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Improving the spectral efficiency of an uplink 5G massive MIMO system using convex optimization approach

[1]Chiamaka B. Nwabanne, [2]Obinna S. Oguejiofor, [3]Scholastica U. Nnebe
[1,2, 3]Department of Electronic and Computer Engineering
[*]Corresponding Author's E-mail: cb.nwabanne@unizik.edu.ng

Abstract
5G deployment faces challenges in achieving high spectral efficiency and managing interference. This paper proposes an optimal beamforming approach for improving the spectral efficiency of an uplink 5G massive MIMO system, considering a quality of service (QoS) constraint. The selection of coordinated beamformers that maximized the weighted sum SE of the system, while fulfilling the QoS constraint, was formulated as a non-convex optimization problem, and reformulated to a convex problem by fixing the interference and noise term to a known constant for all UEs, and solved using CVX. Numerical simulations show that the proposed design is optimal and there is improvement in spectral efficiency when either the number of base stations receive antennas or total number of user equipment increases, or both.

Keywords: beamforming, convex optimization, massive MIMO, spectral efficiency

1. Introduction

The fifth-generation (5G) cellular networks have been developed to meet the increasing demand for high-speed wireless communication. One of the key technologies for achieving the high data rate and low latency requirements of 5G is Massive Multiple Input Multiple Output (Massive MIMO) systems (Marzetta, 2016). Massive MIMO systems use a large number of antennas at the base station to serve multiple users simultaneously, resulting in significant improvements in spectral efficiency and energy efficiency. However, the uplink spectral efficiency of a single cell Massive MIMO system is limited by the inter-user interference (IUI) caused by the transmission of multiple users at the same time and frequency, under space division multiple access scheme (Ngo, 2014). Several techniques have been proposed to reduce the IUI in the uplink, including scheduling algorithms (Ngo, 2017) and precoding methods (Ngo, 2014). However, these methods are often complex and require significant computational resources.

Recently, beamforming and convex optimization techniques have been applied to improve the uplink spectral efficiency of Massive MIMO systems. In "Beamforming for Massive MIMO Systems: A Convex Optimization Approach," (Mehanna, Alkhateeb, & Leus, 2015) proposed a beamforming design for a massive MIMO system that is based on a convex optimization problem. The aim of the work was to maximize the spectral efficiency of the system while taking into account the constraints imposed by the available channel state information and the limited power budget. The authors used simulations to show that their proposed method significantly improves the spectral efficiency compared to other existing methods. The article, "A Low-Complexity Convex Optimization-Based Beamforming Design for Massive MIMO Systems," by (Liu, Qin, & Vucetic, 2016) presented a low-complexity beamforming design for a massive MIMO system that is based on a convex optimization problem. The authors proposed a method to approximate the original problem by a sequence of second-order cone programming (SOCP) sub-problems. The aim of the work was to develop a beamforming design that is both effective and computationally efficient. The authors used simulations to demonstrate that their proposed method outperforms other existing methods.

"A Convex Optimization Framework for Uplink Power Control and Beamforming in Massive MIMO Systems," by (Jafar, Swindlehurst, & Heath, 2017) presented a convex optimization framework for designing the uplink power control and beamforming in a massive MIMO system. The authors proposed a method to jointly optimize the power control and beamforming vectors by solving a convex optimization problem. The authors also proposed an algorithm to solve this problem efficiently and provided the simulation results to show the performance of the proposed method. The article, "Beamforming Design for Massive MIMO Uplink Systems Using Second-Order Cone Programming," by (Zhag, Dai, & Wang, 2018) presented a beamforming design for a massive MIMO uplink system that utilizes second-order cone programming (SOCP) optimization. The authors aimed to maximize the system sum-rate by optimizing the beamforming vectors and power allocation subject to the per-antenna power constraints. The simulation results showed that the proposed method outperforms existing suboptimal algorithms and it was also close to the performance of the optimal algorithm, which has a high computational complexity.

(Beezer & Kirubarajan, 2016) in "Beamforming in Wireless Communication," presented a detailed overview of different beamforming techniques in wireless communications, their advantages and disadvantages, and discussed their various design considerations and trade-offs. (Liu, Li, & Chen, 2018) in "Maximizing Spectral Efficiency of Mobile Communication Systems via Antenna Array: A Rayleigh Quotient Approach," presented a method to optimize the spectral efficiency of mobile communication systems by maximizing the SNR of the received signal. The authors proposed an optimization method based on RQ to optimize the array configuration and beamforming weights of a multi-antenna system. They showed, analytically, that their method can achieve significant improvement in spectral efficiency compared to other methods, and provided numerical results and simulations to validate their methods and demonstrate its effectiveness.

