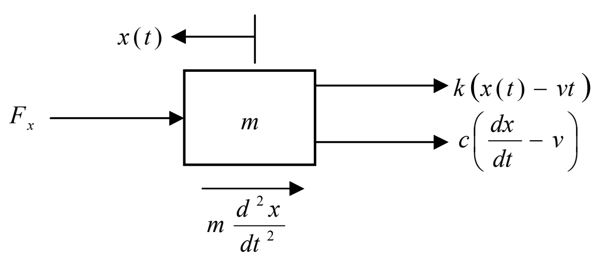
The purpose of this work is to give a detailed stability analysis that leads to the generation of a working chart for turning process with the following parameters; mass $m = 0.0431kg$, Natural frequency $\omega_n = 5700\ rad/sec$, damping factor $\xi = 0.02$, feed speed $\mathrm{v} = 0.0025\mathrm{m/sec}$ and material cutting coefficient $C = 3.5 \times 10^7 \mathrm{Nm}^{\frac{-7}{4}}$. These values are typical for machine turning processes. The stability chart which this work aims to generate is valuable in machine turning processes. With it, it is easy to determine the machining conditions for stability operations. This type of analysis is considered a form of proactive chatter control for machine tools which ultimately results in better products, higher productivity and machine longevity.

2. Equation of vibratory motion of turning tool

Fig.1a represents the turning of an external cylindrical surface. The workpiece at the rotational speed Ω in revolutions per minute of the spindle is clamped in a chuck while the tool is made to transverse it. The mechanical model in fig.1a represents an orthogonal turning process. In orthogonal cutting process, the cutting edge of the tool is perpendicular to the feed motion. The modal parameters of the turning process are; m, mass of tool; c, the equivalent viscous damping coefficient modelling the hysteretic damping of the tool system and k, the stiffness of the tool system. Regenerative waviness on the machined surface is shown magnified for emphasis. In this model a single degree of freedom vibration is assumed in x −direction (feed direction). The tool is fed into the workpiece at a speed v. Response $x(t)$ of the tool system is measured relative to the unloaded equilibrium position of tool (or tool holder axis). The response of the tool $x(t)$ satisfies the equation of motion derived as follows;



$fig. 1a: Mechanical\ model\ of\ orthogonal\ turning$



$fig. 1b: free - body - diagram\ of\ the\ tool$

The free body diagram for the tool system is fig.1b and gives the equation of motion at an arbitrary time of cutting, t as

$$m\ddot{x}(t) + c[\dot{x}(t) - vt] + k[x(t) - vt] + F_x = 0 \quad (1)$$

Where F_x is the x-component of cutting force. F_x could have the empirical form found in the works of Tlusty as reported by Stepan et.al. (2003);

$$F_x = Cwf_a^{\gamma} \quad (2)$$

where C is the cutting coefficient (a property of the workpiece material), w is the depth of cut, f_a is the actual feed rate and γ is an exponent that has typical values $0.8\ and\ {}^{3}/_{4}$ (Stepan and Kalmar-Nagy, 1997). The latter exponent spells the three-quarter rule. Since uniform feed rate f is described as the prescribed movement of the tool cutting edge in $meters\ per\ revolution$ of the workpiece, the actual feed rate f_a could be defined as difference of present and one period delayed position of tool if discrete delay equal to period of revolution is adopted. Thus from fig. 1a it could be seen that

$$f_a = x(t) - x(t - \tau) \quad (3)$$

Putting eq.s (1), (2) and (3) together gives

$$m\ddot{x}(t) + c[\dot{x}(t) - vt] + k[x(t) - vt] + Cw[x(t) - x(t - \tau)]^{\gamma} = 0 \quad (4)$$

To make the derivation more compact the following notations are used; $x(t) = x$ and $(t - \tau) = x_{\tau}$. The notation also applies to any subsequent variable that involves delay. Applying the notation and re-arranging, eq. (4) becomes

$$m\ddot{x} + c\dot{x} + kx = cv + kvt - Cw(x - x_{\tau})^{\gamma} \quad (5)$$

Suppose the motion of the tool is assumed to be a linear superposition of prescribed feed motion vt, static transverse deflection of the tool system $x_t(t)$ and perturbation $z(t)$ (Insperger, 2002). In this work, perturbation is synonymous with regenerative vibrations. Then

$$x(t) = vt + x_t(t) + z(t) \quad (6)$$

Straight-forwardly it goes that $x_t(t) = \frac{-Cwf^{\gamma}}{k}$. Using this in eq. (6) gives eq. (5) as

$$m\ddot{z} + c\dot{z} + kz = Cw(v\tau)^{\gamma} - Cw[v\tau + (z - z_{\tau})]^{\gamma} \quad (7)$$

