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APPLICATION OF MARKOV CHAIN IN MUNICIPAL SOLID WASTE MANAGEMENT

A Case of Awka Municipal Solid Waste Management

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Abstract
The study aimed at applying Markov model in solid waste management. A Markovian solid waste management model formulated as a transition matrix *M* was used in predicting the probable future monthly percentage waste contributions from the various Zones of Awka, the Capital City of Anambra State. *M* contains the probability vectors used in the predictions for the year 2014 as the first year, and for the subsequent years. The prediction gave a long time percentage monthly solid waste contributions to total waste stream from each of the twelve zones of Awka city in future years as: Amawbia = 8.3%, Zik's Avenue = 13.9%, Amaikwo = 8.8%, Amaenyi/Amaku = 8.6%, Udoka Estate = 8.8%, Nibo/Umuawulu = 7.2%, Iyiagu Estate = 6.4%, Okpuno = 8.2%, Enugu/Onitsha Expressway = 6.3%, Emma Nnaemeka Axis = 5.2%, Ifite = 9.2% and Government House = 9.1%. By this prediction, the Anambra State Waste Management Authority may be guided in its plans and decisions on how to schedule its disposal trucks to the Zones and if variable charges are to apply in the area, which zone pays what? The analysis also showed that it does not matter which month of a given year the study made on solid waste management started, provided the same length of period (twelve months, say) is maintained. Besides, it is hopeful that the model application to solid waste management will contribute to knowledge enhancement in the academic world.

Keywords: Analysis, Markov chain, prediction, solid waste, transition matrix

NOMENCLATURE

"=" means "is equal to" or "represents"
λ_i = rate of waste deposit in a roadside dumpsite(s) on the i^{th} day
λ_j = rate of waste deposit in roadside dumpsite j
λ_{ij} = rate of waste deposit in roadside dumpsite j on the i^{th} day
Q = total quantity (bin load-counts) of waste produced (deposited) in an area in a given

period
MntCod = Month code
P = probability vector; P_{ij} = state distribution (data elements) of solid waste production in a given area
M = transition matrix; n = number of repetitions of an experiment or the n^{th} term of a sequence
Zone 1, Zone 2, Zone 3, …, Zone 12 = 1, 2, 3, …, 12 respectively, Year = 0; and Jan = 1', Feb = 2', Mar = 3', …, Dec =12' (see Figure 3)
X_0 = initial probability vector; I_{ij} = final distribution of state
ASWAMA = Anambra State Waste Management Authority;
ANSEPA = Anambra State Environmental Protection Agency
SWM = Solid waste management;
NPC = National Population Commission
CBO = Community based organization
NGO=Non-governmental organization
ICCE = Ihueze Chukwutoo-Chukwumuanya Emmanuel

1.1 Introduction

In recognition of the importance of reliable tools to enable prediction of solid waste generation, characteristics and management costs, various re-searchers have attempted to construct models to predict these parameters. They found that due to variations in consumer behavior and lifestyle the relationships obtained between various parameters vary by country (Rafia et al, 2008). Besides, many waste manage-ment authorities do not effectively manage the quantities of solid waste generated daily in their areas due to non availability of enough fund to prosecute the project, and lack of monitoring/ assessment tools for evaluating their performances and financial transactions made over time. This loss also prevents them from making functional, reliable and sustainable management plans for effective management of the waste generated in their domains. A part in solving this shortcoming is establishing a systematic method of collecting, analyzing and keeping both qualitative and quantitative data on solid waste generation in the locality.

In the study, a stochastic process approach known as Markov chain is used in developing a forecasting model for use in predicting future production of solid waste in Awka Urban City of Anambra State in the next few years. Hereunder, the researchers use the memoryless property of the Markov's model in analyzing the quantitative data on waste obtained in the course of this work. Applications of Markov chain in areas like medicine, sociology, marketing, weather control and fore-casting, gambling, games and distri-bution etc. have become common. In fact, according to Addison-Wesley (2003), it is now a standard tool for medical (and meteorology) decision making.

2.0 Material and methods

Three groups of research agents collected a thirty six month data on solid waste generation and evacuation in Awka Metropolitan city of Anambra State. The data were collected in parts from ASWAMA, ANSEPA, internet, field survey and literature. First group of the agents were those who either had face to face interactions with the staff and management of different organizations or obtained the necessary information from them through a questionnaire, or both; the next group were the Roadside Dumpsite agents who re-corded the rates of filling and evacuation of the public waste bins/containers kept along the streets/lanes of Awka. These agents established phone and physical contacts with some of the persons living around the place where the waste containers were kept; and the third group were the Final Dumpsite Agents who kept daily data on the quantities of waste brought by disposal trucks to the final dump site at Agu-Awka. However, solid waste that were not moved from their points of generation to the roadside or final dumpsites were not considered in the study. The secondary data were obtained from textbooks, journal articles, newsletters, official reports, and the internet. Official reports here-in referred to include reports from private and government recognized institutions like ASWAMA, the local government area authorities, the ministry of information and the NPC in the state, government recognized private contr-actors, NGOs, CBOs, the internet, et cetera.

Due to non-availability of platform scales and such other tools/equipment for more accurate weight-volume or materials mass balance measure of the waste, load-count analysis method was used in the study, with unit of measure in number of waste bin loads per run (Ihueze and Chukwumuanya, 2015). Excel spread sheet and MATLab assisted greatly in the analysis.

2.1 Modeling of Solid Waste Management Processes for Markov Model Application

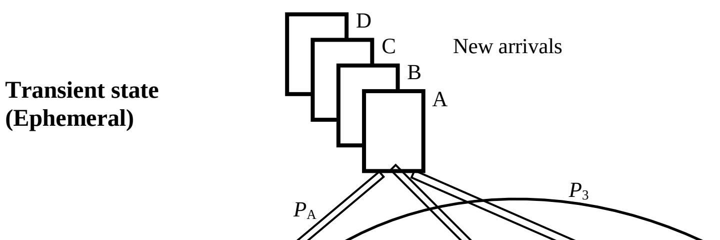
Suppose there are three waste containers kept at different points (see Figure 1) in a given location and waste are randomly dumped into these containers. Arrival of these waste changes the states of the containers in terms of waste mass/volume of accumulation. This means that each of these containers can assume any state from being empty to being filled or overflowing with waste. Figure 1 depicts the various states of the waste disposal system with the probability vectors defined. To see how these proportions would change after each day generation, we use the tree diagram of Figure 2. For example, to find the initial proportions of waste at points 1, 2 and 3 (states 1 - 3) as at the commencement of the experiment (before the three day generation), we add the probabilities:

$$P_{11} + P_{12} + P_{13} = I_{10} \quad (1a)$$

$$P_{21} + P_{22} + P_{23} = I_{20} \quad (1b)$$

$$P_{31} + P_{32} + P_{33} = I_{30} \quad (1c)$$

It is assumed here that the final distribution of states I_{10}, I_{20} and I_{30} (expressed in percentages), came after one generation which emanated from the initial waste dump proportions I_{10} in state 1, I_{20} in state 2, and I_{30}, in state 3. The distribution can also be written as probability vectors with the percentages changed to decimals rounded to the nearest hundredth.