These articles have shown that convex optimization can be used to effectively and efficiently design the beamforming vectors in a massive MIMO system, particularly in the context of 5G uplink systems, with the aim of maximizing the spectral efficiency while taking into account various constraints. Additionally, these works have proposed a number of different methods for solving the convex optimization problem, ranging from low-complexity methods to more general frameworks, and indicate that convex optimization techniques are promising approach to design the beamforming vectors

The aim of this study is to improve the spectral efficiency of a 5G Massive MIMO single cell uplink system by applying convex optimization techniques in the design of beamforming vectors. This study will focus on developing a convex optimization-based technique to reduce IUI in the uplink and improve the spectral efficiency of the system. This work, also, argues that using numerical simulations, similar improvements in spectral efficiency can be achieved as the analytical Rayleigh Quotient method (RQM).

2.0 Material and methods

2.1 Materials

The materials (software) used to actualize this work include CVX, SeDumi and MatLaB. CVX is a modeling system for constructing and solving discipline convex programs. It supports a number of standard problem types, including linear and quadratic programs (LPs/QPs), Second-Order Cone programs (SOCPs), and semidefinite programs (SDPs). CVX can, also, be used to solve much more complex convex optimization problems, including non-differentiable functions, such as l1 norms. In this work, CVX is used to conveniently formulate and solve constrained discipline convex optimization problem. CVX is implemented in MatLab, effectively turning MatLab into an optimization modeling language. SeDumi is one of the solvers CVX uses in solving optimization problems.

2.2 Method: System Model

Here, a simplified system model comprising of only a single cell but with uplink transmission is considered. A system having M active UEs at different locations, and only two transmitting at the same time and frequency to the receiver, a BS, that has an antenna array of N antenna elements is considered, at the time considered. The system model of this scenario is shown in Figure 1 below.

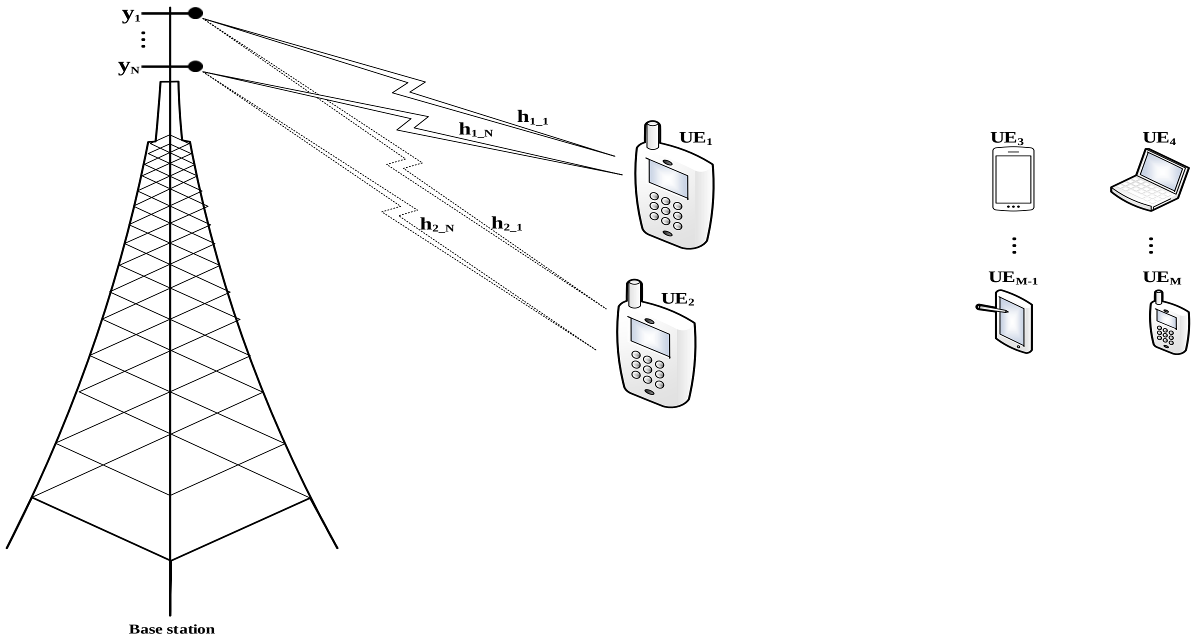

Figure 1: System model of a single cell uplink system with *M* active UEs and only two UEs transmitting to a BS antenna array of *N* antenna elements at a particular time.