Linearizing eq. (7) after being put in Taylor's series about $v\tau$ results

$$m\ddot{z} + c\dot{z} + kz = -hw(z - z_{\tau}) \quad (8)$$

Where $h = C\gamma(v\tau)^{\gamma - 1}$ is the specific force variation (Insperger, 2002). Eq. (8) becomes rearranged into the modal form to give

$$\ddot{z} + 2\xi\omega_n\dot{z} + \left(\omega_n^2 + \frac{hw}{m}\right)z = \frac{hw}{m}z_{\tau} \quad (9)$$

Where the natural frequency and damping ratio are respectively given as $\omega_n = \sqrt{\frac{k}{m}}\ and\ \xi = \frac{c}{2\sqrt{mk}}$. Eq. (9) is seen to represent a delayed oscillator. Eq. (9) is the general equation modelling the regenerative vibration of the tool in turning process. The nature of the solution of eq. (9) is a reflection of stability condition of an operating point. If the perturbation motion or its derivative rises with time, there is chatter instability while bounded response with time implies a stable operation.

With the substitutions $y_1 = z\ and\ y_2 = \dot{z}$ made, eq. (9) could be put in state differential equation form

$$\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\left(\omega_n^2 + \dfrac{hw}{m}\right) & -2\xi\omega_n \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{hw}{m} & 0 \end{bmatrix} \begin{Bmatrix} y_{1,\tau} \\ y_{2,\tau} \end{Bmatrix} \quad (10)$$

Where $y_{i,\tau} = y_i(t - \tau)$ for $i = 1\ and\ 2$. The time domain eq. (10) is the substance of stability characterization of turning process. Eq. (10) could be given in terms of dimensionless parameters $\tilde{\Omega}$ and $\tilde{w}$ as

$$\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2(1 + \tilde{w}) & -2\xi\omega_n \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ \tilde{w}\omega_n^2 & 0 \end{bmatrix} \begin{Bmatrix} y_{1,\tau} \\ y_{2,\tau} \end{Bmatrix} \quad (11)$$

Where $\tilde{\Omega} = \frac{\pi\Omega}{30\omega_n}$ and $\tilde{w} = \frac{wh}{m\omega_n^2}$. By solving eq.(11) numerically, time histories for stable and unstable turning together with their determining dimensionless parameters were calculated and presented in fig.2

