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Optimization and Prediction of Heat Affected Zone Width of Mild Steel Weldment using Response surface methodology

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Abstract
The optimization of welding process parameters to simultaneously control Heat Affected Zone (HAZ) width in mild steel weldments remains a persistent challenge due to complex, nonlinear thermal-metallurgical interactions that are inadequately captured by conventional empirical modeling approaches. This study aimed to develop and validate Response Surface Methodology (RSM) for optimizing Gas Metal Arc Welding (GMAW) of mild steel to enhance prediction accuracy and multi-response optimization capability. The primary objectives were to quantify the influence of welding current, voltage, and travel speed on weld HAZ width,
A Central Composite Design comprising twenty experimental runs was employed to systematically investigate the effects of three input factors: welding current (180–210 A), voltage (22–25 V), and weld speed (2.0–3.5 mm/s). Quadratic polynomial models were developed using Response Surface Methodology in Design-Expert software. Multi-objective optimization was performed using the desirability function approach within RSM. All optimized parameter combinations were experimentally validated through confirmation runs, with statistical diagnostics including the coefficient of determination (R^2), adjusted R^2 (94.52%), predicted R^2, lack-of-fit tests, and residual analysis used to assess model adequacy and predictive reliability.
Keywords: current, voltage, weld speed, weldment, width, polynomial.

1. Introduction

Welding is one of the most widely used manufacturing and fabrication processes in engineering industries due to its effectiveness in joining metallic materials (Sada and Achebo, 2022). Mild steel is widely used in structural, automotive, marine, and construction applications due to its availability, low cost, ductility, and weldability (Etin-osa and Achebo, 2017). However, welding operations often generate excessive heat, which affects the microstructure and mechanical behavior of the welded joint, particularly within the Heat Affected Zone (HAZ) (Sada et al., 2021).
The width of the Heat-Affected Zone significantly influences dimensional shape deviation, residual stresses, distortion, tensile strength, and toughness of welded components (Nweze and Achebo, 2021). Excessive heat input during welding may result in undesirable dimensional inaccuracies and deterioration of weldment performance (Pondi et al., 2021). Therefore, controlling HAZ width and dimensional shape deviation is essential for improving weld quality and structural integrity (Sada and Achebo, 2022).
Traditional optimization methods in welding studies commonly use Response Surface Methodology (RSM) for its ability to establish mathematical relationships between process parameters and responses (Imhansoloeva et al., 2018). Central Composite Design (CCD), a widely used RSM experimental matrix, enables the development of quadratic predictive models for process optimization (Achebo and Odinikuku, 2015).
The Heat-Affected Zone (HAZ) is arguably the most critical region influenced by the welding thermal cycle, as it governs the subsequent mechanical and metallurgical behavior of the entire weldment (Achebo, 2011). The HAZ comprises the portion of the base metal that does not melt during welding but experiences elevated temperatures sufficient to induce microstructural transformations, grain growth, phase changes, and residual stress development. Unlike the fusion zone, which solidifies from the molten state and can be engineered through filler metal selection, the HAZ's properties are determined primarily by the interaction between the welding thermal cycle and the original base metal microstructure (Achebo and Odinikuku, 2015).

2.0 Materials and methods

Figure 1 (a and b) shows the mild steel coupons and the welding machine employed in this investigation were fabricated in strict compliance with ASTM A36/A36M standards for structural steel, ensuring material homogeneity and reproducibility across experimental runs (ASTM International, 2019). Each specimen was precision-cut to dimensions of 150 mm × 50 mm × 6 mm using abrasive water-jet machining to minimize thermal preconditioning of the base metal and thereby preserve the as-received microstructure before welding. Surface preparation involved sequential grinding with 80–240-grit silicon carbide papers, followed by acetone degreasing to remove oxide layers and contaminants that could compromise weld bead consistency or introduce spurious porosity (Kou, 2003). The chemical composition of the mild steel, verified via optical emission spectroscopy, showed carbon content of 0.15–0.25 wt%, manganese of 0.60–1.00 wt%, and balanced iron, parameters critical for ensuring predictable solidification behavior and HAZ evolution during arc welding processes (Messler, 2004

