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Application of Scheffe's model in optimization of compressive strength of lateritic concrete

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Abstract
In this research work, the use of Scheffe's simplex theory for the optimization of the compressive strength of lateritic concrete was investigated. The objective of the study is to develop a model that can predict the mix ratio when the desired compressive strength is known or vice-versa. A total of sixty (60) concrete cubes were cast. For each of the twenty mix ratios, three cubes were cast and the average determined. The first thirty cubes were used to determine the coefficients of the model while the other thirty cubes were used to validate the model (control test). The optimum compressive strength of concrete at 28 days curing was found to be 25.04N/mm^2 and the corresponding mix ratio was 0.6:1:1.75:1.75 (water, cement, laterite, granite). VISUAL BASIC program was used to ease the use of the model and computer model accuracy was tested using Fisher F-test. The model was found to be adequate for prescribing concrete mix ratios, when the desired compressive strength is known and vice-versa. The highest compressive strength predicted by the model is 26.74N/mm^2 when the mix ratio is 0.572:1:1:52:1.54 (water-cement ratio, cement, laterite, granite).

Keywords: Compressive strength, scheffe's model, lateritic concrete, curing.

1. Introduction

Concrete mix design could be carried out using either the empirical or statistical experimental method (Simon etal, 1997). For instance, optimization of mix proportions of mineral aggregates for use in polymer concrete was attempted using statistical techniques (Mohan etal, 2002). There have been some advances in statistical experimental design for performing tests on concrete but these do not explicitly take into consideration the chemistry involved (Simon, 2003). The supplementary cementitious materials optimization system has been developed (Malhorta, etal, 2002). The method is a decision making system that enables the reduction of portland cement in concrete by determining the optimum level of replacement by supplementary cementitious materials. New mix designs for fresh and hardened concrete were developed in order to create constructions materials with high performance (Bloom, Bentur 1995). Some of the statistical experimental methods include simplex design (Sheffe 1958, 1963) and (Obam 1998), axial design, mixture experiments involving process variables, mixture models with inverse terms (Draper, St John 1997) and K-model (Draper, Pukelsheim 1997). Empirical methods are prone to trial and error which results in material wastage whenever they are used (Ezeh, Ibearugbulem 2009). Sequel to this, statistical experimental method could be adopted using simplex design. The materials used in such experiments include water, cement, laterite and granite. There is the need to formulate mathematical models that will prescribe concrete mix ratios, when the desired compressive strength is known and vice-versa. Similarly, the need to determine the combination of the materials that would give the highest compressive strength should be met.

In this paper, the Scheffe's mathematical model was adopted in the optimization of compressive strength of lateritic concrete.

2. Literature Review

Concrete is a mixture of several component such as cement, fine aggregate, coarse aggregate and water. According to (Onyenuga 2001), concrete is known to be a composite inert material comprising of binder course (cement) and mineral filler (body) or aggregate and water. Admixture could be added but for given set o of materials the proportion of the components influences the properties of the concrete mixture, hence, the need to optimize concrete properties such as strength. Mathematical modeling is the process of creating a mathematical representation of some phenomenon in order to gain a better understanding of that phenomenon (Osunade 1994). (Lasis, Ogunjimi 1984) described a model as an abstract that uses mathematical language to control the behaviour of a giving system. According to (Osadebe 2003), modeling is mathematical equation of dependent variable (Response) and independent variable (Predictor). (Manasce etal, 1994) from their studies refers to it as a representation of a system. (Simon etal, 1997) stated that the area of application of mathematical modeling includes engineering and natural sciences.

(Simon etal, 1997) in their studies on high performance concrete, which contains many constituents and which are often subjected to several performance constrains can be a difficult and time consuming task. (Simon etal, 1997. Ezeh, Ibearugbulem 2009. Osadebe 2003) in their different work demonstrated the application of mathematical modeling in Civil Engineering.