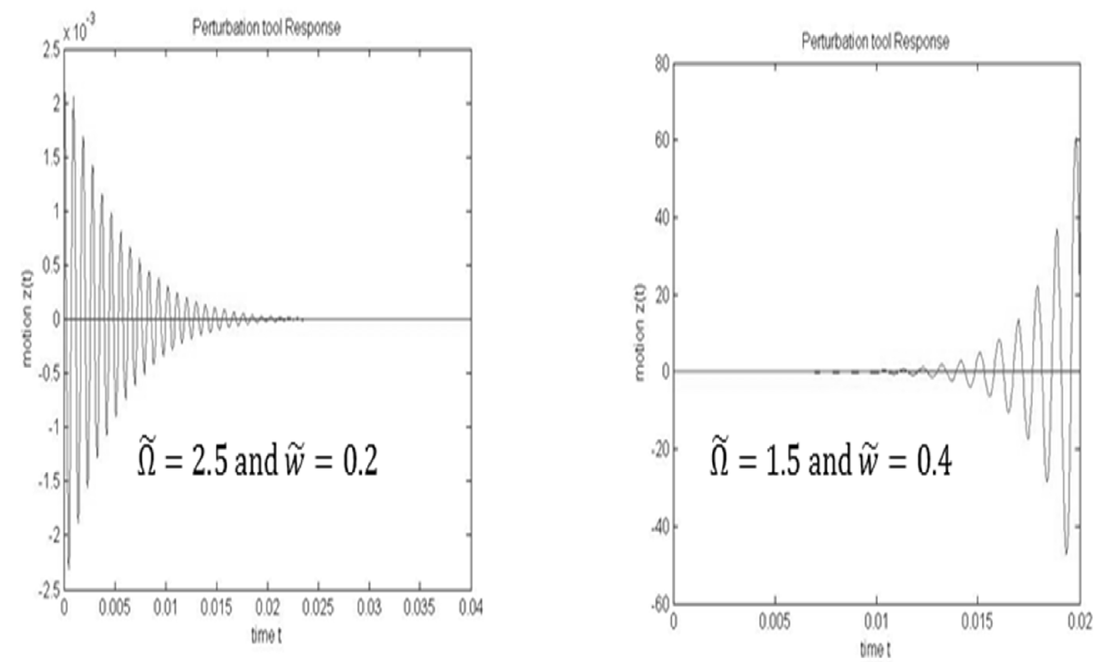


Figure2: Sample turning time histories for stable and unstable operations

3. **Stability analysis of turning**

Eq. (11) has the general form

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\boldsymbol{y}_\tau \quad (12)$$

Where $\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -\left(\omega_n^2 + \frac{hw}{m}\right) & -2\xi\omega_n \end{bmatrix}$ and $= \begin{bmatrix} 0 & 0 \\ \frac{hw}{m} & 0 \end{bmatrix}$. This is an autonomous delay-differential equation with discrete delay. A trial solution of form $\boldsymbol{y}(t) = \boldsymbol{K}e^{\lambda t}$ (Stepan, 1998) put in eq. (12) gives the equation

$$\lambda \boldsymbol{y} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\boldsymbol{y}e^{-\lambda\tau} \quad (13)$$

The characteristic equation of the tool system is seen from eq. (13) to be

$$\left|\lambda \boldsymbol{I} - \boldsymbol{A} - \boldsymbol{B}e^{-\lambda\tau}\right| = 0 \quad (14)$$

Upon simplification equation (14) becomes

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 + \frac{hw}{m}\left(1 - e^{-\lambda\tau}\right) = 0 \quad (15)$$

Expansion of the exponential term of eq. (15) in Maclaurin's series shows that the characteristic equation has infinitely many solutions or *eigen-values* also called *characteristic roots,* with each having the form $\lambda = \sigma + i\omega$. Eigen-values of the system migrate on the complex plane as the cutting parameters are varied. All the roots must have negative real parts for the turning process to be stable; thus operation is critical whenever there exist roots on the imaginary axis. Bifurcation in turning operation could occur when a pair of complex conjugate characteristic roots crosses from the left-half plane to right-half plane of the complex plane. This occurrence is called the Hopf bifurcation of a corresponding non-linear system (Insperger, 2002). The trivial solution to eq. (12) is $z(t) = Ke^{\lambda t}$ *where* $K, \lambda \in C$ (Stepan, 1998). For any pair of complex conjugate roots $\lambda_{1,2}$ there exists a solution $z(t) = K_1e^{\lambda_1 t} + K_2e^{\lambda_2 t}$ being that eq. (12) is linear. Since under bifurcation condition the critical characteristic roots are pure imaginary, this solution becomes

$$z(t) = K_1e^{i\omega t} + K_2e^{-i\omega t} = C\cos(\omega t - \varphi) \quad (16)$$

where $C = \sqrt[2]{(K_1 + K_2)^2 - (K_1 - K_2)^2}$ and $\varphi = \tan^{-1}\left[\frac{i(K_1 - K_2)}{K_1 + K_2}\right]$. Thus ω is seen to be the frequency of the arising chatter vibrations. This is the bifurcation of Hopf type which has been proven experimentally by Shi and Tobias as reported by Insperger (2002) and analytically by Stepan and Kalmar-Nagy (1997) to have subcritical nature.