The channel modelling for the above case is: h_{1_1} represents the channel from transmitting UE$_1$ to antenna element 1, and continues in this manner until h_{1_N}, which represents the channel from transmitting UE$_1$ to antenna element N; while h_{2_1} represents the channel from transmitting UE$_2$ to antenna element 1, and continues in this manner until h_{2_N}, which represents the channel from transmitting UE$_2$ to antenna element *N*.

For the case under consideration, the desired UE is UE$_1$, while the undesired are the other UEs, at the time considered. The thick lines represent strong carriers, while the dotted lines represent strong interferences, for this uplink transmission. The data signal from transmitting UE$_1$ is denoted as s_1 while the data signal from transmitting UE$_2$ is denoted as s_2. The noise in each channel is denoted as n_1, n_2, …, n_N, and the data received at each antenna element is denoted as y_1, y_2, …, y_N respectively, and are expressed thus:

$$y_1 = h_{1\_1}s_1 + h_{2\_1}s_2 + n_1 \quad (1)$$

$$y_2 = h_{1\_2}s_1 + h_{2\_2}s_2 + n_2 \quad (2)$$

$$\vdots$$

$$y_N = h_{1\_N}s_1 + h_{2\_N}s_2 + n_N \quad (3)$$

The canonical form representation for this uplink transmission is given as

$$\boldsymbol{y} = \sum_{i=1}^{2} \boldsymbol{h_i} s_i + \boldsymbol{n} \quad (4)$$

The receiver can apply beamforming weights $w_1, w_2, \dots, w_N$ (because it has multiple antennas) such that the signal that contains only s_1 and not s_2, which means only the signal from UE$_{1,}$ is received, while the signal from UE$_2$ is suppressed. This implies that the BS receives from all active UEs, both wanted and unwanted signals including noise. But if the BS is interested in a particular user only, this can be modeled as shown below.

The signal the BS receives from *k^th^* UE, the desired UE, is given as:

$$\boldsymbol{y}_k = \boldsymbol{h}_k s_k + \sum_{i \neq k} \boldsymbol{h}_i s_i + \boldsymbol{n}_k \quad (5)$$

Where $\boldsymbol{h}_k \in \mathbb{C}^N, and\ \boldsymbol{y}_k \in \mathbb{C}^N$ are vector quantities. The signal post processing performed by the base station to separate the desired signal from the unwanted signal is usually done by the inner product of each term of Equation 5, with the conjugate of an uplink beamforming vector $\boldsymbol{w}_k \in \mathbb{C}^N$. This will yield a new representation of equation as shown below.

$$\check{y}_k = \boldsymbol{w}_k^H \boldsymbol{y}_k = \boldsymbol{w}_k^H (\boldsymbol{h}_k s_k + \sum_{i \neq k} \boldsymbol{h}_i s_i + \boldsymbol{n}_k) \qquad (6)$$

Opening the brackets of Equation 6 will give:

$$\check{y}_k = \boldsymbol{w}_k^H \boldsymbol{h}_k s_k + \sum_{i \neq k} \boldsymbol{w}_k^H \boldsymbol{h}_i s_i + \boldsymbol{w}_k^H \boldsymbol{n}_k \qquad (7)$$

The received noise is often modeled as $\boldsymbol{n}_k \sim \mathcal{N}(0, \sigma_k^2 \boldsymbol{I})$, where the covariance matrix $\sigma_k^2 \boldsymbol{I}$ has the size N by N. The number N is obtained from the total number of antennas present at the base station. Splitting the uplink beamforming vectors ($\boldsymbol{w}_k$) in terms of their uplink receiver powers and beamforming directions, $\boldsymbol{w}_k$ can be represented as $\boldsymbol{w}_k = \sqrt{q_k} \widetilde{\boldsymbol{w}}_k$.

The signal-to-interference-and-noise ratio (SINR) of the k^th UE can, then, be represented as:

$$SINR_k = \frac{q_k |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_k|^2}{\sum_{i \neq k} q_i |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_i|^2 + |\widetilde{\boldsymbol{w}}_k^H \sigma_k^2 \boldsymbol{I}|^2} = \frac{q_k |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_k|^2}{\sum_{i \neq k} q_i |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_i|^2 + \widetilde{\boldsymbol{w}}_k^H \sigma_k^2 \boldsymbol{I} \widetilde{\boldsymbol{w}}_k} \qquad (8)$$

Where $\boldsymbol{s}_k$ is normalized to unit power and modeled as $\boldsymbol{s}_k \sim \mathcal{N}(0, 1)$. Also, the following terms: $q_k |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_k|^2$, $\sum_{i \neq k} q_i |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_i|^2,\ and\ \widetilde{\boldsymbol{w}}_k^H \sigma_k^2 \boldsymbol{I} \widetilde{\boldsymbol{w}}_k$ represent the received desired signal power, the sum of all interfering powers and the noise power respectively.