3. Materials and Methods

Simplex design formulation: The relation between the actual components and pseudo components is according to (Sheffe 1958)

$Z = AX$--------------------------(1)

Z and X are four element vectors, where A is a four by four matrix. The value of matrix A will be obtained from the first four mix ratios. The mix ratios are Z_1 [0.5:1:1:1], Z_2 [0.55:1:1.5:2], Z_3[0.65:1:2:1.5], Z_4[0.6:1:1.5:1.5].

The corresponding pseudo mix ratios are X_1(1:0:0:0], X_2[0:1:0:0], Z_3[0:0:1:0], Z_4[0:0:0:1]. Substitution of X_i and Z_i into equation 1 gives the values of A as

$$A = \begin{pmatrix} 0.5 & 0.55 & 0.65 & 0.6 \\ 1 & 1 & 1 & 1 \\ 1 & 1.5 & 2 & 1.5 \\ 1 & 2 & 1.5 & 1.5 \end{pmatrix} \quad (2)$$

The first four mix ratios are located at the vertices of the tetrahedron simplex. Six other pseudo mix ratios located at mid points of the lines joining the vertices of the simplex are

X_{12} [$^1/_2$: ½:0:0], X_{13} [$^1/_2$:0: $^1/_2$:0], X_{14} [$^1/_2$:0:0: $^1/_2$], X_{23}[0: $^1/_2$: $^1/_2$:0] X_{24}[0: $^1/_2$:0: $^1/_2$],X_{34}[0:0: $^1/_2$: $^1/_2$].

Substituting these values into equation (1) will give the corresponding actual mix ratios, Z as

Z_{12}[0.525:1:1.25:1.5] Z_{13}[0.575:1:1.5:1.25]
Z_{14}[0.55:1:1.25:1.25] Z_{23}[0.6:1:1.75:1.75]
Z_{24}[0.575:1:1.5:1.75], Z_{34}[0.625:1:1.75:1.5]

No pseudo component according to (Sheffe 1958) should be more than one or less than zero. The summation of all the pseudo components in a mix ratio must be equal to one (Sheffe 1958, Obam 1998).

That is

$0 \le X_i \le 1$-------------(3)

$\sum X_i = 1$---------------(4)

The general equation for regression is given as

$Y = b_o + \sum b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ij} k x_i x_j$ XK+--------+ ∑bi1, i2---- inxi, xi2---, xin+e-----------(5)

Where $1 \le i \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le k \le q$ and 1≤i1≤i1≤-----≤in≤q respectively (Simon etal,1997).

Expanding equation (5) up to second order polynomial for four component mixture, we obtain:

$$Y = b_o + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_{11}x_1^2 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{22}x_2^2 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{33}x_3^2 + b_{34}x_3x_4 + b_{44}x^2_4 + e \text{-(6)}$$

Multiplying equation 4 by b_o, we obtain

$b_o = x_1b_o + x_2b_o + x_3b_o + x_4b_o$----------------------(7)

multiplying equation (4) again by x_i and re-arranging we obtain

$x_1^2 = x_i - x_1x_1 - x_2\,x_i$------------------------------(8)

substituting equation (7) and (8) into equation (6) and collecting like terms together, we obtain

$$Y = \theta_1x_1 + \theta_2x_2 + \theta_3x_3 + \theta_4x_4 + \theta_{12}x_1x_2 + \theta_{13}x_1x_3 + \theta_{14}x_1x_4 + \theta_{23}x_2x_3 + \theta_{24}x_2x_4 + \theta_{34}x_3x_4 + e \text{------------------------------(.9)}$$

Where $\theta_i = b_o + b_i + b_{ii}$ and $\theta_{ij} = b_{ij} - b_i - b_{ij}$ without loss of generality, e is the estimated error and could be dropped from equation (9).