The stability boundary curves also called the D-curves or Stability lobes are drawn to separate the stable cutting domain (at which all $\sigma < 0$) from the unstable one (at which some $\sigma > 0$). On the D-curves are roots of the characteristic equation that are most critical; pure imaginary roots or zero. The D-curves could be tracked based on eq. (15) by making the substitution $\lambda = i\omega$. The two equations resulting are

$$-\omega^2 + \omega_n^2 + \frac{hw}{m}(1 - cos\omega\tau) = 0 \qquad (17)$$

$$2\xi\omega_n\omega + \frac{hw}{m} sin\omega\tau = 0 \qquad (18)$$

By method of D-subdivision (Insperger, 2002; Stepan, 1998) eqs (17) and (18) could be solved to give expressions for critical combinations of cutting parameters of depth of cut w and spindle speed Ω as follows;

From eq. (18) results

$$\frac{hw}{m} = \frac{-2\xi\omega_n\omega}{sin\omega\tau}\ , \omega\tau \neq k\pi\ for\ k = 0,1,2 \qquad (19)$$

For positive depth of cut equation (19) gives the condition $sin\omega\tau < 0$. Eq. (19) put into (17) gives

$$-\omega^2 + \omega_n^2 - 2\xi\omega_n\omega\frac{(1 - cos\omega\tau)}{sin\omega\tau} = 0 \qquad (20)$$

Using the trigonometric relationship $\frac{(1-cos\omega\tau)}{sin\omega\tau} = tan\frac{\omega\tau}{2}$, equation (20) becomes

$$-\omega^2 + \omega_n^2 - 2\xi\omega_n\omega tan\frac{\omega\tau}{2} = 0 \qquad (21)$$

Since $(1 - cos\omega\tau) > 0$ then

$$tan\frac{\omega\tau}{2} = \frac{-\omega^2 + \omega_n^2}{2\xi\omega_n\omega} < 0 \qquad (22)$$

It is implied from (19) and (22) that $\frac{\omega\tau}{2}$ lies in any of the intervals

$$\frac{\pi}{2}(2n + 1) < \frac{\omega\tau}{2} < \pi(n + 1)$$
$$\frac{\pi}{2}(2n + 3) < \frac{\omega\tau}{2} < \pi(n + 2) \qquad (23)$$

Where $n = 0,1,2,3,4, \dots\dots\dots\dots$ From eq.(24), sign inversion gives

$$-tan\frac{\omega\tau}{2} = \frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega} > 0 \qquad (24)$$

It could be seen from eq. (24) that it holds under the constraint

$$0 < tan^{-1}\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right) < \frac{\pi}{2}$$
$$\pi < tan^{-1}\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right) < \frac{3\pi}{2} \qquad (25)$$

Then eq. (24) gives

$$\frac{\omega\tau}{2} = j\pi - tan^{-1}\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right), j = 1,2,3, \dots \qquad (26)$$

Therefore

$$\tau = \frac{2}{\omega}\left\{j\pi - tan^{-1}\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right)\right\} \qquad (27)$$

Though positive depth of cut is assumed in arriving at eq. (27), the same result is achieved for negative depth of cut following a similar argument.

Eq. (26) can be written as follows;

$$sin\omega\tau = sin2[j\pi - tan^{-1}\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right)]$$

The above equation can be simplified to

$$sin\omega\tau = \frac{-2\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right)}{\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right)^2 + 1}$$

It then follows from result of eq.(19) that

$$\frac{hw}{m} = \xi\omega_n\omega\left[\frac{\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right)^2 + 1}{\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right)}\right]$$

This becomes re-arranged to give the expression for boundary depth of cut as

$$w = \frac{m}{2h}\left\{\frac{(\omega^2 - \omega_n^2)^2 + 4\xi^2\omega_n^2\omega^2}{\omega^2 - \omega_n^2}\right\} \qquad (28)$$

Stability charts are most often given in terms of cutting parameters like spindle speed Ω and depth of cut w to give the range of technological parameters for non-chatter cutting. The pair of equations for stability analysis of turning thus becomes;

$$\Omega = \frac{60}{\tau} = \frac{30\omega}{j\pi - tan^{-1}\left(\frac{\omega^2 - \omega_n^2}{2\xi\omega_n\omega}\right)} \qquad (29)$$

$$w = \frac{m}{2h}\left\{\frac{(\omega^2 - \omega_n^2)^2 + 4\xi^2\omega_n^2\omega^2}{\omega^2 - \omega_n^2}\right\} \qquad (30)$$

Put in dimensionless form eqs. (29) and (30) become

$$\tilde{\Omega} = \frac{\pi\Omega}{30\omega_n} = \frac{\pi\omega}{\omega_n\left\{j\pi - tan^{-1}\left(\frac{\omega^2-\omega_n^2}{2\xi\omega_n\omega}\right)\right\}} \qquad (31)$$

$$\tilde{w} = \frac{wh}{m\omega_n^2} = \frac{(\omega^2-\omega_n^2)^2 + 4\xi^2\omega_n^2\omega^2}{2\omega_n^2(\omega^2-\omega_n^2)} \qquad (32)$$

Where the dimensionless speed $\tilde{\Omega}$ is seen to be the boundary frequency ratio of the tool since ω is the circular frequency of arising loss of stability vibrations.