Simplifying further by dividing each term by q_k, Equation 8 can be represented as:

$$SINR_k = \frac{|\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_k|^2}{\sum_{i \neq k} \frac{q_i}{q_k} |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_i|^2 + \widetilde{\boldsymbol{w}}_k^H \frac{\sigma_k^2}{q_k} \boldsymbol{I} \widetilde{\boldsymbol{w}}_k} \qquad (9)$$

Further expansion of Equation 9 will make it to be represented as:

$$SINR_k = \frac{\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_k \boldsymbol{h}_k^H \widetilde{\boldsymbol{w}}_k}{\widetilde{\boldsymbol{w}}_k^H \left(\frac{1}{q_k} \sum_{i \neq k} q_i \, \boldsymbol{h}_i \boldsymbol{h}_i^H + \frac{\sigma_k^2}{q_k} \boldsymbol{I} \right) \widetilde{\boldsymbol{w}}_k} \qquad (10)$$

$SINR_k$ can be maximized by taking derivative of Equation 10 with respect to $\widetilde{\boldsymbol{w}}_k$, to give Equation 11.

$$\boldsymbol{h}_k \boldsymbol{h}_k^H \widetilde{\boldsymbol{w}}_k = SINR_k \left(\frac{1}{q_k} \sum_{i \neq k} q_i \, \boldsymbol{h}_i \boldsymbol{h}_i^H + \frac{\sigma_k^2}{q_k} \boldsymbol{I} \right) \widetilde{\boldsymbol{w}}_k \qquad (11)$$

Dividing both sides of Equation 11 by $\left(\frac{1}{q_k} \sum_{i \neq k} q_i \, \boldsymbol{h}_i \boldsymbol{h}_i^H + \frac{\sigma_k^2}{q_k} \boldsymbol{I} \right)$ will yield a new equation.

$$\left(\frac{1}{q_k} \sum_{i \neq k} q_i \, \boldsymbol{h}_i \boldsymbol{h}_i^H + \frac{\sigma_k^2}{q_k} \boldsymbol{I} \right)^{-1} \times \boldsymbol{h}_k \boldsymbol{h}_k^H \widetilde{\boldsymbol{w}}_k = SINR_k \times \widetilde{\boldsymbol{w}}_k \qquad (12)$$

Equation 12 is an eigenvector equation with $SINR_k$ being the Eigen values. The Eigen vector associated with the largest Eigen value is the optimum beamforming vector $\widetilde{\boldsymbol{w}}_k^{opt}$, applying the generalized Rayleigh quotient formula (Beezer R. , 2016). Equation 13 was obtained using this popular traditional method, which is an analytical version of obtaining the optimal beamformers needed to maximize the spectral efficiency of the 5G system. However, using numerical method, this thesis provides an alternative way to solve the same problem.

$$\widetilde{\boldsymbol{w}}_k^{opt} = \frac{\left(\frac{1}{q_k} \sum_{i \neq k} q_i \boldsymbol{h}_i \boldsymbol{h}_i^H + \frac{\sigma_k^2}{q_k} \boldsymbol{I} \right)^{-1} \times \boldsymbol{h}_k}{\left\| \left(\frac{1}{q_k} \sum_{i \neq k} q_i \boldsymbol{h}_i \boldsymbol{h}_i^H + \frac{\sigma_k^2}{q_k} \boldsymbol{I} \right)^{-1} \times \boldsymbol{h}_k \right\|} \qquad (13)$$

The optimal spectral efficiency for an uplink single cell 5G system can only be achieved using the optimum beamforming vectors as obtained in Equation 14. The optimal spectral efficiency of each UE in this system can, then, be modelled as:

$$SE_k = \log_2(1 + SINR_k) bits/s/Hz \qquad (14)$$

2.3 Optimization Problem Formulation and Solution

The goal of this optimization is to select $\{\boldsymbol{w}_k\} \forall k = 1, \ldots, M,$ that will maximize the weighted sum SE of this system, while fulfilling the quality of service (QoS) constraint for all UEs. This optimization problem is, therefore, formulated as

$$\underset{\{w_k\}\forall k}{\text{maximize}} \sum_{k=1}^{M} u_k SE_k$$

$$\text{Subject to} \quad SINR_k \geq \gamma_k \qquad (15)$$

Where the utility function represents the weighted sum spectral efficiency of the system, with u_k denoting a positive weight assigned to individual UEs, chosen to reflect varying level of the individual channel gain. The constraint represents the desired quality of service constraint, upper bounded by a threshold denoted as γ_k for UE k.

