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Efficacy of Orthogonal Polynomial Displacement Functions in Dynamic Analysis of Rectangular Plates.

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Abstract

In this paper, the derivation of orthogonal polynomial displacement functions based on static deflection profiles for rectangular plates with various boundary conditions was carried out. The completeness characteristics for appropriate shape functions were taken into consideration; and the derived polynomial shape functions satisfied the homogeneous boundary conditions. Furthermore, the efficacy of the derived polynomial shape functions in dynamic analysis of rectangular plates was determined by using the derived polynomial shape functions to determine the fundamental frequencies of rectangular plate with various boundary conditions in Ritz method. The numerical values for the fundamental frequencies of rectangular plates as computed were compared with the results from previous work in literature; and it was discovered that 76.19% of the boundary conditions of rectangular plates showed good convergence to results in literature. Equally observed, the set of boundary conditions containing two or more free edges conditions exhibited poor convergence to exact results. Nonetheless, the mean percentage difference between the present study's results and those results from the previous work in literature is 12.581, which in view of statistical interpretation is in close agreement with results in literature.

Keywords: Orthogonal Polynomial, Shape Functions, Boundary Conditions, Ritz Functional, Fundamental Frequency, Rectangular Plate

Notation

a length of rectangular plate
b width of rectangular plate
t thickness of plate
E Young modulus
ρ mass density of plate materials
μ Poisson's ratio
D flexural rigidity
M mass per unit area of the plate
$w(\cdot,\cdot)$ polynomial displacement function
$w'(\cdot,\cdot)$ first derivative with coordinate axis
$w''(\cdot,\cdot)$ second derivative
$w'''(\cdot,\cdot)$ third derivative

1.0 Introduction

Rectangular plates, in their engineering applications, are characterized with edge constraints; and altogether, there are twenty one simple combinations from principal edge constraints (*i.e.* clamped edge, simply supported edge and free edge), which are identified for rectangular plates. But unfortunately, closed –form solutions to all the boundary conditions of rectangular plates are possible only but for a few cases. Consequent upon, many researchers on rectangular plate problems resort to the use of numerical and approximation techniques in their investigations. The approximation techniques which are popular among researchers are the Rayleigh-Ritz method, Galerkin method, the energy method in Hamilton principle, improved Kantorovich method and the like.

However in approximation techniques, most of the mathematical models formulated require that characteristic shape functions (displacement functions) are to be substituted into the derived equations; from which their solutions are expected to approximate exact solutions. Thus, the efficacy of a given set of shape functions is desirous to yield results that are in good agreement with exact results. Consequently, many investigators used to employ different shape functions in bid to seek for most appropriate displacement functions that can yield excellent results that are comparable with exact results. For instance, Leissa (1969); Leissa and Qatu (2011) and Birman (2011), in their individual texts, used trigonometric functions in Fourier series to solve free vibration problems of rectangular plates in Ritz method. Other investigators who applied trigonometric functions as shape functions in their individual investigations into dynamical problems of rectangular plates are Mindlin *et al.* (1951), Johnson and Bauld (1968), Cheung *et al.* (1998), Janevski (2002), Shimpi (2002), Shooshtari amd Khadem (2007), Hao *et al.* (2011), Nefovska-Danilovié and Petronijevié (2014), Saheb and Rao (2014), and Mamandi *et al.* (2015). On the other hand, Reddy (2007) used polynomial displacement functions as shape functions to determine flexural natural frequencies of rectangular plate with various boundary conditions in Ritz method. But the author did not give any method on how to derive the selected polynomial displacements he used; and therefore such application lacked in academic systematization. However, Ezeh *et al.* (2013) used Taylor series to generate polynomial displacement functions for dynamic analysis of isotropic SSSS plate in Galerkin method.