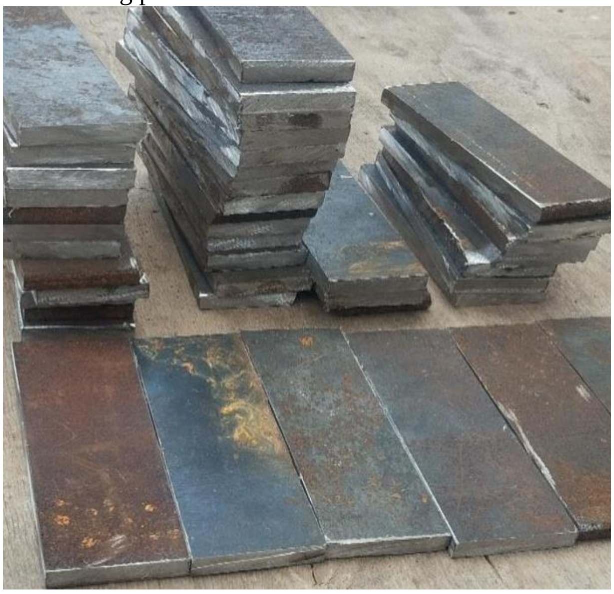


Figure 1a: Mild Steel Coupons Figure 1b: Welding Machine

Table 1 presents the coded and uncoded levels of the three independent welding process variables, current (A), voltage (B), and weld speed (C), selected for systematic investigation based on preliminary screening experiments and extant literature on GMAW of low-carbon steels. The factor ranges (Current: 180–210 A; Voltage: 22–25 V; Weld speed: 2.0–3.5 mm/s).

Table.1: Research Factors

Factor	Name	Units	Type	SubType	Minimum	Maximum	Coded Low	Coded High	Mean	Std. Dev.
A	Current	I	Numeric	Continuous	180.00	210.00	-1 ↔ 180.00	+1 ↔ 210.00	195.00	11.47
B	Voltage	V	Numeric	Continuous	22.00	25.00	-1 ↔ 22.00	+1 ↔ 25.00	23.50	1.15
C	Weld speed	mm/s	Numeric	Continuous	2.00	3.50	-1 ↔ 2.00	+1 ↔ 3.50	2.75	0.5735

2.1.RSM Methods

Central Composite Design (CCD) experimental matrix employed to systematically investigate the effects of welding current (A), voltage (B), and weld speed (C) on heat affected zone width responses. The design comprised twenty experimental runs structured according to a face-centered CCD configuration, incorporating eight factorial points (±1 levels), six axial points (±α, where α = 1.0 for rotatability), and six replicated center points to estimate pure experimental error and assess lack-of-fit (Montgomery, 2017).

3.0 Result and Discussion

Table 2 presents the complete experimental data set obtained from the twenty-run Central Composite Design matrix, documenting the measured responses for each unique combination of welding current (180–210 A), voltage (22–25 V), and weld speed (2.0–3.5 mm/s). The tabulated results reveal pronounced sensitivity of heat affected zone width to variations in the input parameters, with notable trends emerging across the design space: for instance, Run 13 (210 A, 25 V, 2.0 mm/s) yielded the minimum HAZ width (2.68 mm) and the maximum heat input (2021.25 J/mm), attributable to the synergistic effect of high current and voltage coupled with low travel speed, which collectively elevate arc energy density and prolong thermal exposure (Kou, 2003; Messler, 2004).