Maximizing the weighted sum spectral efficiency of a system under a given constraint is generally regarded as a non-convex, non-polynomial hard optimization problem because it is very difficult to get an efficient algorithm that could solve it in polynomial time. In order to find the actual cause of non-convexity of the optimization problem, analysis of the utility function and constraint function in Equation 15 are carried out. Firstly, the utility function represents a concave function which is being maximized, though it depends on the SINRs of the UEs in the system. The SINR constraint function is a non-convex function of beamforming vectors $\{\boldsymbol{w}_k\}_{k=1}^{M}$, because it cannot be classified as a second-order cone constraint or a semi-definite constraint. The following steps are taking to make the constraint function convex.

First, Equation 15 is rewritten as follows:

$$\underset{\{w_k\}\forall k}{\text{maximize}} \sum_{k=1}^{M} u_k SE_k$$

$$\text{Subject to} \quad |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_k|^2 \geq \gamma_k(\mathrm{T}_k) \quad \forall_k \qquad (16)$$

Where $\mathrm{T}_k = \sum_{i \neq k} \frac{q_i}{q_k} |\widetilde{\boldsymbol{w}}_k^H \boldsymbol{h}_i|^2 + \widetilde{\boldsymbol{w}}_k^H \frac{\sigma_k^2}{q_k} \boldsymbol{I} \widetilde{\boldsymbol{w}}_k$

Considering discipline convex optimization, the multiplication of T_k, which is not a constant to the SINR threshold, makes it non-convex. We resolved this by fixing the interference and noise term, T_k, to a known constant for all *k*. This disciplined convex optimization problem can now be solved using CVX. The set of uplink beamformers obtained from this optimization is optimal, and will be shown in the results that the proposed set of beamformers obtained through this method is almost the same as the one obtained through the Rayleigh Quotient Method.

2.4 Numerical Simulation

The deployment scenario considered is termed Urban-Macro for 5G New Radio. The carrier frequency adopted is around 4GHz (C-Band) with a 100MHz bandwidth. A simple simulation setting, is considered, with a minimum of ten UEs in the coverage area of a MBS. These UEs are served by the MBS and are uniformly distributed between 210m and 260m from the MBS. Other system parameters are also based on the 3GPP simulation parameters and can be found in (3GPP, 2022). The total BS transmit powers for MBS is 46dBm and the UE transmit power is 23dBm. The simulation setting will be used except otherwise indicated.

3.0 Results and Discussions

The following results were obtained by running the Matlab codes generated based on the above simulation settings. For uplink transmission, *N* = Number of Base station antenna element, and *M* = total number of UEs in the cell. In the single cell 5G uplink transmission, the proposed method which provides an alternative numerical solution to the analytical solution by Rayleigh Quotient Method (RQM) were compared to show that both schemes are optimal.

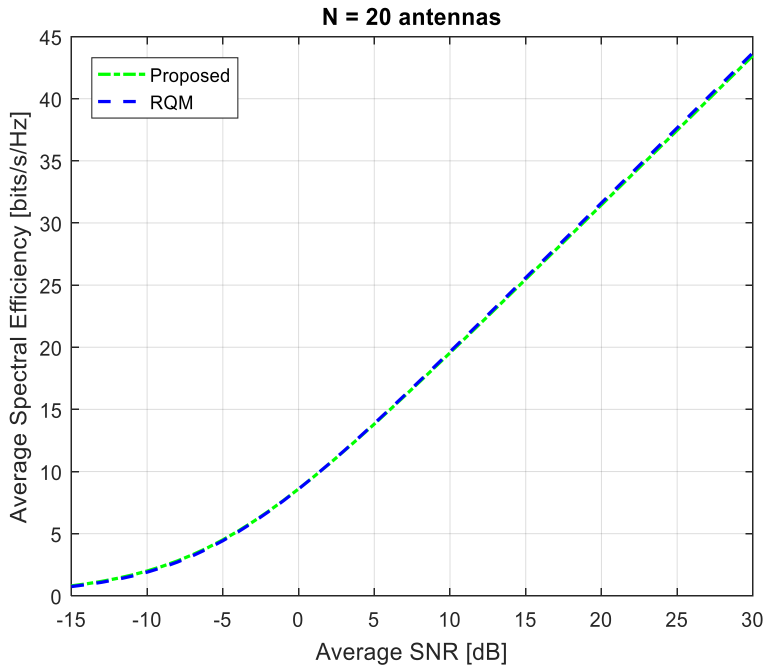

Figure 1: Average spectral efficiencies achievable at different SNR for *N* = 20, *M* = 4