Hence

$$Y = \theta_1x_1 + \theta_2x_2 + \theta_3x_3 + \theta_4x_4 + \theta_{12}x_1x_2 + \theta_{13}x_1x_3 + \theta_{14}x_1x_4\ \theta_{23}x_2x_3 + \theta_{24}x_2x_4 + \theta_{23}x_3x_4 \text{-- (10)}$$

Let n_i be the experimental compressive cube strength of any of the first four mix ratios, and n_{ij} be the experimental compressive strength of the remaining six mix ratios that were used in this model formulation. Substituting for n_i and the corresponding pseudo mix ratio into equation (10) gives

$n_i = \theta_i$ ---------------(11)

In the same way substituting n_{ij} and the correspond pseudo mix ratio into equation (10) gives $n_{ij} = 0.50_i + 0.05\theta_j + 0.25\theta_{ij}$ -------------------(12) Rearranging equations (11) and (12), we obtain $\theta_i = n_i$ --------(13), $\theta_{ij} = 4n_{ij} - 2n_i - 2n_j$-------------(14)

Substituting equation (13) into equation (14) gives

$\theta_{ij} = 4n_{ij} - 2n_i - 2n_j$-------------(15)

Substituting equation (13) and (15) into equation (10) and collecting like terms will give $F(x) = n_1x_1(1 - 2x_2 - 2x_3 - 2x_4) + n_2x_2(1-2x_1 - 2x_3-2x_4) + n_3x_3(1-2x_1 - 2x_2 - 2x_4) + n_4x_4(1-2x_1 - 2x_2 - 2x_3) + 4n_{12}x_1x_2 + 4n_{13}x_1x_3 + 4n_{14}x_1x_4 + 4n_{23}x_2x_3 + 4n_{24}x_2x_4 + 4n_{34}x_3x_4$------------- (16)

Now, multiplying equation (4) by 2 and subtracting 1 from both sides, we obtain

$2x_1 + 2x_2 + 2x_3 + 2x_4 - 1 = 1$---------------------(17)

Rearranging equation (17), we obtain

$2x_2 - 1 = 1 - 2x_1 - 2x_2 - 2x_4$------------------------[18]

Similarly

$2x_2 - 1 = 1 - 2x_1 - 2x_3 - 2x_4$----------------------(19)

$2x_3 - 1 = 1 - 2x_1 - 2x_2 - 2x_4$------------------ (20)

$2x_4 - 1 = 1 - 2x_1 - 2x_2 - 2x_3$------------------- (21)

Substituting equation (18), (19), (20) and (21) into equation (16), we obtain

$F(x) = n_1x_1(2x_1-1) + n_2x_2(2x_2-1) + n_3x_3(2x_3-1) + n_4x_4(2x_4-1) + 4n_{12}x_1x_2 + 4n_{13}x_1x_3 + n_{14}x_1x_4 + 4n_{23}x_2x_3 + 4n_{24}x_2x_4 + 4n_{34}x_3x_4$-------------- (22)

Equation (22) is the mathematical model equation.

To validate the model, extra ten mix ratios (control) were determined and used in the ANOVA test. The aim of the test was to ascertain whether the different between the results of compressive strength from experiment and model was significant or not. If the different between the two results is significant, alternative hypothesis will be adopted. If the different between the two results is not significant, null hypothesis will be adopted. The mix ratios are shown in Table 1:

Table 1: Pseudo and actual mix ratios for the control test

Points	**Ratio of materials**							
	Water	Cement	Laterite	Granite	Water	Cement	Laterite	Granite
	X_1	X_2	X_3	X_4	Z_1	Z_2	Z_3	Z_4
C_1	0	0.5	0	0.5	0.575	1	1.5	1.75
C_2	0.15	0.15	0.35	0.35	0.595	1	1.6	1.5
C_3	0.4	0.1	0.2	0.3	0.565	1	1.4	1.35
C_4	0.1	0.1	0.4	0.4	0.34	1	1.65	1.5
C_5	0.25	0.3	0.25	0.2	0.5725	1	1.5	1.525
C_6	0.16	0.34	0.22	0.28	0.578	1	1.53	1.59
C_7	0.17	0.31	0.25	0.27	0.58	1	1.54	1.57
C_8	0.32	0.18	0.33	0.17	0.5755	1	1.505	1.43
C_9	0.26	0.34	0.30	0.10	0.572	1	1.52	1.54
C_{10}	0.5	0.25	0.25	0	0.55	1	1.375	1.375