Table 2: Experimental Results

		Factor 1	Factor 2	Factor 3	Response 1
Std	Run	A:Current	B:Voltage	C:Weld speed	HAZ-W-Pre
		I	V	mm/s	Mm
5	1	190	23	2.5	3.89
8	2	210	22	3.5	3.2
1	3	180	25	3.5	3.72
11	4	190	23	2.5	4.1
17	5	190	23	2.5	4.07
15	6	190	25	2.5	2.73
20	7	210	25	3.5	2.78
7	8	190	23	2.5	4.12
6	9	180	25	2	3.38
9	10	200	23	3	3.72
18	11	210	22	2	4.47
16	12	180	24	3	4.62
14	13	210	25	2	2.68
2	14	200	22	3	3.44
10	15	210	24	3	3.9
19	16	200	24	2	3.85
4	17	200	24	3.5	3.32
12	18	180	22	2	4.63
13	19	180	22	3.5	3.25
3	20	200	24	3	3.53

RSM Results and Analysis

Quadratic models developed using RSM demonstrated satisfactory predictive capability for Heat affected zone width responses.

Heat Affected Zone Width (HAZ-W)

Table 3 presents the sequential model sum of squares analysis used to identify the most appropriate polynomial order for modeling the welding Heat Affected Zone width response. The quadratic model was statistically recommended ("Suggested") based on multiple diagnostic criteria: the sequential p-value (< 0.0001) confirmed that adding quadratic terms significantly improved model fit beyond the two-factor interaction (2FI) formulation, while the lack-of-fit p-value (0.3091) indicated no significant unexplained variation relative to pure experimental error, a critical requirement for model adequacy (Montgomery, 2017). Furthermore, the quadratic model achieved an Adjusted R^2 of 0.9452 and a Predicted R^2 of 0.8210, with their difference (0.1242) falling well below the 0.20 threshold recommended for ensuring model robustness and predictive reliability (Anderson & Whitcomb, 2016).

Table 3: Fit Summary for Heat Affected Zone Width

Source	Sequential p-value	Lack of Fit p-value	Adjusted R^2	Predicted R^2	
Linear	0.0323	0.0105	0.3034	-0.0412	
2FI	0.0387	0.0181	0.5401	0.1740	
Quadratic	< 0.0001	0.3091	0.9452	0.8210	Suggested
Cubic	0.3091		0.9675		Aliased

Table 4 provides complementary statistical metrics that further substantiate the selection of the quadratic model for HAZ-W prediction. The quadratic formulation yielded the lowest standard deviation of residuals (0.1368 mm), indicating superior precision in capturing experimental variability compared to linear (0.4875 mm) or 2FI (0.3961 mm) alternatives. The coefficient of determination ($R^2 = 0.9711$) confirmed that 97.11% of the total variation in HAZ-W was explained by the model, while the close agreement between Adjusted R^2 (0.9452) and Predicted R^2 (0.8210) reinforced the model's generalizability to unseen data points (Box & Draper, 1987). Additionally, the Predicted Residual Error Sum of Squares (PRESS = 1.16) for the quadratic model was substantially lower than competing formulations, demonstrating enhanced cross-validation performance and reduced susceptibility to influential observations (Khuri & Mukhopadhyay, 2010). The cubic model's PRESS statistic remained undefined due to leverage values of 1.0000 at axial points, a common limitation in saturated higher-order designs that precludes reliable external validation.

Table 4: Model Summary Statistics for Heat Affected Zone Width (HAZ-W)

Source	Std. Dev.	R^2	Adjusted R^2	Predicted R^2	PRESS	
Linear	0.4875	0.4134	0.3034	-0.0412	6.75	
2FI	0.3961	0.6853	0.5401	0.1740	5.35	
Quadratic	0.1368	0.9711	0.9452	0.8210	1.16	Suggested
Cubic	0.1054	0.9949	0.9675		*	Aliased

Table 5 presents the Analysis of Variance (ANOVA) for the quadratic Response Surface Methodology model predicting pre-welding Heat Affected Zone width, confirming the statistical significance and explanatory power of the developed regression equation. The overall model F-value of 37.38 ($p < 0.0001$) indicates that the model explains a statistically significant proportion of the response variability relative to random noise, with less than a 0.01% probability that such an F-ratio could arise by chance alone. Individual term analysis reveals that welding voltage (B) exerted the strongest linear effect ($F = 84.92$, $p < 0.0001$), followed by weld speed (C; $F = 43.14$) and current (A; $F = 27.67$), underscoring the dominant influence of arc energy density on thermal diffusion and HAZ formation kinetics (Kou, 2003). Critically, the lack-of-fit test yielded a non-significant F-value of 1.98 ($p = 0.3091$), demonstrating that unexplained variation was attributable to pure experimental error rather than model misspecification, a prerequisite for reliable prediction and optimization (Box & Draper, 1987).