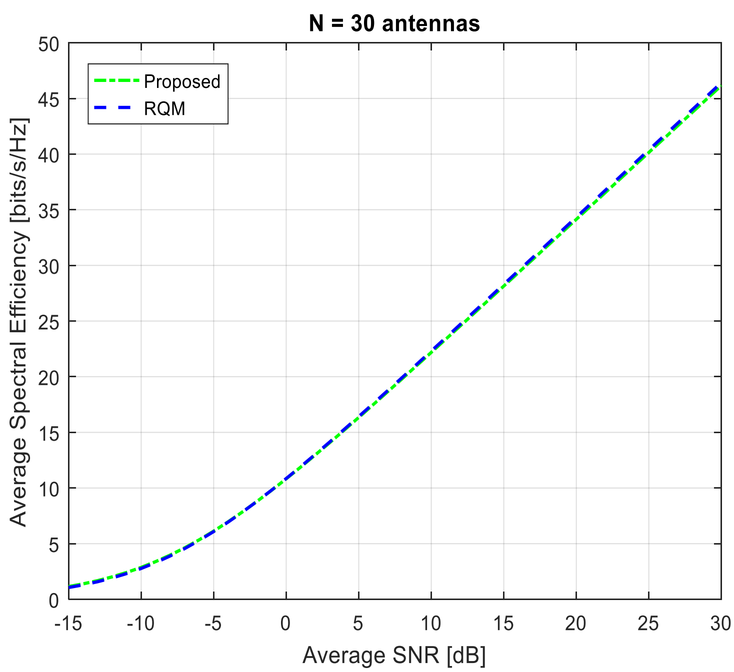

Figure 2: Average Spectral efficiencies achievable at different SNR for *N* = 30, *M* = 4

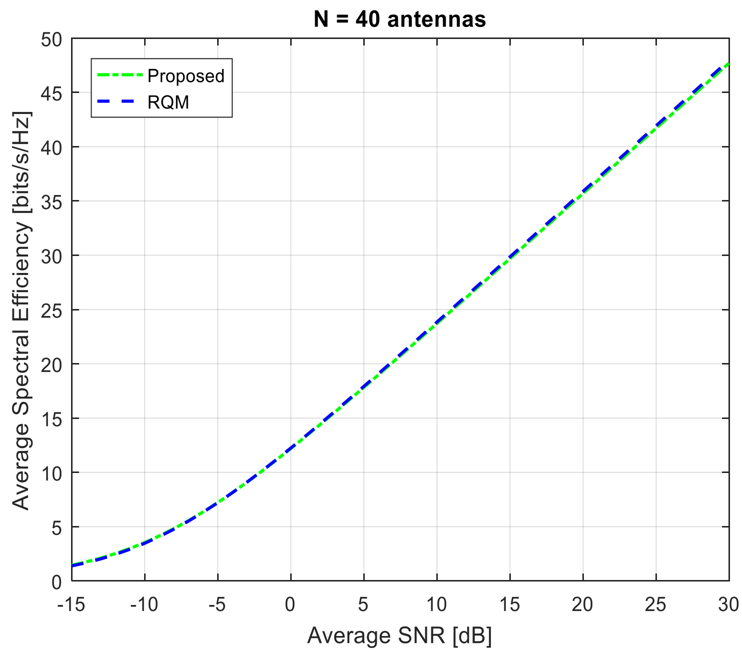

Figure 3: Average Spectral efficiencies achievable at different SNR for *N* = 40, *M* = 4