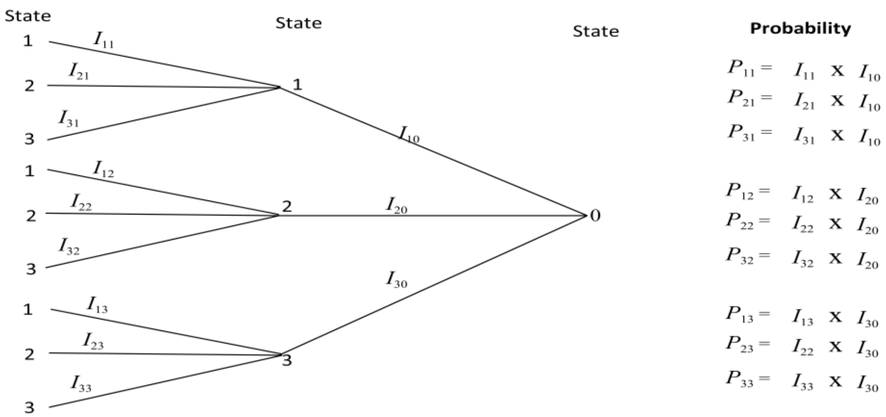


Figure 2: A tree diagram of the state distribution

2.2 Modeling of Awka Municipal Solid Waste Management for Markov Model Application

Extending the information contained in Figure 1 to the entire ASWAMA Zones in Awka, Table 1 is formulated as a transition table for quantities of solid waste produced in each of the zones for each month of a year. Each of these waste productions is represented by *P* (probability vectors), expressed in percentage of the total monthly generations from all the zones in each month rounded up to the nearest reasonable number. The table consists of twelve orthogonal arrays (twelve rows and twelve columns) of numerical values, with P_{ij} as the data elements for the twelve Zones in a (twelve months) year.

Table 1: Transition table for quantities of solid waste produced in Awka area

Zone Codes →		Jan 1	Feb 2	Mar 3	Apr 4	May 5	Jun 6	Jul 7	Aug 8	Sep 9	Oct 10	Nov 11	Dec 12
↓	**States** →	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	**1** ↓	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{110}	P_{111}	P_{112}
2	**2**	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	P_{210}	P_{211}	P_{212}
3	**3**	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}	P_{39}	P_{310}	P_{311}	P_{312}
4	**4**	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	P_{410}	P_{411}	P_{412}
5	**5**	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	P_{56}	P_{57}	P_{58}	P_{59}	P_{510}	P_{511}	P_{512}
6	**6**	P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_{66}	P_{67}	P_{68}	P_{69}	P_{610}	P_{611}	P_{612}
7	**7**	P_{71}	P_{72}	P_{73}	P_{74}	P_{75}	P_{76}	P_{77}	P_{78}	P_{79}	P_{710}	P_{711}	P_{712}
8	**8**	P_{81}	P_{82}	P_{83}	P_{84}	P_{85}	P_{86}	P_{87}	P_{88}	P_{89}	P_{810}	P_{811}	P_{812}
9	**9**	P_{91}	P_{92}	P_{93}	P_{94}	P_{95}	P_{96}	P_{97}	P_{98}	P_{99}	P_{910}	P_{911}	P_{912}
10	**10**	P_{101}	P_{102}	P_{103}	P_{104}	P_{105}	P_{106}	P_{107}	P_{108}	P_{109}	P_{1010}	P_{1011}	P_{1012}
11	**11**	P_{111}	P_{112}	P_{113}	P_{114}	P_{115}	P_{116}	P_{117}	P_{118}	P_{119}	P_{1110}	P_{1111}	P_{1112}
12	**12**	P_{121}	P_{122}	P_{123}	P_{124}	P_{125}	P_{126}	P_{127}	P_{128}	P_{129}	P_{1210}	P_{1211}	P_{1212}

Data in Table 1 is next rewritten as a transition matrix M in which it is assumed that a given monthly waste production comes from any of the twelve discrete states represented by the rows and columns numbers 1, 2, 3, ..., 12. This same information is shown in the ICCE solar-tree diagram of Figure 3 (compare with Figure 2).

States	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{110}	P_{111}	P_{112}
2	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	P_{210}	P_{211}	P_{212}
3	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}	P_{39}	P_{310}	P_{311}	P_{312}
4	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	P_{410}	P_{411}	P_{412}
5	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	P_{56}	P_{57}	P_{58}	P_{59}	P_{510}	P_{511}	P_{512}
6	P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_{66}	P_{67}	P_{68}	P_{69}	P_{610}	P_{611}	P_{612}
7	P_{71}	P_{72}	P_{73}	P_{74}	P_{75}	P_{76}	P_{77}	P_{78}	P_{79}	P_{710}	P_{711}	P_{712}
8	P_{81}	P_{82}	P_{83}	P_{84}	P_{85}	P_{86}	P_{87}	P_{88}	P_{89}	P_{810}	P_{811}	P_{812}
9	P_{91}	P_{92}	P_{93}	P_{94}	P_{95}	P_{96}	P_{97}	P_{98}	P_{99}	P_{910}	P_{911}	P_{912}
10	P_{101}	P_{102}	P_{103}	P_{104}	P_{105}	P_{106}	P_{107}	P_{108}	P_{109}	P_{1010}	P_{1011}	P_{1012}
11	P_{111}	P_{112}	P_{113}	P_{114}	P_{115}	P_{116}	P_{117}	P_{118}	P_{119}	P_{1110}	P_{1111}	P_{1112}
12	P_{121}	P_{122}	P_{123}	P_{124}	P_{125}	P_{126}	P_{127}	P_{128}	P_{129}	P_{1210}	P_{1211}	P_{1212}

$= M$

Meanwhile, M can be seen as representing the probability of a change in monthly waste generation (production) in the twelve zones. The notation P_{ij} is used to denote the change from state i to state j. To forecast the probable future waste percentage productions in the twelve zones under review, M is raised to an index number greater than unity. For example, the percentage contributions each of these zones will likely make to the total waste in the next one year can be investigated. To achieve the result, M *is* squared and the matrix solved by method of matrix multiplication. That is to say,

$$M^2 = M \cdot M \qquad (2)$$

In doing so, the memoryless property of the Markov chain must have been utilized, i.e. the data for the chosen year is used to predict the outcome of the subsequent year(s).