Commonly observed from the previous works reviewed herein, 88.24% of them used trigonometric functions as shape functions. Probably the urge to adopt trigonometric functions as shape functions is informed by the fact that trigonometric functions have the advantage of being continuous in their derivatives. Nevertheless, the application of trigonometric functions as displacement functions has some shortcomings in dynamic analysis of rectangular plates, especially in nonlinear dynamic analysis. Due to orthogonality properties of trigonometric functions, only but a few cases of nonlinear vibrations of thin rectangular plates could be successfully solved by using trigonometric functions as shape functions. Moreover from the previous works reviewed herein, the application of polynomial displacement functions has not been adequately explored in dynamic analysis of rectangular plates based on energy methods. Anyway, polynomial displacement functions are popular in finite element and finite difference analyses. Notwithstanding with their unpopularity as shape functions in energy methods, polynomial displacement functions are very easy to handle with respect to differentiation and integration. Therefore, the present work wants to fill the identified gaps herein with the objectives to derive the appropriate orthogonal polynomial displacement functions based on static deflection profiles for the twenty-one boundary conditions of rectangular plates; and to assess the effectiveness of orthogonal polynomial displacements as shape functions by applying the derived orthogonal polynomial displacement functions in evaluating the fundamental frequency for each of the rectangular plates.

2.0 Polynomial Displacement Functions

A rectangular plate (Figure 1) can be idealized as a system consisting of series of orthogonal beam network; and the deflection configuration of the plate is assumed to be a composition of series of orthogonal beam deflection configurations. Equally, it is assumed that each beam element running along any coordinate direction is a good representative of other series of beams that run along the same direction.

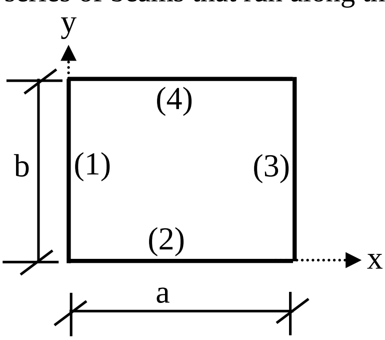

Figure 1: A Rectangular Plate with Arbitrary Edge Constraints, showing Coordinate Dimensions a and b.

Furthermore, a rectangular plate (Figure 1) is constrained at each edge either as clamped edge, or simply supported edge or free edge; and a minimum of two homogeneous boundary conditions must be satisfied on each edge. For instance, for clamped edge, the boundary conditions are:

$$w(\cdot,\cdot) = w'(\cdot,\cdot) = 0 \qquad (1)$$

And for simply supported edge, the boundary conditions are:

$$w(\cdot,\cdot) = w''(\cdot,\cdot) = 0 \quad (2)$$

And for free edge, the boundary conditions are:

$$w'(\cdot,\cdot) = w''(\cdot,\cdot) = w'''(\cdot,\cdot) = 0 \quad (3)$$

Suppose in Figure 1, the edge (1) is clamped; the edge (2) is simply supported; the edge (3) is free and the edge (4) is simply supported; then the boundary conditions for the plate are thus:

$$w(0, y) = w'(0, y) = 0 \quad (4)$$

$$w''(a, y) = w'''(a, y) = 0 \quad (5)$$

$$w(x, 0) = w''(x, 0) = 0 \quad (6)$$

$$w(x, b) = w''(x, b) = 0 \quad (7)$$

Let us define the five-term polynomial deflection configurations in x- and y-directions respectively as:

$$w(x) = \sum_{i=0}^{4} c_i x^i \quad (8)$$

$$w(y) = \sum_{j=0}^{4} d_j y^j \quad (9)$$

Where

$c_i \; and \; d_j$ are undetermined coefficients

By applying boundary conditions of Equations (4) and (5) on Equation (8), the deflection function that is complete in nature is:

$$w(x) = c_4 * a^4 \left[6\left(\frac{x}{a}\right)^2 - 4\left(\frac{x}{a}\right)^3 + \left(\frac{x}{a}\right)^4\right] \quad (10)$$