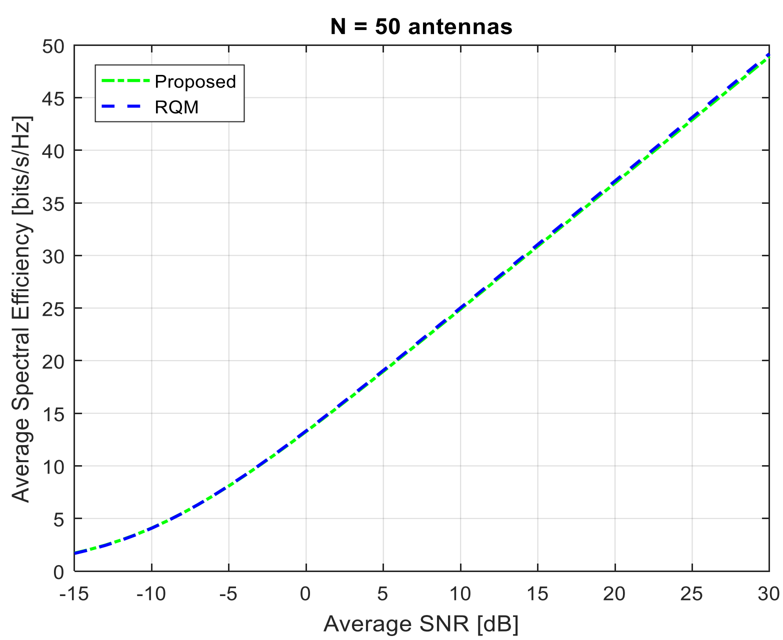

Figure 4: Average Spectral efficiencies achievable at different SNR for *N*=50, *M*=4

In Figure. 1, the average cell spectral efficiency achievable at SNR = 30dB, N = 20 and M = 4 is approximately 44 bits/s/Hz for both the proposed and the Rayleigh Quotient Method (RQM), which is an analytical method used to obtain the optimal beamformers in the uplink of a cell. This, also, is proof that the method proposed in this work is optimal, since similar values were gotten by the proposed method and the RQM, despite the proposed method being a numerical method.

Using the same parameters (M = 4, and SNR = 30dB), Figures 2, 3 and 4 respectively show that slight improvements were achieved in the cell spectral efficiency (46bits/s/Hz, 47.5bits/s/Hz and 49bits/s/Hz) when the base station receive antennas, N, was increased. This, also, shows that for a meaningful improvement in the spectral efficiency of a cell, the UE component needs to increase as well as the base station receive antennas, as illustrated in Figure 5 and Figure 6.

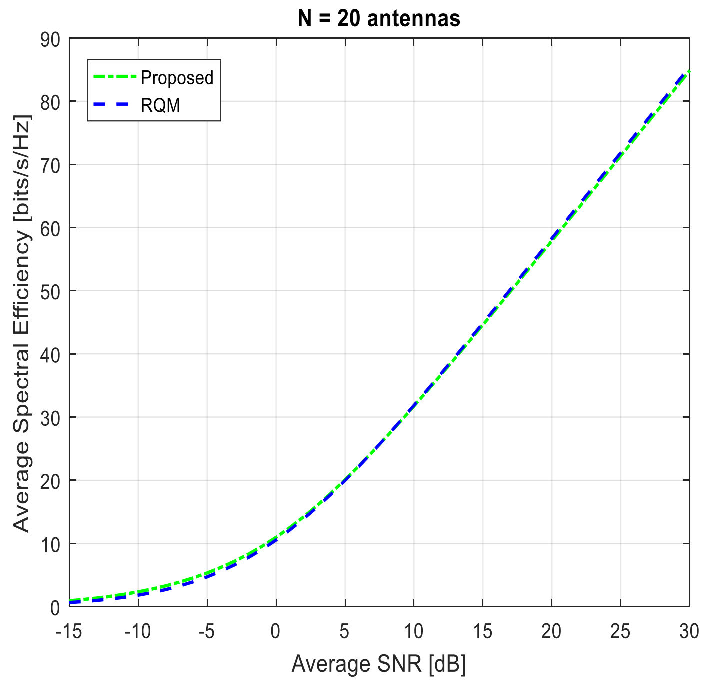

Figure 5: Average Spectral efficiencies achievable at different SNR for *N* = 20, *M* = 9

In Figure 5, the average cell spectral efficiency is improved to 85bits/s/Hz based on the following parameters (*SNR* = 30 dB, *N* =20, *M* = 9). When compared to Figure 1 which has similar parameters with exception in the total number of UEs in the cell (*M*), one can see that the difference in the spectral efficiency achieved in both figures at SNR = 30dB is 41 bits/s/Hz.