This work is, therefore, summarized as follows:

M^n gives the probabilities of a transition from one state to another after n repetitions of an experiment.

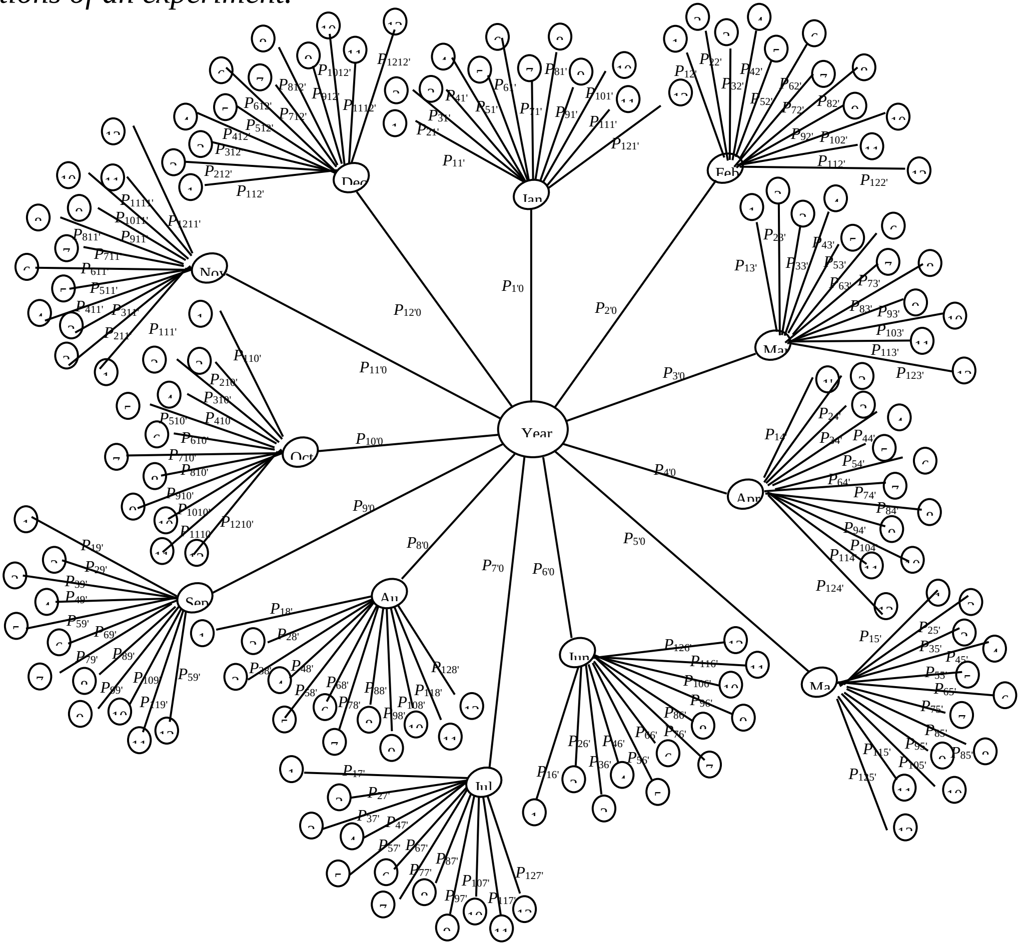


Figure 3: An ICCE solar-tree diagram showing the probability distribution of waste production states at the end of different months in a year.

An alternative approach to arriving at the same result as above is to multiply matrix M with an initial probability vector. Recall that a probability vector is a matrix with only one row, having non-negative entries, with the sum of the entries equal to 1. In the study, it is assumed that the initial probability vector, X_0, is the matrix that represents the annual total productions from each of the twelve zones. Let this initial vector be:

$$X_0 = [P_{01}, P_{02}, P_{03}, P_{04}, P_{05}, P_{06}, P_{07}, P_{08}, P_{09}, P_{010}, P_{011}, P_{012}] \quad (4)$$

From the foregoing, one can safely find the distribution of states after a number of years (future productions) from the relations:

$$M^2 = X_0.M \quad (5a)$$

$$M^3 = X_0.M^2 \quad (5b)$$

. . . .

$$\dot{M}^{n} = \dot{X}_{0}.\dot{M}^{n} \qquad (5c$$

3.0 Model Application, Results and Discussions

Awka Urban City of Anambra State was divided into twelve Zones namely Amawbia, Zik's Avenue, Amaikwo, Amaenyi/Amaku, Udoka Estate, Nibo/Umuawulu, Iyiagu Estate, Okpuno, Enugu/Onitsha Expressway, Emma Nnaemeka, Ifite and Government House to enable effective solid waste management in the area. These zones are respectively represented with the italicized numbers in the first row of Table 2. Of all the quantities of solid waste produced in the zones and dumped at Agu-Awka final dumpsite during a thirty six month field study, it was noticed that each zone contributed a certain quantity every month as depicted in Table 2 following the explanations given on Figure 2.

Table 2: Approximate quantities of solid waste dumped at the monitored points in the 12 zones of Awka city