Then for series of beams running in the same direction, we have:

$$w(x) = c_4 * a^4 \left[6\left(\frac{x}{a}\right)^{m+1} - 4\left(\frac{x}{a}\right)^{m+2} + \left(\frac{x}{a}\right)^{m+3}\right] \quad (11)$$

for m $= 1,2,3,\cdots$

Similarly, by applying boundary conditions of Equations (6) and (7) on Equation (9), the deflection function that is complete in nature is:

$$w(y) = d_4 * b^4 \left[\left(\frac{y}{b}\right) - 2\left(\frac{y}{b}\right)^3 + \left(\frac{y}{b}\right)^4\right] \quad (12)$$

Then for series of beams running in the same direction, we have:

$$w(y) = d_4 * b^4 \left[\left(\frac{y}{b}\right)^n - 2\left(\frac{y}{b}\right)^{n+2} + \left(\frac{y}{b}\right)^{n+3}\right] \quad (13)$$

for n $= 1,2,3,\cdots$

Thus, the orthogonal polynomial displacement functions for CSFS rectangular plate based on static deflection configuration are:

$$w(x, y) = A_{mn}\left[6\left(\frac{x}{a}\right)^{m+1} - 4\left(\frac{x}{a}\right)^{m+2} + \left(\frac{x}{a}\right)^{m+3}\right]\left[\left(\frac{y}{b}\right)^n - 2\left(\frac{y}{b}\right)^{n+2} + \left(\frac{y}{b}\right)^{n+3}\right] \quad (14)$$

Where A_{mn} is the amplitude of deflection, defined as expressed in Equation (15):

$$A_{mn} = (c_4 * a^4) * (d_4 * b^4) \quad (15)$$

Moreover, suppose in Figure 1, the edge (1) is clamped; the edge (2) is free; the edge (3) is clamped and the edge (4) is free, then the associated boundary conditions for the plate are:

$$w(0, y) = w'(0, y) = 0 \quad (16)$$

$$w(a, y) = w'(a, y) = 0 \quad (17)$$

$$w''(x, 0) = w'''(x, 0) = 0 \quad (18)$$

$$w'(x, b) = w'''(x, b) = 0 \quad (19)$$

Then the five-term polynomial deflection configurations in x- and y-directions are defined respectively as:

$$w(x) = \sum_{i=0}^{4} c_i x^i \quad (20)$$

$$w(y) = \sum_{j=1}^{5} d_j y^j \quad (21)$$

By applying the boundary conditions of Equations (16) and (17) on Equation (20), the deflection function that is complete in nature is:

$$w(x) = c_4 * a^4 \left[\left(\frac{x}{a}\right)^2 - 2\left(\frac{x}{a}\right)^3 + \left(\frac{x}{a}\right)^4\right] \quad (22)$$

Then for series of beams running in the same direction, we have:

$$w(x) = c_4 * a^4 \left[\left(\frac{x}{a}\right)^{m+1} - 2\left(\frac{x}{a}\right)^{m+2} + \left(\frac{x}{a}\right)^{m+3}\right] \quad (23)$$

for $m = 1,2,3,\cdots$

Similarly, by applying the boundary conditions of Equations (18) and (19) on Equation (21), the deflection function that is complete is:

$$w(y) = d_5 * b^5 \left[5\left(\frac{y}{b}\right) - \frac{5}{2}\left(\frac{y}{b}\right)^4 + \left(\frac{y}{b}\right)^5\right] \quad (24)$$

Then for series of beams running in the same direction, we have:

$$w(y) = d_5 * b^5 \left[5\left(\frac{y}{b}\right)^n - \frac{5}{2}\left(\frac{y}{b}\right)^{n+3} + \left(\frac{y}{b}\right)^{n+4}\right] \quad (25)$$

for $n = 1,2,3,\cdots$

Thus, the orthogonal polynomial displacement functions for CFCF rectangular plate based on static deflection configuration are:

$$w(x, y) = A_{mn}\left[\left(\frac{x}{a}\right)^{m+1} - 2\left(\frac{x}{a}\right)^{m+2} + \left(\frac{x}{a}\right)^{m+3}\right]\left[5\left(\frac{y}{b}\right)^n - \frac{5}{2}\left(\frac{y}{b}\right)^{n+3} + \left(\frac{y}{b}\right)^{n+4}\right] \quad (26)$$

Where

$$A_{mn} = (c_4 * a^4) * (d_5 * b^5) \quad (27)$$

Thus procedurally, the orthogonal polynomial displacement functions, w(x,y) for twenty-one known boundary conditions of rectangular plates were derived and presented in Table A.1 of Appendix A.