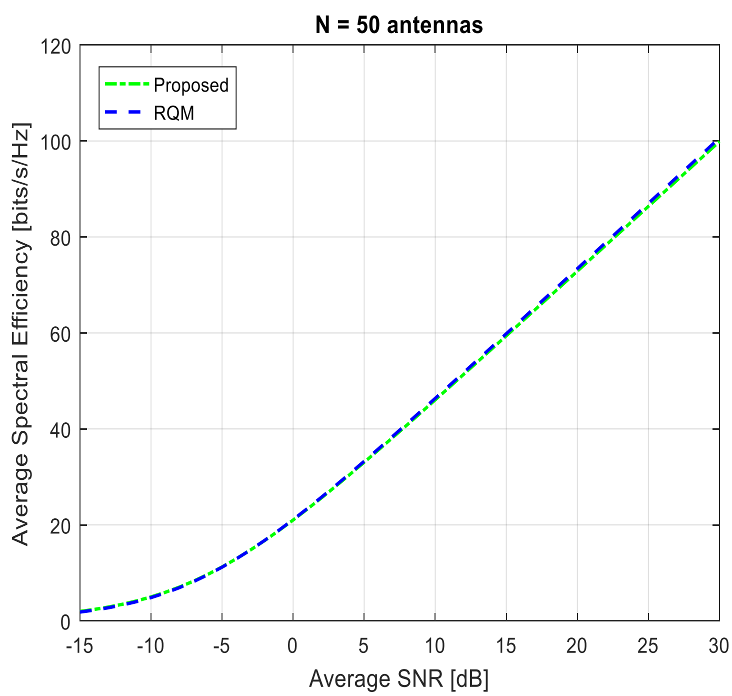

Figure 6: Average Spectral efficiencies achievable at different SNR for *N* = 50, *M* = 9

In Figure 6, the average cell spectral efficiency achievable at SNR = 30dB is improved to 100bits/s/Hz. When compared to Figure 4 at the same SNR = 30dB, having the same number of receive antenna elements, but lesser number of UEs, one can see that the spectral efficiency achieved for Figure 4 although 49bits/s/Hz, is less than that obtained in Figure 6. So, the gain in spectral efficiency when comparing Figure 6 to Figure 4 stood at 51 bits/s/Hz.

The simulation results presented in this research demonstrate the effectiveness of the proposed method in improving the spectral efficiency of a single cell 5G uplink system. The proposed method, which provides an alternative numerical solution to the analytical solution by Rayleigh Quotient Method (RQM), was found to be optimal in achieving similar results as the RQM. This indicates that the proposed method can be used as a reliable and efficient method for obtaining optimal beamformers in the uplink of a cell.

The results, also, show that increasing the number of BS receive antennas can lead to an improvement in spectral efficiency. Research in the field of 5G uplink systems also confirms that increasing the number of antennas in base stations improves spectral efficiency (Gao, 2019); (Khan, 2019); (Li, 2019). These works agree with the simulation results that increasing the number of BS receive antennas can lead to an improvement in spectral efficiency. But it will not be a significant improvement unless the number of UEs is also increased. This is consistent with previous research that suggests that increasing the number of antennas in base stations can improve the spectral efficiency of a 5G uplink system. This highlights the importance of considering the number of UEs in the design of 5G uplink systems, as it can have a significant impact on the overall performance of the system. Research has also highlighted the importance of considering the number of UEs in the design of 5G uplink systems, as it can have a significant impact on the overall performance of the system (Jiang, 2018); (Yuan, Zhang, Chen, & Chen, 2018). These works agree with the simulation results that increasing the number of UEs in the cell can lead to a significant improvement in spectral efficiency.

4.0. Conclusion

In this paper, a new beamforming design approach for a 5G Massive MIMO single cell uplink system based on convex optimization that maximize the spectral efficiency of a 5G Massive MIMO system, while fulfilling a QoS constraint, was proposed and developed. The results obtained show that, one, the proposed method is optimal, two, slight improvement in the cell spectral efficiency is obtained when the number of base station receive antennas is increased while meaningful improvement in the cell spectral efficiency is obtained when the total number of UEs in the cell increases as well, and three, there is significant difference in achievable spectral efficiency between two cells having the same SNR and number of BS receive antennas, but different total number of UEs in their cells, with the cell having more users achieving greater spectral efficiency.

5.0 Recommendation

It would be useful to examine the potential for integrating the convex optimization approach with other optimization techniques, such as game theory or control theory, to further improve the performance of 5G Massive MIMO systems.

Acknowledgements

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Nomenclatures

[$(\cdot)^H$ is the transpose conjugate operation, $(\cdot)^T$ is the transpose operation, $\|\cdot\|_2$ is the Euclidean norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $\mathbb{R}^n$ denotes the set of real n-vectors. $\mathbb{C}$ denotes the set of complex numbers, while $\mathbb{C}^n$ denotes the set of complex n-vectors. Uppercase boldface letters are used to represent matrices while lower-case boldface for vectors.]

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