Year	Month	Mnt_ Code	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*Total*
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_T
	Jan	1	119.2	210.6	134.2	133.0	133.6	117.9	93.0	120.4	84.0	74.6	144.4	132.9	*1497.8*
	Feb	2	123.9	189.0	126.5	114.9	131.3	110.5	103.1	112.8	87.9	80.3	132.3	126.8	*1439.3*
	Mar	3	112.2	208.4	138.5	144.0	137.4	96.9	94.7	133.7	101.6	72.1	148.7	142.0	*1530.1*
	Apr	4	136.6	237.0	141.0	119.6	151.0	128.9	114.8	130.2	107.1	78.3	143.5	149.9	*1637.9*
	May	5	117.4	206.4	144.2	156.9	143.1	111.5	109.6	136.4	92.3	92.7	153.3	131.3	*1594.8*
2012	Jun	6	145.5	241.3	152.1	143.8	118.8	118.3	86.9	139.5	115.9	91.0	143.0	164.8	*1660.9*
	Jul	7	116.8	210.3	140.2	128.5	151.8	105.1	93.5	128.5	93.5	70.1	140.2	151.8	*1530.3*
	Aug	8	142.8	214.4	116.7	136.1	135.8	107.0	104.7	121.1	99.8	68.3	142.4	132.7	*1521.9*
	Sep	9	118.6	229.5	142.2	131.2	144.0	129.4	114.8	146.2	97.5	87.5	155.1	148.3	*1644.3*
	Oct	10	147.1	244.0	153.9	145.5	120.1	119.6	87.8	141.1	117.2	92.0	144.6	166.6	*1679.6*
	Nov	11	131.6	195.7	120.3	135.2	150.8	110.2	111.9	114.9	88.0	85.1	143.8	137.1	*1524.5*
	Dec	12	144.8	224.7	136.6	130.4	135.7	108.4	107.5	122.0	94.3	88.1	137.1	126.3	*1555.8*
	Jan	13	153.9	227.1	122.5	143.5	144.8	119.3	111.7	127.6	106.5	72.9	151.9	141.5	*1623.1*
	Feb	14	130.6	242.7	161.3	167.7	160.0	112.8	110.3	155.7	118.3	83.9	173.2	165.4	*1781.9*
	Mar	15	157.3	260.8	164.4	155.5	128.4	127.8	93.9	150.8	125.3	98.4	154.6	178.1	*1795.2*
	Apr	16	171.8	266.5	162.0	154.7	160.9	128.6	127.5	144.7	111.8	104.5	162.6	149.9	*1845.5*
	May	17	149.9	264.8	168.7	167.2	168.0	148.2	116.9	151.4	105.6	93.8	181.5	167.0	*1883.1*
2013	Jun	18	138.5	243.4	170.1	185.1	168.8	131.5	129.2	160.9	108.9	109.3	180.8	154.8	*1881.3*
	Jul	19	147.2	219.1	134.6	151.3	168.7	123.4	125.2	128.6	98.4	95.2	160.9	153.4	*1706.2*
	Aug	20	142.6	247.4	147.2	124.8	157.6	134.6	119.9	135.9	111.8	81.7	149.8	156.4	*1709.8*
	Sep	21	133.6	258.6	160.2	147.8	162.3	145.8	129.3	164.7	109.9	98.6	174.7	167.1	*1852.6*
	Oct	22	130.3	234.6	156.4	143.4	169.4	117.3	104.3	143.4	104.3	78.2	156.4	169.4	*1707.2*
	Nov	23	145.9	222.6	149.0	135.3	154.6	130.2	121.4	132.9	103.6	94.6	155.8	149.3	*1695.0*
	Dec	24	168.9	280.1	176.6	166.9	137.8	137.3	100.8	161.9	134.6	105.6	166.0	191.2	*1927.8*
	Jan	25	174.6	301.4	180.0	146.9	188.9	162.8	145.0	164.5	135.3	98.9	181.2	189.3	*2068.7*
	Feb	26	198.8	329.7	207.8	196.5	162.2	161.5	118.7	190.6	158.4	124.3	195.4	225.1	*2268.9*
	Mar	27	179.3	316.7	201.8	200.0	200.9	177.3	139.9	181.1	126.4	112.2	217.1	199.8	*2252.5*
	Apr	28	167.7	311.6	207.0	215.3	205.4	144.8	141.6	199.9	151.9	107.8	222.4	212.3	*2287.7*
	May	29	169.3	327.7	203.1	187.3	205.7	184.8	163.9	208.7	139.2	124.9	221.4	211.8	*2347.7*
2014	Jun	30	163.5	287.4	200.8	218.5	199.2	155.2	152.6	189.9	128.6	129.0	213.4	182.8	*2220.8*
	Jul	31	180.3	268.2	164.8	185.3	206.6	151.0	153.3	157.5	120.5	116.6	197.0	187.8	*2088.9*
	Aug	32	169.0	304.4	202.9	186.1	219.7	152.2	135.3	186.1	135.3	101.5	202.9	219.7	*2215.1*
	Sep	33	190.6	290.7	194.6	176.7	201.9	170.0	158.5	173.6	135.3	123.5	203.4	195.0	*2213.7*
	Oct	34	201.0	311.7	189.5	180.9	188.2	150.5	149.2	169.2	130.8	122.2	190.2	175.3	*2158.7*
	Nov	35	192.4	319.2	201.2	190.2	157.1	156.4	114.9	184.5	153.3	120.4	189.1	217.9	*2196.6*
	Dec	36	235.0	346.9	187.2	219.2	221.2	182.2	170.6	194.9	162.6	111.3	232.1	216.2	*2479.4*
		Averages:	*154.13*	*258.18*	*162.78*	*160.14*	*163.66*	*135.26*	*121.01*	*152.94*	*116.55*	*96.93*	*171.17*	*169.08*	*1861.79*

The average quantity of solid waste contributed by each of the zones in the corresponding months - i.e. $^1/_3$(Jan 2012 + Jan 2013 + Jan 2014), $^1/_3$ (Feb 2012 + Feb 2013 + Feb 2014), ..., $^1/_3$ (Dec 2012 +Dec 2013 + Dec 2014) - of the three year-period were calculated and copied into Table 3.

Table 3: Average monthly waste generation calculated for the same months of Jan 1, 2012 to Dec 31, 2015

Year	Month Code	1 $\bar{\lambda}_1$	2 $\bar{\lambda}_2$	3 $\bar{\lambda}_3$	4 $\bar{\lambda}_4$	5 $\bar{\lambda}_5$	6 $\bar{\lambda}_6$	7 $\bar{\lambda}_7$	8 $\bar{\lambda}_8$	9 $\bar{\lambda}_9$	10 $\bar{\lambda}_{10}$	11 $\bar{\lambda}_{11}$	12 $\bar{\lambda}_{12}$	**Average Total Dump**
2012 to 2014	1	149.2	246.4	145.6	141.1	155.8	133.3	116.6	137.5	108.6	82.1	159.2	154.6	1729.9
	2	151.1	253.8	165.2	159.7	151.2	128.3	110.7	153.0	121.5	96.2	167.0	172.4	1830.1
	3	149.6	262.0	168.2	166.5	155.6	134.0	109.5	155.2	117.8	94.2	173.5	173.3	1859.3
	4	158.7	271.7	170.0	163.2	172.4	134.1	128.0	158.3	123.6	96.9	176.2	170.7	1923.7
	5	145.5	266.3	172.0	170.5	172.3	148.2	130.1	165.5	112.4	103.8	185.4	170.0	1942.0
	6	149.2	257.4	174.3	182.5	162.3	135.0	122.9	163.4	117.8	109.8	179.1	167.5	1921.0
	7	148.1	232.5	146.5	155.0	175.7	126.5	124.0	138.2	104.1	94.0	166.0	164.3	1775.1
	8	151.5	255.4	155.6	149.0	171.0	131.3	120.0	147.7	115.6	83.8	165.0	169.6	1815.5
	9	147.6	259.6	165.7	151.9	169.4	148.4	134.2	161.5	114.2	103.2	177.7	170.1	1903.6
	10	159.5	263.4	166.6	156.6	159.2	129.1	113.8	151.2	117.4	97.5	163.7	170.4	1848.5
	11	156.6	245.8	156.8	153.6	154.2	132.3	116.1	144.1	115.0	100.0	162.9	168.1	1805.5
	12	182.9	283.9	166.8	172.2	164.9	142.6	126.3	159.6	130.5	101.7	178.4	177.9	1987.7