3.0 Variational Functional in Ritz Method

Under free vibrations, the linear equation of equilibrium of motion for thin, isotropic and homogeneous rectangular plate was given by Leissa and Qatu (2011) as:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{\rho t \omega^2}{D} w = 0 \quad (28)$$

The variational statement (Weak-form) for Equation (28) is:

$$0 = \int_0^a \int_0^b V(x,y)\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - KM\omega^2 w\right] dxdy \quad (29)$$

Where

V(x,y) = weight function

$$K = \frac{1}{D} \quad (30)$$

$$M = \rho * t \quad (31)$$

Then by performing integration by parts on Equation (29) to trade differentiation from w(x,y) to V(x,y), the functional in Ritz method is:

$$0 = \int_0^a \int_0^b \left[\frac{\partial^2 V}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 V}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 V}{\partial y^2}\frac{\partial^2 w}{\partial y^2} - \omega^2 MKVw\right] dxdy \quad (32)$$

Where

V = V(x,y

4.0 Linearly Fundamental Frequency

For first mode of vibration of rectangular plate, the variables m and n contained in the expression for orthogonal polynomial displacement functions are individually equal to unity; and V equals w, hence Equation (32) becomes:

$$0 = \int_0^a \int_0^b \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 - \omega^2 MKw^2\right] dxdy \quad (33)$$

That is

$$\omega^2 MK \int_0^a \int_0^b w^2 dxdy = \int_0^a \int_0^b \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] dxdy \quad (34)$$

Thus by substituting for the orthogonal polynomial displacement functions of any set of boundary conditions as required by Equation (34); and then performing direct integration as required, the unknown frequency parameter in ω is solved for.

5.0 Numerical Examples

In linearly dynamic analysis of rectangular plates, the rigidity characteristics are considered to be constant for all small deflections (Volmir, 1974). Therefore the amplitude of displacement, A_{mn} is taken to be equal to unity without loss in generality.

The plate parameters adopted for the numerical examples are:

$E = 10.92 * 10^6 Nm^{-2};\ \rho = 100Kgm^{-3};\ \mu = 0.3;\ a = b = 1.0m;\ t = 0.01m$

By performing direct integration term by term on Equation (34) we have:

$$\int_0^a \int_0^b \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dxdy = u_1 \tag{35}$$

$$\int_0^a \int_0^b \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 dxdy = u_2 \tag{36}$$

$$\int_0^a \int_0^b \left(\frac{\partial^2 w}{\partial y^2}\right)^2 dxdy = u_3 \tag{37}$$

And

$$T_1 = u_1 + 2 * u_2 + u_3 \tag{38}$$

$$\int_0^a \int_0^b w^2 dxdy = J_1 \tag{39}$$

Then

$$\omega^2 MKJ_1 = T_1 \tag{40}$$

That is:

$$\omega^2 = \frac{1}{MK}\frac{T_1}{J_1} \tag{41}$$

Therefore

$$\omega_{11} = \sqrt{\frac{T_1}{J_1}} * \sqrt{\frac{D}{M}} \tag{42}$$

Or

$$\omega_{11} = \lambda \sqrt{\frac{D}{M}} \tag{43}$$

Where

λ = nondimensional frequency parameter

ω_{11} = fundamental frequency

$$D = \frac{E * t^3}{12(1 - \mu^2)} \tag{44}$$

The numerical results for the boundary conditions of rectangular plates are presented in Table A.2 of Appendix A.