Table 5: ANOVA for Heat Affected Zone Width (HAZ-W)

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	6.30	9	0.6995	37.38	< 0.0001	Significant
A-Current	0.5178	1	0.5178	27.67	0.0004	
B-Voltage	1.59	1	1.59	84.92	< 0.0001	
C-Weld speed	0.8072	1	0.8072	43.14	< 0.0001	
AB	0.3065	1	0.3065	16.38	0.0023	
AC	0.0089	1	0.0089	0.4758	0.5060	
BC	1.13	1	1.13	60.48	< 0.0001	
A^2	0.7978	1	0.7978	42.63	< 0.0001	
B^2	1.42	1	1.42	76.03	< 0.0001	
C^2	0.0180	1	0.0180	0.9630	0.3496	
Residual	0.1871	10	0.0187			
Lack of Fit	0.1538	7	0.0220	1.98	0.3091	not significant
Pure Error	0.0333	3	0.0111			
Cor Total	6.48	19				

The key goodness-of-fit and predictive performance metrics that collectively validate the robustness and practical utility of the quadratic RSM model for HAZ-W. The coefficient of determination ($R^2 = 0.9711$) indicates that 97.11% of the total variability in experimental HAZ width measurements was captured by the model, while the Adjusted R^2 (0.9452) accounts for model complexity and confirms that the inclusion of nine regression terms was statistically justified without over fitting (Anderson & Whitcomb, 2016). The Predicted R^2 of 0.8210, derived via leave-one-out cross-validation using the PRESS statistic, demonstrates strong external predictive capability, with the difference between Adjusted and Predicted R^2 (0.1242) falling well below the 0.20 threshold recommended for ensuring model generalizability (Khuri & Mukhopadhyay, 2010). The low standard deviation of residuals (0.1368 mm) relative to the response mean (3.67 mm) yields a coefficient of variation (C.V. = 3.73%), reflecting high experimental precision and minimal unexplained scatter (Montgomery, 2017). Most notably, the

Equation (1) constitutes the empirically derived second-order polynomial model for predicting the Heat Affected Zone width as a function of the three independent GMAW process parameters expressed in their natural measurement units. The negative linear coefficients for current (−0.874978) and weld speed (−6.61187) suggest that, within the investigated experimental domain, increasing these parameters tends to reduce HAZ width, whereas the positive voltage coefficient (+17.93102) indicates an opposing trend that must be interpreted in conjunction with the significant negative quadratic voltage term (−0.370682B2), which introduces concave curvature and implies the existence of an optimal intermediate voltage level that minimizes HAZ expansion. The interaction term +0.328470BC reveals a synergistic effect wherein the influence of voltage on HAZ width is amplified at higher travel speeds, while the negligible AC interaction (−0.002899) suggests current and weld speed operate largely independently in governing thermal diffusion. Critically, this equation enables direct prediction of HAZ-W for any combination of A, B, and C within the design space (180–210 A, 22–25 V, 2.0–3.5 mm/s), facilitating process optimization and sensitivity analysis; however, as noted in the model documentation, the coefficients are scaled to accommodate the disparate units and ranges of the input factors, and the intercept does not correspond to the geometric center of the design space, thereby precluding direct interpretation of coefficient magnitudes as measures of relative factor importance, for such assessments, the coded-factor model (Montgomery, 2017; Anderson & Whitcomb, 2016).

$$\mathrm{HAZ-W} = -107.52822 - 0.874978A + 17.93102B - 6.61187C - 0.008548AB - 0.002899AC + 0.328470BC + 0.002742A^2 - 0.370682B^2 - 0.164819C^2 \quad (1)$$