In order to use the memoryless property of the Markov model in attending to the above stated problem, these calculated average values are taken as data in the present for determining the ones in the future. Actual values of the data in Table 3 are plotted in graph (Bar chart) of Figure 4. All the twelve zones of ASWAMA are clearly shown in the figure, with the quantities of waste produced in the same month in each of the zones represented by the various bars verged with the same colour. It can be vividly seen that the bars in each zone vary in lengths, showing variations in waste production - i.e. a stochastic process.

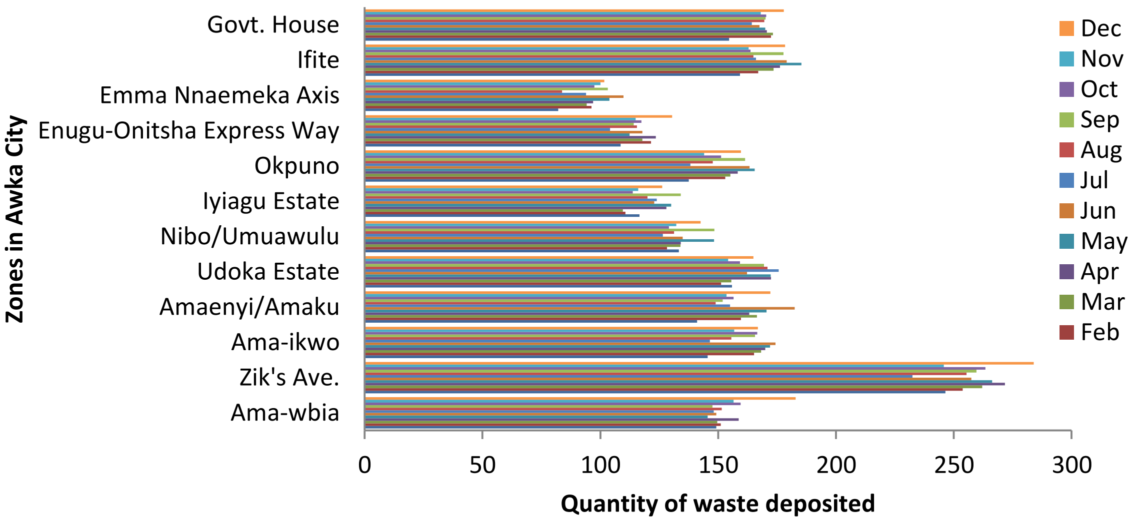
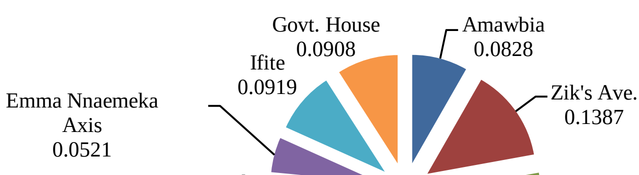


Figure 4: Average Monthly Solid Waste Generation in each of the twelve Zones of Awka Urban City

Figure 5 is a plot of the data in Table 3 represented as monthly fractional contri-butions from the respective zones. It is easily seen from the data in Table 3, represented in Figures 4 and 5 that Zik's Avenue produces more waste than the rest of the ASWAMA Zones of



Awka Municipality, followed by Ifite Zone. Nnamemeka Axis produces the smallest quantities.

3.1 Determining a Long-Range Trend of Awka Urban solid Waste Generation

Data in Table 3 were converted to percentages, expressed as probability vectors of the average monthly total generations, with the transition states defined. The percents were changed to decimals rounded up to the nearest one-thousandth. Eqn 3 is then applied and Table 4 generated.

Table 4: Percentage monthly contributions by each ASWAMA Zone to the average total solid waste produced in Awka city annually

States	1	2	3	4	5	6	7	8	9	10	11	12	Total
1	0.086	0.142	0.084	0.082	0.090	0.077	0.067	0.079	0.063	0.047	0.092	0.089	1.000
2	0.083	0.139	0.090	0.087	0.083	0.070	0.060	0.084	0.066	0.053	0.091	0.094	1.000
3	0.080	0.141	0.090	0.090	0.084	0.072	0.059	0.083	0.063	0.051	0.093	0.093	1.000
4	0.082	0.141	0.088	0.085	0.090	0.070	0.067	0.082	0.064	0.050	0.092	0.089	1.000
5	0.075	0.137	0.089	0.088	0.089	0.076	0.067	0.085	0.058	0.053	0.095	0.088	1.000
6	0.078	0.134	0.091	0.095	0.084	0.070	0.064	0.085	0.061	0.057	0.093	0.087	1.000
7	0.083	0.131	0.083	0.087	0.099	0.071	0.070	0.078	0.059	0.053	0.094	0.093	1.000
8	0.083	0.141	0.086	0.082	0.094	0.072	0.066	0.081	0.064	0.046	0.091	0.093	1.000
9	0.078	0.136	0.087	0.080	0.089	0.078	0.070	0.085	0.060	0.054	0.093	0.089	1.000
10	0.086	0.143	0.090	0.085	0.086	0.070	0.062	0.082	0.064	0.053	0.089	0.092	1.000
11	0.087	0.136	0.087	0.085	0.085	0.073	0.064	0.080	0.064	0.055	0.090	0.093	1.000
12	0.092	0.143	0.084	0.087	0.083	0.072	0.064	0.080	0.066	0.051	0.090	0.090	1.000

Using eqn 2 (or 5), it is possible to predict the probable future monthly percentage waste contributions from the various Zones of Awka city. As stated earlier, it is taken that ***M*** is for the year (2014) as the first year and the second year in our prediction is 2015, the third year is 2016, et cetera. This prediction will assist the Awka waste manager, ASWAMA, in making decision on how to schedule its disposal trucks to the zones and if variable charges are to apply in the area, which zone pays what? Converting the values in Table 4 as the probability vectors of matrix M, we have:

M = (6a)