Compressive strength test: The materials used for this test includes:

- Granite, which is free from deleterious substance with a maximum size of 20mm.
- The cement used is dangote cement which is a brand of Ordinary Portland Cement and conform to (BS 12, 1978).
- The laterite used was obtained from Nekede in Owerri North L.G.A of Imo State; Nigeria.
- The water used is clean water from bore-hole.

The materials were batched by weight. Mixing was done manually using spade and hand trowel. 150mm x 150mm x 150mm concrete moulds were used for casting the concrete cubes. The concrete cubes were cured in water for 28 days. At the end of the hydration period the cubes were crushed and their compressive strength were determined according to the requirement of (BS 1881,1986).

4. Results and Discussions

The compressive strength test results are shown in table 2. Higher compressive strength was recorded at point 23 (point of observation of the mixture in the factor space) (25.04N/mm^2). This is the highest strength recorded when the compressive strength of concrete ingredients located at the various points in the factor space where considered. Although the mix ratio of 0.6:1:1.75:1.75 had low cement content, the high water/cement ratio of 0.6 seemed to be the major reason for this. Point 1 should have given higher compressive strength going by its high cement content but it had low water cement ratio of 0.5 it could be observed that water/cement ratio and cement content were not the only factors responsible for the behaviour of the compressive strength. This is so because the strength at point 23 is higher than that at point 1 irrespective of the fact that point 1 has lower water/cement ratio and higher cement content than point 23. According to Ezeh, Ibearugbulem (2009), the highest compressive strength predicted by the application of Sheffe's model in optimization of compressive strength of River Stone aggregate concrete is 37.62N/mm^2 when the mix ratio is 0.5:1:2.4:3.6 (water-cement ratio, cement, river sand, river stone). Also the result obtained when Osadebe's model (2003) was used to predict the compressive strength of concrete containing water-cement ratio, cement, fine aggregate, coarse aggregate in a mix ratio of 0.6:1:0.7:2.5 is 25.39N/mm^2. These results shows that both Sheffe and Osadebe's model can be used in the optimization of compressive strength of concrete containing four or five mixture ingredients. According to Umuonyiagu and Onyeyili (2011), the highest compressive strength predicted by the application of Scheffe's mathematical model for the prediction of the compressive strength characteristics of concrete made with unwashed local gravel is 21.328N/mm^2 when the mix ratio is 0.575:1:1:375:2 (water cement ratio, cement, fine aggregate, coarse aggregate). Also, according to the same authors (2011), the highest compressive strength predicted by the application of Scheffe's mathematical model for the predictions of the compressive strength characteristics of concrete made with granite chipping is 31.025N/mm^2 when the mix ratio is 0.575:1:1:375:2 (water cement ratio, cement, fine aggregate, coarse aggregate).

The average compressive strength of the two research works of Umuonyiagu and Onyeyili is 26.18N/mm^2 when the mix ratio is 0.575:1:1:375:2 (water cement ratio, cement, fine aggregate, coarse aggregate). But the result gotten in this research work where laterite is used instead of fine aggregate is 26.74N/mm^2 when the mix ratio is 0.572:1:1.52:1.54. (water cement ratio, cement, laterite, granite). These results shows that there is no significant difference in the compressive strength predicted by the Scheffe's model despite the fact that there is a little variation in the mix ratios used.