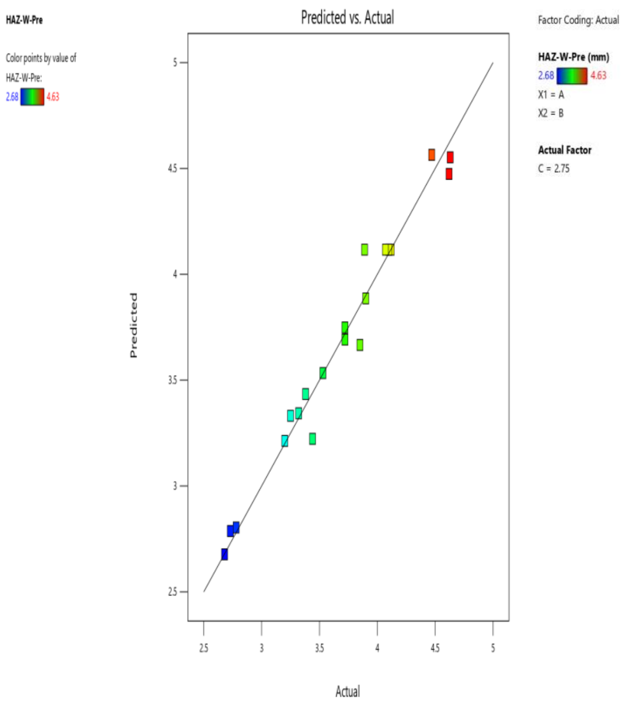
where A is welding current (A), B voltage (V), and C weld speed (mm/s).

Table 6: Predicted verses Actual for Heat Affected Zone Width (HAZ-W)

Run Order	Actual Value	Predicted Value	Residual	Standard Order
1	3.89	4.12	-0.2267	5
2	3.20	3.21	-0.0131	8
3	3.72	3.69	0.0281	1
4	4.10	4.12	-0.0167	11
5	4.07	4.12	-0.0467	17
6	2.73	2.79	-0.0575	15
7	2.78	2.80	-0.0240	20
8	4.12	4.12	0.0033	7
9	3.38	3.43	-0.0547	6
10	3.72	3.75	-0.0289	9
11	4.47	4.56	-0.0944	18
12	4.62	4.47	0.1458	16
13	2.68	2.68	0.0028	14
14	3.44	3.22	0.2173	2
15	3.90	3.89	0.0139	10
16	3.85	3.67	0.1837	19
17	3.32	3.34	-0.0239	4
18	4.63	4.55	0.0773	12
19	3.25	3.33	-0.0818	13
20	3.53	3.53	-0.0038	3

Table 6 shows the result of the predicted against the actual value

Figure 2: presents the Predicted versus Actual plot for the welding Heat Affected Zone width, serving as a primary visual diagnostic for evaluating the predictive fidelity of the quadratic Response Surface Methodology model against experimental measurements. Data points are tightly clustered along the 45° diagonal reference line with minimal systematic deviation, visually corroborating the high coefficient of determination (R^2 = 0.9711) and confirming strong agreement between model predictions and observed values across the entire experimental domain. The color gradient mapping response magnitude (2.68–4.63 mm) demonstrates that prediction accuracy remains consistent regardless of HAZ width severity, indicating that the model successfully captures the underlying nonlinear thermal-metallurgical relationships without exhibiting heteroscedastic error patterns or region-specific bias.



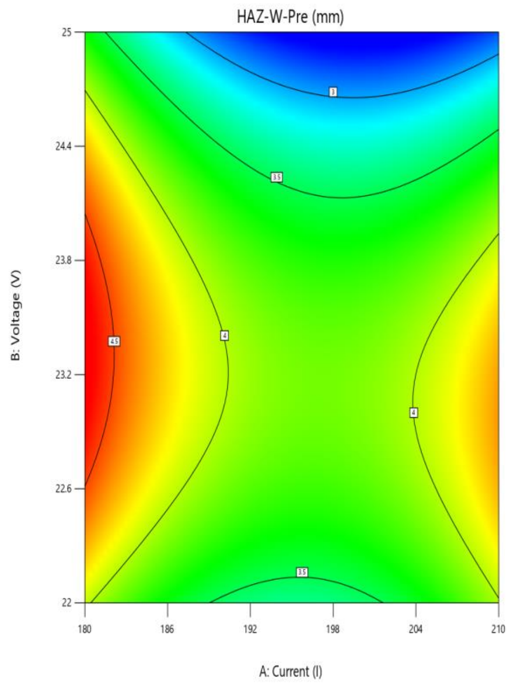


Figure 2: Prediction Vs Actual plot Figure 3: Contour Plot for (HAZ-W)