States	1	2	3	4	5	6	7	8	9	10	11	12
1	0.086	0.142	0.084	0.082	0.090	0.077	0.067	0.079	0.063	0.047	0.092	0.089
2	0.083	0.139	0.090	0.087	0.083	0.070	0.060	0.084	0.066	0.053	0.091	0.094
3	0.080	0.141	0.090	0.090	0.084	0.072	0.059	0.083	0.063	0.051	0.093	0.093
4	0.082	0.141	0.088	0.085	0.090	0.070	0.067	0.082	0.064	0.050	0.092	0.089
5	0.075	0.137	0.089	0.088	0.089	0.076	0.067	0.085	0.058	0.053	0.095	0.088
6	0.078	0.134	0.091	0.095	0.084	0.070	0.064	0.085	0.061	0.057	0.093	0.087
7	0.083	0.131	0.083	0.087	0.099	0.071	0.070	0.078	0.059	0.053	0.094	0.093
8	0.083	0.141	0.086	0.082	0.094	0.072	0.066	0.081	0.064	0.046	0.091	0.093
9	0.078	0.136	0.087	0.080	0.089	0.078	0.070	0.085	0.060	0.054	0.093	0.089
10	0.086	0.143	0.090	0.085	0.086	0.070	0.062	0.082	0.064	0.053	0.089	0.092
11	0.087	0.136	0.087	0.085	0.085	0.073	0.064	0.080	0.064	0.055	0.090	0.093
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$M^7 =$ (6c)

States	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0826	0.1384	0.0873	0.0859	0.0873	0.0722	0.0644	0.0818	0.0628	0.0517	0.0916	0.0907
2	0.0828	0.1387	0.0874	0.0861	0.0874	0.0724	0.0645	0.0820	0.0629	0.0518	0.0918	0.0909
3	0.0827	0.1385	0.0873	0.0860	0.0873	0.0723	0.0645	0.0819	0.0628	0.0517	0.0917	0.0908
4	0.0828	0.1387	0.0874	0.0861	0.0874	0.0724	0.0645	0.0820	0.0629	0.0518	0.0918	0.0909
5	0.0828	0.1387	0.0874	0.0861	0.0874	0.0724	0.0645	0.0820	0.0629	0.0518	0.0918	0.0909
6	0.0827	0.1385	0.0873	0.0860	0.0873	0.0723	0.0645	0.0819	0.0628	0.0517	0.0917	0.0908
7	0.0829	0.1388	0.0875	0.0862	0.0875	0.0725	0.0646	0.0821	0.0630	0.0518	0.0919	0.0910
8	0.0827	0.1385	0.0873	0.0860	0.0873	0.0723	0.0645	0.0819	0.0628	0.0517	0.0917	0.0908
9	0.0827	0.1385	0.0873	0.0860	0.0873	0.0723	0.0645	0.0819	0.0628	0.0517	0.0917	0.0908
10	0.0829	0.1389	0.0876	0.0863	0.0876	0.0725	0.0647	0.0822	0.0630	0.0519	0.0920	0.0911
11	0.0827	0.1385	0.0873	0.0860	0.0873	0.0723	0.0645	0.0819	0.0628	0.0517	0.0917	0.0908
12	0.0829	0.1389	0.0876	0.0863	0.0876	0.0725	0.0647	0.0822	0.0630	0.0519	0.0920	0.0911

$M^2 =$ (6b)

States	1	2	3	4	5	6	7	8	9	10	11	12
1	0.0826	0.1384	0.0873	0.0861	0.0874	0.0724	0.0645	0.0819	0.0628	0.0518	0.0917	0.0908
2	0.0829	0.1389	0.0875	0.0862	0.0875	0.0725	0.0646	0.0821	0.0630	0.0518	0.0918	0.0910
3	0.0828	0.1387	0.0875	0.0861	0.0873	0.0724	0.0645	0.0820	0.0630	0.0518	0.0918	0.0909
4	0.0828	0.1388	0.0875	0.0862	0.0876	0.0725	0.0646	0.0821	0.0629	0.0519	0.0919	0.0910
5	0.0828	0.1388	0.0876	0.0863	0.0875	0.0724	0.0646	0.0821	0.0630	0.0519	0.0919	0.0910
6	0.0828	0.1387	0.0875	0.0861	0.0875	0.0723	0.0645	0.0820	0.0629	0.0518	0.0918	0.0909
7	0.0829	0.1389	0.0876	0.0863	0.0877	0.0726	0.0648	0.0821	0.0630	0.0519	0.0920	0.0911
8	0.0827	0.1386	0.0874	0.0861	0.0874	0.0724	0.0646	0.0820	0.0629	0.0518	0.0918	0.0909
9	0.0827	0.1386	0.0874	0.0862	0.0875	0.0724	0.0645	0.0820	0.0629	0.0519	0.0918	0.0909
10	0.0831	0.1392	0.0877	0.0863	0.0877	0.0726	0.0647	0.0823	0.0631	0.0519	0.0920	0.0912
11	0.0828	0.1387	0.0874	0.0861	0.0874	0.0724	0.0645	0.0820	0.0629	0.0518	0.0918	0.0909
12	0.0831	0.1391	0.0877	0.0863	0.0877	0.0727	0.0648	0.0822	0.0631	0.0519	0.0920	0.0912

A close look at the values in each of the columns in *M* for the 2nd and 7th years seem to converge to certain specific values. After many (say, 50) years the values in the columns (rounded up to the nearest 1000th) seem to have almost fully converged to their target end numbers.

$M^{50} =$ (6d)

States	1	2	3	4	5	6	7	8	9	10	11	12
1	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
2	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
3	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
4	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
5	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
6	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
7	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
8	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
9	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
10	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
11	0.083	0.137	0.087	0.085	0.087	0.073	0.065	0.082	0.063	0.052	0.092	0.090
12	0.082	0.138	0.087	0.086	0.087	0.072	0.064	0.081	0.062	0.051	0.092	0.090

The results obtained in M^{50} provides ASWAMA with formulae for sharing its responsibilities (job scheduling, revenue generation and expenditure, just name it) among the twelve zones of Awka municipality in the approximated ratio shown in Table 5.

Table 5: Sharing formula for allocation of ASWAMA resources and job schuling in Awka area.