4. Results and Discussions

The typical machine turning parameters had earlier been given as: $m = 0.0431kg; \omega_n = \frac{5700rad}{sec}$ $and\ \xi = 0.02;\ C = 3.5\ \times 10^7\mathrm{Nm}^{\frac{-7}{4}}$ and $\mathrm{v} = 0.0025\mathrm{m/sec}$ (Ozoegwu, 2011). To generate the stability chart for the system assumed to have the parameters stated above, use was made of eqs (31) and (32), and the results are shown in fig.3.

ẅ = Dimensionless Depth of Cut

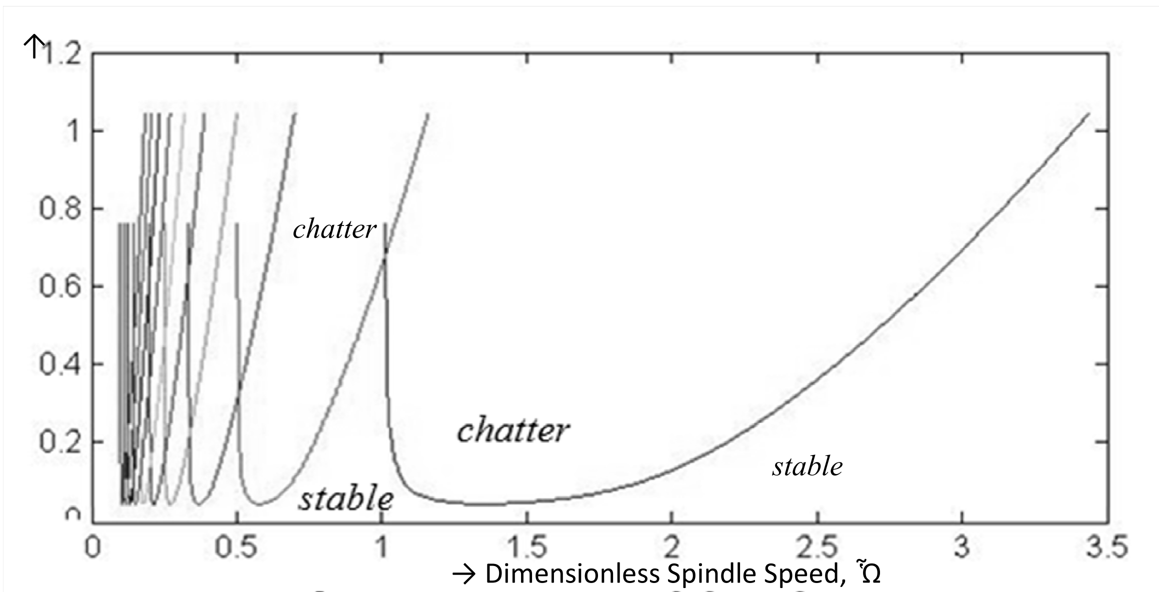


figure3: Turning stability chart

Making use of the extended Routh-Hurwitz stability criteria for infinite dimensional system, one arrives at the result that, points above each curve will produce chatter while those below each curve will result in stable cutting. The stability chart of fig. 3 makes the work of an operator on a turning machine systematic since regions of stable cutting are made clear, and choices can now be made. Recalling that $\tilde{\Omega} = \frac{\Omega}{60f_n} = \frac{\Omega\pi}{30\omega_n}$, $\tilde{w} = \frac{wh}{m\omega_n^2}$, $h = C\gamma(v\tau)^{\gamma-1}$ and $\tau = \frac{2\pi}{\omega_n\tilde{\Omega}}$, any point on the plane of dimensionless parameters $(\tilde{\Omega}, \tilde{w})$ becomes given as a point $\left(\frac{30\omega_n\tilde{\Omega}}{\pi}, \frac{m\omega_n^2\tilde{w}}{C\gamma\left(v\frac{2\pi}{\omega_n\tilde{\Omega}}\right)^{\gamma-1}}\right)$ on the Ω - w plane. For example the choice of a point (0.9, 0.3) on the $\tilde{\Omega} - \tilde{w}$ plane which is equivalent to the point (48988, 0.00067) on the Ω - w plane is an asymptotically stable turning operation for the system while the choice of operation (0.35, 0.4) on the $\tilde{\Omega} - \tilde{w}$ plane which is equivalent to the point (19051, 0.0011) on the Ω - w plane is an unstable one. This is confirmed by the time histories of this system generated by solving eq. (10) at these points as shown on fig. 4.