Figure 3 presents the contour plot for pre-welding HAZ width as a function of welding current (A) and voltage (B) at a constant weld speed of 2.75 mm/s, providing a two-dimensional topographic representation of the response surface. The elliptical and saddle-shaped contour lines, color-coded according to HAZ width magnitude (blue for minimum ~2.68 mm, red for maximum ~4.63 mm), clearly delineate regions of optimal and suboptimal thermal exposure. The pronounced curvature and asymmetry of the contours indicate significant interaction effects between current and voltage, consistent with the statistically significant AB interaction term identified in the ANOVA, while the central green zone represents the parameter combination yielding minimal HAZ expansion. Peripheral red and blue regions highlight boundary conditions where excessive or insufficient arc energy respectively degrade dimensional stability. This graphical representation facilitates rapid visual identification of feasible operating windows and serves as a practical decision-support tool for welding engineers seeking to balance heat input against microstructural constraints in real-time fabrication environments.
The three-dimensional surface plot (Figure 4) provides a comprehensive geometric visualization of the quadratic RSM model for pre-welding HAZ width across the current–voltage design plane at a fixed travel speed of 2.75 mm/s. The undulating saddle-shaped topology distinctly captures the combined linear, interaction, and quadratic effects of the two primary process variables, with the surface minima located near the intermediate current and high voltage regions. The color gradient mapping response magnitude aligns precisely with the contour plot, enabling intuitive interpretation of response gradients, curvature directionality, and ridge/valley formations that characterize the welding process landscape.

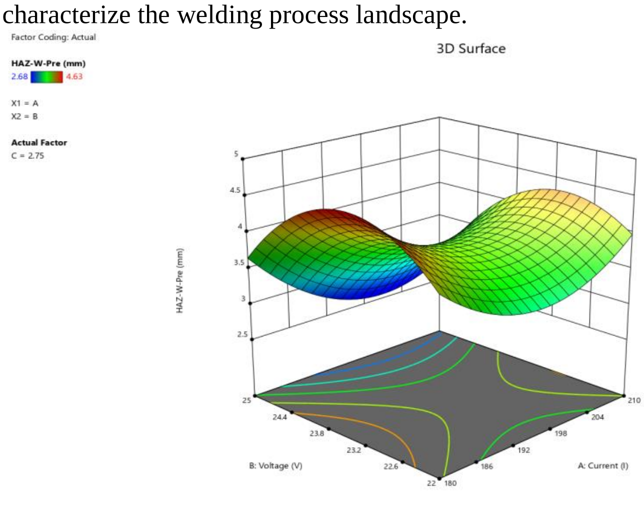


Figure 4: three-dimensional surface plot.

4.0. Conclusion

This study successfully investigated the optimization and prediction of Heat Affected Zone width in mild steel weldments through the systematic implementation of Response Surface Methodology was rigorously applied to model, predict, and optimize heat affected zone width welding responses. The results conclusively demonstrated that welding parameters, particularly current, voltage, and travel speed, exert statistically significant and nonlinear influences on weld quality, with voltage emerging as the dominant factor in the RSM modeling, Experimental validation under optimized conditions confirmed that the identified parameter window (Current ≈194–195 A, Voltage = 25 V, Weld Speed ≈2.7–3.5 mm/s)

5.0 Recommendation

Other methods should be used and the results compared to RSM

Nomenclature

A= Analysis of variance
CCD=Central composite design
GMAW=Gas Metal Arc Welding
HAZ= Heat affected zone
RSM= Response surface methodology

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