1 Ama-wbia	*2* Zik's Ave.	*3* Ama-ikwo	*4* Amaenyi / Amaku	*5* Udoka Estate	*6* Nibo/ Umu-awulu	*7* Iyiagu Estate	*8* Okpuno	*9* Enugu-Onitsha Express Way	*10* Emma Nnaemeka Axis	11 Ifite	*12* Govt. House
8.3%	13.7%	8.8%	8.6%	8.8%	7.2%	6.4%	8.2%	6.3%	5.2%	9.2%	9.1%

3.2 Determination of the Month of Best Fit to Start a Field Study in SWM

Meanwhile, one of the initial problems that arose because of monthly and seasonal variations in waste production was determining which of the months of the year would serve as the best time to start the field study. Nevertheless, the researchers later decided to start the research on 1st January, 2012. The field study ended on 31st December, 2014, lasting for 1096 days. Assuming that the various quantities of waste contributed by the zones represent the transition states of the waste, what is the long-range trend of the Markov chain in the waste generation? Verify if the month of January, or any other month of the year, chosen by the researchers is the most appropriate period to start the research.

In attempting a solution to the above problem, it is seen that the last column of Table 3 gives the mean values of the monthly total quantities of solid waste generated in all the zones in Awka city. Assuming here that these mean quantities are the actual generations in the area and that the waste were allowed to keep accumulating till the end of the year. These assumptions enabled Table 6 to be constructed. The table shows the transition states of the total waste generated monthly in the area. At the end of the month of January of a given year, only the quantity of waste generated in the month is recorded; no waste is generated in the future months. At the end of the month of February of the same year, only the quantity of waste generated in the months of January and February accumulated and were recorded; no waste is generated in the future months. By the same line of thinking, at the end of the month of March only the waste generated in the months of January to March were recorded as in the previous cases.

Table 6: Stepwise consideration of data collected on waste generated monthly in Awka City

Month (Present ↓)	**Jan**	**Feb**	**Mar**	Future → **Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**	**Average Total Dump**
Jan	1729.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1729.9
Feb	1729.9	1830.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3560.0
Mar	1729.9	1830.1	1859.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5419.3
Apr	1729.9	1830.1	1859.3	1923.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7343.0
May	1729.9	1830.1	1859.3	1923.7	1942.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	9285.0
Jun	1729.9	1830.1	1859.3	1923.7	1942.0	1921.0	0.0	0.0	0.0	0.0	0.0	0.0	11206.0
Jul	1729.9	1830.1	1859.3	1923.7	1942.0	1921.0	1775.1	0.0	0.0	0.0	0.0	0.0	12981.1
Aug	1729.9	1830.1	1859.3	1923.7	1942.0	1921.0	1775.1	1815.5	0.0	0.0	0.0	0.0	14796.6
Sep	1729.9	1830.1	1859.3	1923.7	1942.0	1921.0	1775.1	1815.5	1903.6	0.0	0.0	0.0	16700.2
Oct	1729.9	1830.1	1859.3	1923.7	1942.0	1921.0	1775.1	1815.5	1903.6	1848.5	0.0	0.0	18548.7
Nov	1729.9	1830.1	1859.3	1923.7	1942.0	1921.0	1775.1	1815.5	1903.6	1848.5	1805.5	0.0	20354.2
Dec	1729.9	1830.1	1859.3	1923.7	1942.0	1921.0	1775.1	1815.5	1903.6	1848.5	1805.5	1987.7	22341.9

The data in Table 6, converted into percentages, expressed as probability vectors are represented in Table 7

Table 7: Values in Table 6 converted to percentages for application in Markovian analysis

States	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total**
1	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.000
2	0.4859	0.5141	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.000

3	0.3192	0.3377	0.3431	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.000
4	0.2356	0.2492	0.2532	0.2620	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.000
5	0.1863	0.1971	0.2002	0.2072	0.2092	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.000
6	0.1544	0.1633	0.1659	0.1717	0.1733	0.1714	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.000
7	0.1333	0.1410	0.1432	0.1482	0.1496	0.1480	0.1367	0.0000	0.0000	0.0000	0.0000	0.0000	1.000
8	0.1169	0.1237	0.1257	0.1300	0.1312	0.1298	0.1200	0.1227	0.0000	0.0000	0.0000	0.0000	1.000
9	0.1036	0.1096	0.1113	0.1152	0.1163	0.1150	0.1063	0.1087	0.1140	0.0000	0.0000	0.0000	1.000
10	0.0933	0.0987	0.1002	0.1037	0.1047	0.1036	0.0957	0.0979	0.1026	0.0997	0.0000	0.0000	1.000
11	0.0850	0.0899	0.0913	0.0945	0.0954	0.0944	0.0872	0.0892	0.0935	0.0908	0.0887	0.0000	1.000
12	0.0774	0.0819	0.0832	0.0861	0.0869	0.0860	0.0795	0.0813	0.0852	0.0827	0.0808	0.0890	1.000

Finally, Table 7 was converted to a regular transition matrix P as shown hereunder:

$P =$ (7a)

States	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.4859	0.5141	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.3192	0.3377	0.3431	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.2356	0.2492	0.2532	0.2620	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.1863	0.1971	0.2002	0.2072	0.2092	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.1544	0.1633	0.1659	0.1717	0.1733	0.1714	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.1333	0.1410	0.1432	0.1482	0.1496	0.1480	0.1367	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.1169	0.1237	0.1257	0.1300	0.1312	0.1298	0.1200	0.1227	0.0000	0.0000	0.0000	0.0000
9	0.1036	0.1096	0.1113	0.1152	0.1163	0.1150	0.1063	0.1087	0.1140	0.0000	0.0000	0.0000
10	0.0933	0.0987	0.1002	0.1037	0.1047	0.1036	0.0957	0.0979	0.1026	0.0997	0.0000	0.0000
11	0.0850	0.0899	0.0913	0.0945	0.0954	0.0944	0.0872	0.0892	0.0935	0.0908	0.0887	0.0000
12	0.0774	0.0819	0.0832	0.0861	0.0869	0.0860	0.0795	0.0813	0.0852	0.0827	0.0808	0.0890

Following the same steps as used for the transition matrix M, P^2, P^{18}, P^{25} *and* P^{50} were determined as follows:

$P^2 =$ (7b)