The forgoing means that a choice of spindle speed of 48988 rpm and depth of cut of 0.67mm represents a good cutting condition while that of 19051 rpm and depth of cut of 1.1 mm is a bad one for the machine. It is instructive to note that relative to an unstable operation at spindle speed of 32659rpm and depth of cut 0.00019757mm, there is a much more productive and economical (in terms of spindle power requirement) operation for spindle speed of 24494rpm and depth of cut 0.39806mm which are stable turning operations. It must be pointed out that machine operation at such high speeds could become interrupted with loss of contact effects, limiting the practical application of stability chart of fig.3 at high speed domain of the chart. It can also be seen from the stability chart that the machine has very low productivity at low spindle speed requiring very small dept of cut for stable operation. For example at a spindle speed of 2000rpm the maximum depth of cut for stable operation is about 0.25mm. This result points to the need of chatter stability enhancement for low speed range turning machine through any means of chatter suppression.

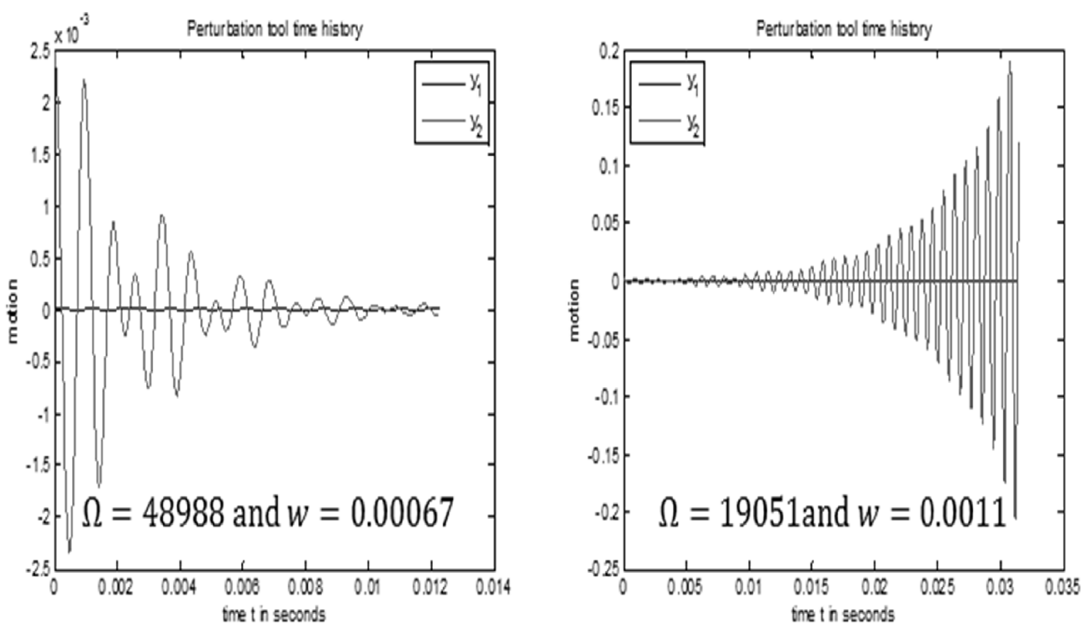


Figure4: Time histories of turning tool confirming the stability chart

5. Conclusion

The first conclusion drawn in this work is that the extent to which achieving high productivity and high surface quality are considered opposed demands depends on the turning machine's stability information at the disposal of the machinist. This conclusion is drawn for the relatively high speed region of the stability chart under the assumption that full-immersion condition persists. The second conclusion drawn is that the turning machine has very low productivity at low spindle speeds thus requiring chatter stability enhancement through any means of chatter suppression. By the use the stability chart, a machinist is able to choose machine turning parameters that will ensure stable operation.

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