6.0 Results and Discussion

The orthogonal polynomial displacement functions for rectangular plate with various boundary conditions are presented in Table A.1; and the linearly fundamental frequencies evaluated based on the derived orthogonal polynomial displacement functions for rectangular plate with various boundary conditions are also presented in Table A.2 along side with results from previous work due to Leissa and Qatu (2011). The orthogonal polynomial displacement functions presented in Table A.1 are physical functions which satisfy both the completeness characteristics of good shape functions and homogeneous boundary conditions. From the results presented in Table A.2 with special reference to column 4, the present study's results were all in the upper bound; and the convergence of the results from the derived orthogonal polynomial displacement functions with respect to majority of the boundary conditions was very good; but nevertheless, some showed poor convergent results, which were common among the boundary conditions involving two or more free edge conditions. In this work, the boundary conditions that showed poor convergence include CCFF, SFSF, CSFF, SSFF, CFSF and CFFF rectangular plates. The reason behind the poor convergence may be attributed on one hand to stability of the derived polynomial displacement functions used as shape functions. On the other hand, it may be attributed to the fact that the integral functions (the functionals) defining the rectangular plate problems are not self-adjoint equations; and if they were not, their approximations using the Ritz energy method are stubborn. In fact, the same observation was pointed out in the previous work of Leissa (1969). Despite the odds, the mean percentage difference in the results from the present work and the results from previous works in literature was 12.581, which invariably is statistically acceptable.

7.0 Conclusion

In this paper, the orthogonal polynomial displacement functions for rectangular plate with various boundary conditions have been derived based on static deflection profiles; and the majority of the fundamental frequencies evaluated for various boundary conditions of rectangular plates were in good agreement with the results from previous works in literature. Furthermore, an average percentage difference in the results from the present work and the results from previous works in literature was 12.58, which was statistically acceptable for majority of engineering precisions. Therefore, it is here concluded that the application of orthogonal polynomial displacement functions as shape functions provides satisfactory results in dynamic analysis of rectangular plates.

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Appendix A

Tables of Results

Table A.1 Orthogonal Polynomial Shape Functions for Various Boundary Conditions of Rectangular Plates

Boundary Conditions	Polynomial Shape Functions, w(x,y)
CCCC	$A_{mn}\left[\left(\frac{x}{a}\right)^{m+1}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[\left(\frac{y}{b}\right)^{n+1}-2\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
SSSS	$A_{mn}\left[\left(\frac{x}{a}\right)^{m}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[\left(\frac{y}{b}\right)^{n}-2\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CSCS	$A_{mn}\left[\left(\frac{x}{a}\right)^{m+1}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[\left(\frac{y}{b}\right)^{n}-2\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CSSS	$A_{mn}\left[\frac{3}{2}\left(\frac{x}{a}\right)^{m+1}-\frac{5}{2}\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[\left(\frac{y}{b}\right)^{n}-2\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CCSS	$A_{mn}\left[\frac{3}{2}\left(\frac{x}{a}\right)^{m+1}-\frac{5}{2}\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[\frac{3}{2}\left(\frac{y}{b}\right)^{n+1}-\frac{5}{2}\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CSFS	$A_{mn}\left[6\left(\frac{x}{a}\right)^{m+1}-4\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[\left(\frac{y}{b}\right)^{n}-2\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CCCS	$A_{mn}\left[\left(\frac{x}{a}\right)^{m+1}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[\frac{3}{2}\left(\frac{y}{b}\right)^{n+1}-\frac{5}{2}\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CCCF	$A_{mn}\left[\left(\frac{x}{a}\right)^{m+1}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[6\left(\frac{y}{b}\right)^{n+1}-4\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
SSSF	$A_{mn}\left[\left(\frac{x}{a}\right)^{m}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[8\left(\frac{y}{b}\right)^{n}-4\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CCSF	$A_{mn}\left[\frac{3}{2}\left(\frac{x}{a}\right)^{m+1}-\frac{5}{2}\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[6\left(\frac{y}{b}\right)^{n+1}-4\left(\frac{x}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CSSF	$A_{mn}\left[\frac{3}{2}\left(\frac{x}{a}\right)^{m+1}-\frac{5}{2}\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[8\left(\frac{y}{b}\right)^{n}-4\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$