Table 2: Compressive strength results

Points	Replicate 1 (N/mm^2)	Replicate 2 (N/mm^2)	Replicate 3 (N/mm^2)	Average compressive strength (N/mm^2)
1	19.69	23.40	21.36	21.48
2	13.58	11.82	15.50	13.63
3	17.10	14.00	13.78	14.96
4	17.00	19.30	17.91	18.07
12	18.90	20.90	19.76	19.85
13	25.40	17.15	21.00	21.18
14	12.60	20.20	16.09	16.30
23	24.76	22.05	28.31	25.04
24	20.45	15.78	26.00	20.74
34	17.72	24.15	19.90	20.59
C_1	17.52	26.10	20.81	21.48
C_2	29.10	22.85	28.92	26.96
C_3	21.90	28.10	23.33	24.44
C_4	5.60	3.70	4.03	4.44
C_5	20.45	29.10	26.90	25.48
C_6	25.60	18.21	25.98	23.26
C_7	26.41	24.00	19.81	23.41
C_8	20.75	25.15	27.00	24.30
C_9	19.85	27.74	23.96	23.85
C_{10}	23.90	19.08	25.90	22.96

The ANOVA test of the generated data are shown in table 3.
The aim of the test was to ascertain whether the different between the results of compressive strength from experiment and model was significant or not.

$$S_E^2 = \frac{\sum(YT - YAT)^2}{N-1} = \frac{157.5374}{9} = 17.504$$

$$S_T^2 = \frac{\sum(YE - YAE)^2}{N-1} = \frac{364.4661}{9} = 40.496$$

$$Hence, S_T^2 = S_1^2 = 40.496$$

$$S_E^2 = S_2^2 = 17.504$$

$$\therefore \frac{S_1^2}{S_2^2} = \frac{40.496}{17.504} = 2.31$$

$$F - value\ from\ table\ is\ F_{\propto}(V_1, V_2)$$

$$= F_{0.05} = (9,9) = 3.18$$

$$\frac{1}{F} = \frac{1}{3.18} = 0.3145$$

$$Consequently, \frac{1}{F} \le \frac{S_1^2}{S_2^2} \le F$$

$$\therefore 0.3145 \le 2.31 \le 3.18$$

Therefore, the difference between the experiment result and the model result was not significant.

Hence, the model is adequate for use in predicting the probable compressive strength when the mix ratio is known and vice-versa.

Table 3: ANOVA test

Points	YE	YT	YE-YAE	YT-YAT	$(YE-YAE)^2$	$(YT-YAT)^2$
C_1	21.48	21.48	-0.578	1.696	0.3341	2.6764
C_2	26.96	13.63	4.902	-6.154	24.030	37.8717
C_3	24.44	14.96	2.382	-4.824	5.6739	23.2718
C_4	4.44	18.07	-17.618	-1.714	310.394	2.9378
C_5	25.48	19.85	3.422	0.066	11.710	0.00436
C_6	23.26	21.18	1.202	1.406	1.4448	1.9768
C_7	23.41	16.30	1.352	-3.484	1.8279	12.1383
C_8	24.30	25.04	2.242	5.256	5.0265	27.6255
C_9	23.85	26.74	1.792	6.956	3.2113	48.3859
C_{10}	22.96	20.59	0.902	0.806	0.8136	0.6496
Total	**220.58**	**197.84**			**364.4661**	**157.5374**
Average	**22.058**	**19.784**				

Legend: YE = experimental strength; YAE = average of the experimental strength.
YT = the model strength; YAT = average of the model strength
N = number of points of observation
V = degree of freedom, α = significant level.

4.0 Conclusion

It was concluded that the highest compressive strength predicted by this model is 26.74N/mm^2. The corresponding mix ratio is 0.572:1:1.52:1.54 [water cement ratio, cement, laterite, granite]. Again, the scheffe's model formulated was adequate and reliable at 5% risk for predicting the compressive strength of concrete made with the above mentioned materials. A model is said to be adequate for use if it is 95% correct and 5% incorrect. The 5% incorrect is the risk involved in predicting the adequacy of a model for use.

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