States	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0000	0	0	0	0	0	0	0	0	0	0	0
2	0.7357	0.2643	0	0	0	0	0	0	0	0	0	0
3	0.5928	0.2895	0.1177	0	0	0	0	0	0	0	0	0
4	0.4992	0.2789	0.1532	0.0686	0	0	0	0	0	0	0	0
5	0.4338	0.2618	0.1630	0.0976	0.0438	0	0	0	0	0	0	0
6	0.3859	0.2449	0.1635	0.1103	0.0660	0.0294	0	0	0	0	0	0
7	0.3514	0.2307	0.1607	0.1155	0.0774	0.0456	0.0187	0	0	0	0	0
8	0.3226	0.2176	0.1565	0.1173	0.0840	0.0559	0.0311	0.0151	0	0	0	0
9	0.2976	0.2053	0.1513	0.1170	0.0877	0.0627	0.0397	0.0257	0.0130	0	0	0
10	0.2773	0.1947	0.1462	0.1157	0.0894	0.0668	0.0453	0.0329	0.0219	0.0099	0	0
11	0.2602	0.1853	0.1413	0.1138	0.0899	0.0692	0.0490	0.0379	0.0283	0.0171	0.0079	0
12	0.2439	0.1761	0.1362	0.1114	0.0897	0.0707	0.0517	0.0418	0.0333	0.0229	0.0144	0.0079

$P^{18} =$ (7c)

States	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.9998	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	0.9999	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

$P^{25} =$ (7d)

States	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

P^{50}

States	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(7e)

The implication of the result obtained in P^{50} is that it does not really matter in which month of the year the said study was started. This claim is what all the values (states) in the first column that have converged to unity indicate.

4.0. Conclusion

From the foregoing discussions, therefore, we conclude as follows:

- A Markov chain in matrix *M* was formulated from the study as a SWM model.
- The Markovian SWM model was used to predict a long time percentage monthly solid waste contributions to total waste stream from each of the twelve zones of Awka city in future years as: Amawbia = 8.3%, Zik's Avenue = 13.9%, Amaikwo = 8.8%, Amaenyi/Amaku = 8.6%, Udoka Estate = 8.8%, Nibo/Umuawulu = 7.2%, Iyiagu Estate = 6.4%, Okpuno = 8.2%, Enugu/Onitsha Expressway = 6.3%, Emma Nnaemeka Axis = 5.2%, Ifite = 9.2% and Government House = 9.1%. As stated earlier, it is taken that *M* used the year 2014 as the first year in our prediction, the second year is 2015, the third year is 2016, et cetera.
- This prediction should assist ASWAMA in making plans and decisions on how to schedule its disposal trucks to the zones and if variable charges are to apply in the area, which zone pays what?
- The analysis also showed that it does not matter which month of a given year the study made on solid waste management started, provided the same length of period (twelve months) is maintained.
- The steps followed in applying Markov chain in Awka municipal solid waste management include:

Step 1: Divide the study area into ***n*** number of zones and the study period, into ***n*** number of equal parts. Assign unique names, numbers or codes to each division of the zones and to each division of the study time periods. Using serial numbers 1, 2, 3, …, *n* as codes is recommended.

Step 2: In a spreadsheet (or any of the such) create a table that has $n + 3$ number of columns and $n+3$ number of rows

Step 3: Label cell of row 1, column 1 as "Time" (or "Periods") and insert a horizontal arrow pointing rightwards into cells of row 1, columns 2 and 3. Insert another arrow pointing downwards into cell of rows 2 and 3, column 1 and serially number column 4 to ***n*** of row 1 and rows 4 to ***n*** of column 1 as are the names or codes given to the time periods in step 1.

Step 4: Label cell of row 2, column 2 as "Zone" and insert a horizontal arrow pointing rightwards into cells of row 2, column 3. Insert another arrow pointing downwards into cell of row 3, column 2 and serially number column 4 to ***n*** of row 2 and rows 4 to ***n*** of column 2 as are the names or codes given to the zones in step 1.

Step 5: Label cell of row 3, column 3 as "States" and serially number column 4 to ***n*** of row 2 and rows 4 to ***n*** of column 2 as 1, 2, 3. …, ***n***.

Step 6: For each zone, enter its percent-age contribution to the total waste stream in periods 1to n in row 5 of columns 5 to ***n*** and sum up these values in column n+1, which should be equal to 1.

Step 7: Write the ***n*** x ***n*** (i.e. rows 5 to n x columns 5 to n) part of the table as a transition matrix M raised to an index number k greater than unity and solve until all the values in each column of the matrix converge to the same unique numbers at the k^{th} transposition of the matrix. Matlab, Scilab or any of such application software may be very helpful in doing this.

Step 8: Stop the matrix operations and write out the various converged numbers as the probable future waste percentage productions in k^{th} (future) time from each of the respective ***n*** number of zones under review.

5.0 Recommendation

From the foregoing analysis and results obtained therefrom, it is clearly seen that Markov chain has been successfully applied in municipal solid management of Awka Municipality, Anambra State. Consequently, it is recommended that waste managers in Anambra State or any other state should start applying Markov model as one of the tools used in their solid waste inventory management.

Bibliography

Alexander Volfovsky (2007). *Markov Chains and Applications.* - University of Chicago. www.math.uchicago.edu/~may/VIGRE/VIGRE2007/REUPapers/FINALFULL/. An Addison-Wesley Product. *Markov Chains.* Copyright © 2003. Pearson Education Inc.

Basharin Gely P., Langville Amy N., and Naumov Valeriy A. (2003). *The Life and Work of A. A. Markov*. Linear Algebra and its Applica-tions, Vol 386, pp 3-26. www.sciencedirect.com doi:10.1016/j.laa.2003.12.041

Ihueze C. C. and Chukwumuanya E. O. (2015). Cost Estimation and Waste Production Forecast Models for Sustain-able Solid Waste Management: A Study on Anambra State Waste Management System. *International Journal of Applied Sciences & Engineering,* Vol 3(1): pp 14-18. www.ijapscengr.com

James Bamford (1982). *The Puzzle Palace: A Report on America's Most Secret Agency*. Houghton Mifflin Company, Boston.

Leonard E. Baum, Ted Petrie, George Soules, and Norman Weiss (1970). A maximization technique occurring in the statistical analysis of probabilistic functions of Markov chains. *The Annals of Mathematical Statistics.* Vol. 41, No. 1, pp. 164-171.
http://www.jstor.org/stable/2239727 Accessed: 28-04-2015 09:37 UTC

Nancy Blachman, Eric Fredricksen, Fritz Schneider (2003). *How to Do Everything with Google*. McGraw-Hill. www.amazon.com/Everything-Google-Fritz-Schneider-2003-11.

Rafia Afroz, Keisuke Hanaki, Kiyo. H. Kurisu (2008). *Factors Affecting Waste Generation and Willingness to Recycle: a Study in a Waste Management Program in Dhaka city, Bangladesh*. Working Paper Series No. 0803.

Rebecca Atherton (2005). *A Look at Markov Chains and Their Use in Google*. Iowa State University. MSM Creative Component Summer

Yee Whye Teh, *Markov Chains and Markov Chain Monte Carlo*, Department of Statistics. http://www.stats.ox.ac.uk/~