CSCF	$A_{mn}\left[\left(\frac{x}{a}\right)^{m+1}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[8\left(\frac{y}{b}\right)^{n}-4\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CCFF	$A_{mn}\left[6\left(\frac{x}{a}\right)^{m+1}-4\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[6\left(\frac{y}{b}\right)^{n+1}-4\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CFCF	$A_{mn}\left[\left(\frac{x}{a}\right)^{m+1}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[5\left(\frac{y}{b}\right)^{n}-\frac{5}{2}\left(\frac{y}{b}\right)^{n+3}+\left(\frac{y}{b}\right)^{n+4}\right]$
SFSF	$A_{mn}\left[\left(\frac{x}{a}\right)^{m}-2\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[5\left(\frac{y}{b}\right)^{n}-\frac{5}{2}\left(\frac{y}{b}\right)^{n+3}+\left(\frac{y}{b}\right)^{n+4}\right]$

Table A.1 Orthogonal Polynomial Shape Functions for Various Boundary Conditions of Rectangular Plates, (Cont'd).

CSFF	$A_{mn}\left[6\left(\frac{x}{a}\right)^{m+1}-4\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[8\left(\frac{y}{b}\right)^{n}-4\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
SSFF	$A_{mn}\left[8\left(\frac{x}{a}\right)^{m}-4\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[8\left(\frac{y}{b}\right)^{n}-4\left(\frac{y}{b}\right)^{n+2}+\left(\frac{y}{b}\right)^{n+3}\right]$
CFSF	$A_{mn}\left[\frac{3}{2}\left(\frac{x}{a}\right)^{m+1}-\frac{5}{2}\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[5\left(\frac{y}{b}\right)^{n}-\frac{5}{2}\left(\frac{y}{b}\right)^{n+3}+\left(\frac{y}{b}\right)^{n+4}\right]$
CFFF	$A_{mn}\left[6\left(\frac{x}{a}\right)^{m+1}-4\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[5\left(\frac{y}{b}\right)^{n}-\frac{5}{2}\left(\frac{y}{b}\right)^{n+3}+\left(\frac{y}{b}\right)^{n+4}\right]$
SFFF	$A_{mn}\left[8\left(\frac{x}{a}\right)^{m}-4\left(\frac{x}{a}\right)^{m+2}+\left(\frac{x}{a}\right)^{m+3}\right]\left[5\left(\frac{y}{b}\right)^{n}-\frac{5}{2}\left(\frac{y}{b}\right)^{n+3}+\left(\frac{y}{b}\right)^{n+4}\right]$
FFFF	$A_{mn}\left[5\left(\frac{x}{a}\right)^{m}-\frac{5}{2}\left(\frac{x}{a}\right)^{m+3}+\left(\frac{x}{a}\right)^{m+4}\right]\left[5\left(\frac{y}{b}\right)^{n}-\frac{5}{2}\left(\frac{y}{b}\right)^{n+3}+\left(\frac{y}{b}\right)^{n+4}\right]$

Table A2 Linearly Fundamental Frequencies of Rectangular Plate with various Boundary Conditions

Boundary Conditions	Present Work $[rad/s]$	Leissa and Qatu (2011) $[rad/s]$	Percentage Difference [%]
CCCC	36.0000	35.99	-0.0278
SSSS	19.7476	19.7392	-0.043
CSCS	28.9560	28.9509	-0.018
CSSS	23.6795	23.6463	-0.140
CCSS	27.1285	27.06	-0.253
CSFS	13.7213	12.6874	-8.149
CCCS	31.8681	31.83	-0.120
CCCF	24.6475	24.02	-2.612
SSSF	12.3435	11.6845	-5.640

CCSF	18.3969	17.62	-4.409
CSSF	17.347	16.86	-2.893
CSCF	23.8605	23.46	-1.707
CCFF	8.0335	6.942	-15.723
CFCF	23.8695	22.270	-7.182
SFSF	12.3595	9.6314	-28.325
CSFF	6.3658	5.364	-18.676
SSFF	4.9369	3.369	-46.539
CFSF	17.3598	15.280	-13.611
CFFF	6.3892 3.8863†	3.492	-82.967 -11.293†

†Result evaluated by using functional in Galerkin's method.